

UNIVERSITY OF GREATER MANCHESTER

SCHOOL OF ENGINEERING

**B.ENG (HONS) AUTOMOTIVE PERFORMANCE
ENGINEERING (MOTORSPORT)**

SEMESTER TWO EXAMINATION 2024/2025

ENGINEERING SCIENCE 2

MODULE NO: MSP5024

Date: Thursday 15 May 2025

Time: 14:00 – 16:00

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer FOUR questions in total.

Answer ONE question from SECTION A and THREE questions from SECTION B

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached after questions).

SECTION A – Answer One Question from this Section

- Q1)** Figure 1a and b show a simplified model of a truck chassis. Part of the structure can be represented by a simply supported beam. It is assumed that the engine weight acts as a point load towards the front of the beam and the fuel and tank act as a point load towards the rear of the beam. the vehicle's body weight can be assumed to act as a uniformly distributed load across the entire span. The frame is made from high strength steel with $E = 200 \text{ GPa}$.

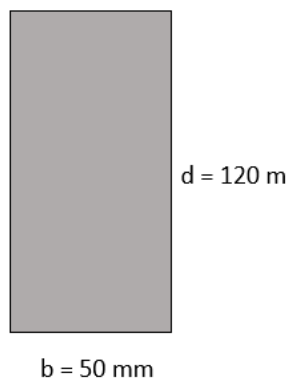


Figure 1a- Cross section

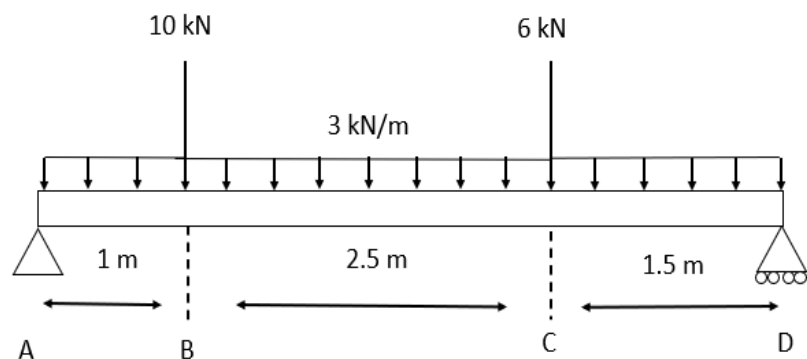


Figure 1b- Beam loading

- Find the support reactions at A and D.
(4 marks)
- Calculate and plot the bending moment distribution across the beam. Using calculus, show that the maximum bending moment is 18.9 kNm and find the position along the beam where this bending moment exists.
(15 marks)
- Using the maximum moment from part b) find the maximum bending stress in the beam.
(6 marks)
- Using the bending moment equation generated in part b) Use Macauley's method to find the deflection at the centre of the beam.
(15 marks)

(Total marks 40)

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- Q2)** Figure 2 shows the cross section of a design for a right-angle mounting plate. The plate will act as a beam and will exhibit unsymmetrical bending when a load is applied.

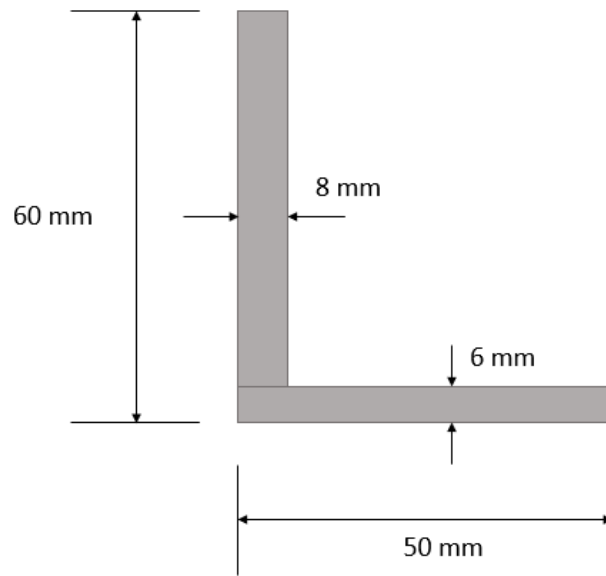


Figure 2- cross section of L angle section

- a) Find the position of the centroid for the section in Figure 2. **(8 marks)**
- b) Use the parallel axis theorem to find the inertia about the centroidal axis in the x and y direction and find the product inertia (I_{xx} , I_{yy} and I_{xy}) **(18 marks)**
- c) Find the angle of inclination of the principal axis to the x, y plane. **(6 marks)**
- d) Find the maximum and minimum inertias about the principal axis (I_u , I_v). **(8 marks)**

(Total marks 40)

END OF SECTION A

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SECTION B – Please Answer 3 Questions from this Section

Q3) Consider a **spherical** pressure vessel that carries a pressurised gas. The vessel has a radius of 1500 mm and thickness of 12 mm. It is constructed from a titanium alloy with a yield strength of 600 MPa.

a) Show that the vessel above can be considered to have a thin wall
(2 marks)

b) If the factor of safety of the vessel should not exceed 2, determine the maximum internal pressure that can be applied.
(4 marks)

A new **cylindrical** vessel is to be made from steel and has an internal diameter of 400 mm and an outer diameter of 500 mm. The external pressure is assumed to be 30 MPa. The internal pressure is 90 MPa. Consider this vessel thick walled.

Take $E = 200 \text{ GPa}$. And Poisson's ratio, $\nu = 0.3$ for the steel.

c) Calculate the hoop stress and radial stress at both the inner and outer radius.
(10 marks)

d) Calculate the longitudinal stress in the cylindrical vessel.
(4 marks)

(Total marks 20)

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Q4) Consider the cross section of the strut in figure 3.

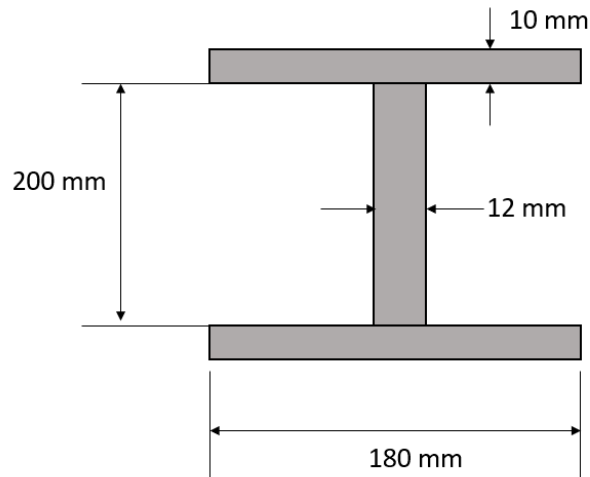


Figure 3-Strut

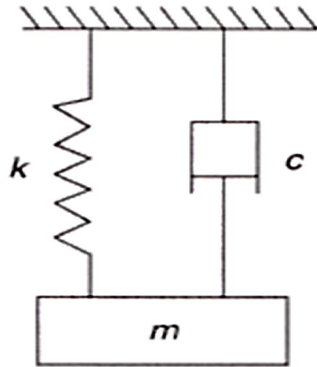
A solid, symmetrical column cast iron is placed vertically and has both ends fully fixed; the column is 5 m in height. Take modulus of elasticity, $E = 70 \text{ GPa}$ and material constant $c = 1/1600$, elastic limit stress as $\sigma_c = 550 \text{ MPa}$.

- Find the second moment of area in both the x and y direction for the strut.
(6 marks)
- Use the result in part (a) to find the least radius of gyration, k .
(3 marks)
- Determine the Euler crippling load, P_{CR} , and Euler crippling stress, σ_{CR} , of the cylindrical column.
(6 marks)
- Calculate the critical Load, P_R , using Rankine-Gordon's formula and compare this to the Euler crippling Load found in part c) suggest which value is most accurate in this instance.
(5 marks)

(Total marks 20)

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Q5)



The diagram in figure 4 is a simplified suspension system.

The spring constant $k = 35 \text{ kN/m}$ and the damping value $c = 4000 \text{ Ns/m}$.
The mass is $m = 1500 \text{ kg}$

Figure 4-Spring, Mass and Damper System

- a) Find the damping ratio for the system in figure 4. State whether the system is underdamped, critically damped or overdamped. **(4 marks)**
- b) Sketch a typical response (displacement vs time) for the suspension system when an initial disturbing force is applied. **(2 marks)**
- c) After an initial disturbance the system vibrates freely with the first amplitude of vibration as 0.2 m . Calculate the amplitude of the next vibration. **(5 marks)**
- d) Find the damped frequency in Hertz. **(3 marks)**

The mass, spring and damper system is now subjected to a forced vibration, which consists of a force with magnitude 8 kN and a forcing frequency of 1.5 Hz .

- e) Calculate the amplitude of the resulting vibration and the phase angle that exists between the forcing frequency and that of the resulting system. **(6 marks)**

(Total marks 20)

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Q6) A pressure vessel is subjected to a complex stress system. The stress in the x direction is 120 MPa in tension, 50 MPa compressive in the y direction, and there is a shear stress of 35 MPa.

Poisson's ratio is $\nu = 0.3$ and the modulus of elasticity is $E = 200 \text{ GPa}$ for the material.

a) Find the inclination of the principal plain, with respect to the x axis, for which maximum normal stress occurs and the inclination of the principal plain for which shear stress occurs. **(4marks)**

b) Hence calculate the principal normal stresses and maximum shear stresses. **(4marks)**

A 45 degrees rosette strain gauge is now connected to the surface of the vessel as shown in Figure 5. The pressure vessel undergoes new loading and the readings on gauges a, b and c are: 500 μ strain, -200 μ strain and 450 μ strain respectively.

c) Determine the orientation of the plain of principal strain, with respect to the x axis, and then find the principal strains (ϵ_1, ϵ_2). **(8marks)**

d) Find the new principal stresses. **(4marks)**

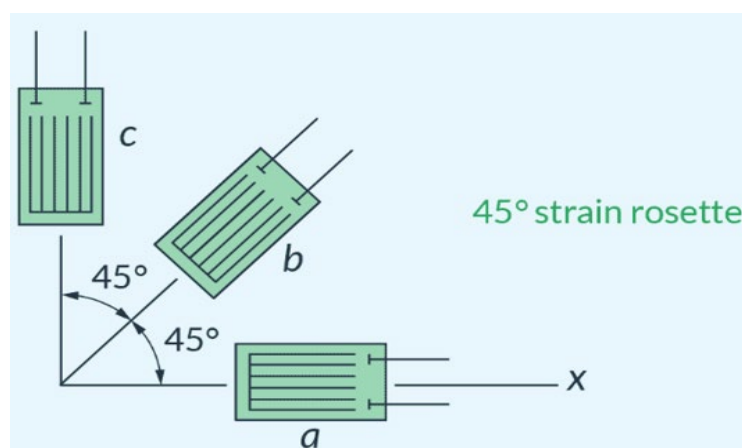


Figure 5- 45-degree strain rosette

(Total marks 20)

END OF QUESTIONS

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FORMULA SHEET

Bending And Deflection 2nd moment of area for basic shapes:

Solid circle: $I_x = I_y = \frac{\pi d^4}{64}$

Hollow circle: $I_x = I_y = \frac{\pi}{64} (D_{outer}^4 - D_{inner}^4)$

Rectangle: $I_x = \frac{bd^3}{12}$ $I_y = \frac{db^3}{12}$

Bending equation: $\sigma = \frac{My}{I} = \frac{E}{\rho}$

Beam deflection (v): $\frac{d^2v}{dx^2} = \frac{M(x)}{EI}$

Method of moments for centroid: $\bar{x} = \frac{\sum a_i x_i}{\sum a_i}$ $\bar{y} = \frac{\sum a_i y_i}{\sum a_i}$

Parallel axis theorem: $I_{xx} = I_{\bar{x}\bar{x}} + Ay^2$ $I_{yy} = I_{\bar{y}\bar{y}} + Ax^2$

Product Inertia: $I_{xy} = \sum I_{\bar{x}\bar{y}} + \sum A_{xy}$

For principal Inertias: $I_{u,v} = \frac{1}{2} (I_{xx} + I_{yy}) \pm \frac{1}{2} (I_{xx} - I_{yy}) \sec 2\theta$

$$I_u + I_v = I_{xx} + I_{yy}$$

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

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Pressure Vessels

Thin-Walled Pressure Vessels (sphere): $\sigma_T = \frac{Pr}{2t}$

Thin-Walled Pressure Vessels (cylinder): $\sigma_h = \frac{Pr}{t}$ $\sigma_l = \frac{Pr}{2t}$

For Thick-Walled vessels stresses and strains:

$$\sigma_h = \frac{P_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2} \quad \text{or} \quad \sigma_h = A + \frac{B}{r^2}$$

$$\sigma_r = \frac{P_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2} \quad \text{or} \quad \sigma_r = A - \frac{B}{r^2}$$

$$\sigma_l = \frac{P_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} \quad \text{or} \quad \sigma_l = A$$

Where;

$$A = \frac{P_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} \quad \text{and} \quad B = \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2)}$$

$$\varepsilon_L = \frac{1}{E} [\sigma_L - \nu(\sigma_R + \sigma_h)]$$

$$\varepsilon_h = \frac{1}{E} [\sigma_h - \nu(\sigma_L + \sigma_r)]$$

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$$\varepsilon_r = \frac{1}{E} [\sigma_r - \nu(\sigma_h + \sigma_L)]$$

Vibrations

$$f = \frac{1}{T}$$

$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}}$$

$$c = \zeta 2m\omega_n \quad c_c = 2m\omega_n$$

$$\zeta = \frac{c}{c_c}$$

$$f_{damp} = f_n \sqrt{1 - \zeta^2}$$

$$\text{Amplitude reduction factor } \Delta = \ln \frac{x_1}{x_r} = \frac{2\pi(r-1)\zeta}{\sqrt{1-\zeta^2}}$$

where; $r = \text{position of maxima}$

Forced Vibration

$$X_0 = \frac{\frac{F}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

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$$\varphi = \tan^{-1} \left(\frac{2\zeta \left(\frac{\omega}{\omega_n} \right)}{1 - \left(\frac{\omega}{\omega_n} \right)^2} \right)$$

Struts

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

$$\text{Slenderness ratio}(SR) = \frac{L_e}{k}$$

Support Condition	Equivalent length l_e
Both ends pinned	$l_e = l$
One end fixed and one end free	$l_e = 2l$
One end fixed and one end pinned	$l_e = l/\sqrt{2}$
Both ends fixed	$l_e = l/2$

$$\text{Critical Euler stress} = \frac{\pi^2 E}{\left(\frac{l_e}{k} \right)^2}$$

$$\text{Critical Euler Load} = \frac{\pi^2 EI}{l_e^2}$$

$$\text{Critical Rankine Stress} = \frac{\sigma}{1 + c \left(\frac{l_e}{k} \right)^2}$$

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$$\text{Critical Rankine Load} = \frac{\sigma A}{1 + c \left(\frac{l_e}{k} \right)^2}$$

Complex Stresses and Transformations

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos(2\theta) - \tau \sin(2\theta)$$

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin(2\theta) + \tau \cos(2\theta)$$

$$\tan(2\theta_p) = \frac{-2\tau}{\sigma_x - \sigma_y}$$

$$\tan(2\theta_s) = \frac{\sigma_x - \sigma_y}{2\tau}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

$$\tau_{max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau^2}$$

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Strain Rosette 45-degree

$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \varepsilon_c$$

$$\gamma_{xy} = 2\varepsilon_b - (\varepsilon_a + \varepsilon_c)$$

Complex Strains and Transformations

$$\tan(2\theta_p) = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan(2\theta_s) = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}$$

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\sigma_1 = \frac{E}{1 - \nu^2} (\varepsilon_1 + \nu\varepsilon_2)$$

$$\sigma_2 = \frac{E}{1 - \nu^2} (\varepsilon_2 + \nu\varepsilon_1)$$

END OF EXAM