

**UNIVERSITY OF GREATER MANCHESTER**  
**NATIONAL CENTRE FOR MOTORSPORT**  
**ENGINEERING**  
**BEng (HONS) AUTOMOTIVE PERFORMANCE**  
**ENGINEERING (MOTORSPORT)**  
**SEMESTER 2 EXAMINATION 2024/2025**  
**ADVANCED VEHICLE SYSTEMS**  
**MODULE NUMBER MSP6011**

Date: Monday 12<sup>th</sup> May 2025

Time: 2:00pm – 4:00pm

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**INSTRUCTIONS TO**  
**CANDIDATES**

This paper has SIX questions  
The marks for each question are shown in  
brackets  
Attempt ALL questions

Electronic calculators may be used  
provided that data and program storage  
memory is cleared prior to the  
examination

Mobile telephones or cellular telephones  
may-not be used as calculators

Formula sheet attached

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**Question 1**

A simplified model of a vehicle's suspension system can be described as:

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

where:

$$m = 400 \text{ kg},$$

$$c = 3200 \text{ Ns/m},$$

$$k = 20000 \text{ N/m}.$$

(a) Derive the transfer function  $G(s) = \frac{X(s)}{F(s)}$

**(5 marks)**

(b) Calculate the natural frequency  $\omega_n$  and the damping ratio  $\zeta$ .

**(5 marks)**

(c) Classify the damping and describe the expected time response of the system.

**(5 marks)**

(d) Sketch the expected shape of the step response, labelling key features.

**(5 marks)**

(e) The damping coefficient is increased to 4800 Ns/m while all other parameters remain the same.

Without performing full recalculations, explain qualitatively how this affects:

- The damping ratio
- The step response characteristics
- Ride comfort in practical terms

**(5 marks)**

**Total 25 marks**

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## Question 2

A DC motor behaves as a first-order system and is subjected to a step input from 0V to 6V. The motor speed increases from 0 rpm to 3000 rpm. After 1 second, the motor reaches approximately 2600 rpm.

(a) Determine the system gain  $K$  and time constant  $\tau$  based on the given data and develop the transfer function for the motor.

**(6 marks)**

(c) Sketch the expected shape of the speed response, labelling key features such as steady-state value and approximate time to reach steady state.

**(6 marks)**

**Total 12 marks**

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### Question 3

A cruise control system uses a PID controller described by:

$$C(s) = K_p + \frac{K_i}{s} + K_d \cdot s$$

The plant is a first-order system:

$$G(s) = \frac{1}{\tau s + 1}$$

(a) Briefly explain the roles of the proportional (P), integral (I), and derivative (D) terms in the controller.

**(6 marks)**

(b) Discuss the effect of increasing each gain ( $K_p$ ,  $K_i$ ,  $K_d$ ) on system performance.

**(6 marks)**

(c) For  $K_p = 2$ ,  $K_i = 5$ ,  $K_d = 0$  and  $\tau = 1$ , determine the closed-loop transfer function assuming unity feedback.

**(3 marks)**

(d) Based on your result in (c), classify the damping of the system and explain whether it will overshoot or not. Justify your reasoning.

**(3 marks)**

**Total 18 marks**

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**Question 4**

A system has the following transfer function:

$$G(s) = \frac{s + 3}{s^2 + 4s + 8}$$

(a) Identify the locations of the poles and zero and plot them on the complex plane.

**(4 marks)**

(b) Describe how the poles and zero influence the system's step response.

**(3 marks)**

(c) Explain the expected effect on system response if the zero were instead located in the right-half plane (e.g.,  $s = +3$ ).

**(3 marks)**

(d) The zero is now moved from  $s = -3$  to  $s = -0.2$ , much closer to the origin. Without recalculating the response, explain qualitatively how this change would affect:

- Rise time
- Overshoot
- Controller design complexity

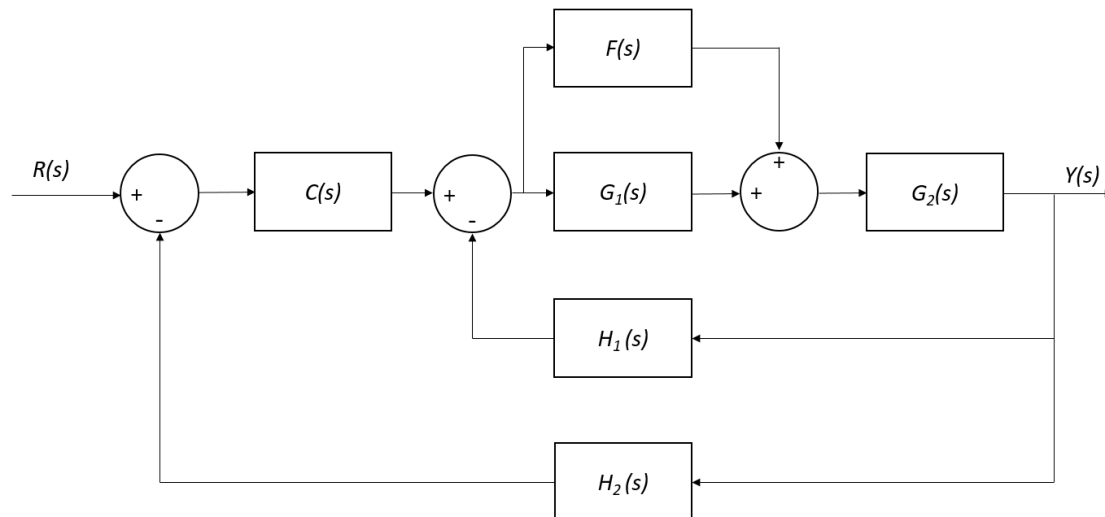
**(5 marks)**

**Total 15 marks**

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### Question 5

Figure Q5 shows a block diagram representation of a linear automotive system with input  $R(s)$  and output  $Y(s)$ .



Using block diagram reduction techniques, find the transfer function of the system,  $\frac{Y(s)}{R(s)}$ .

(18 marks)

Total 18 marks

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### Question 6

A quarter-car model consists of:

A sprung mass  $m_b$

An unsprung mass  $m_w$

Suspension stiffness  $k_s$

Damping  $b$

Tyre stiffness  $k_t$

(a) Derive the coupled differential equations of motion for the system by applying Newton's Second Law to each mass.

**(12 marks)**

**Total 12 marks**

**END OF QUESTIONS**

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### **FORMULA SHEET**

1st Order Systems:

$$\tau \dot{y}(t) + y(t) = ku(t)$$

$$y(t) = k(1 - e^{-t/\tau}), t \geq 0$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{\tau s + 1}$$

2<sup>nd</sup> Order Systems:

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 u(t)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$t_s \approx \frac{4}{\zeta\omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$M_p = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$$

**END OF PAPER**