UNIVERSITY OF GREATER MANCHESTER

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE

BENG (HONS) MECHANICAL ENGINEERING

SEMESTER TWO EXAMINATION 2024/2025

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

MODULE NO: AME6016

Date: Tuesday, 20 May 2025 **Time: 10:00 am – 12:00 pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE (5) questions on the paper.

Answer ANY FOUR (4) questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is erased or cleared prior to the examination.

CANDIDATES REQUIRE: **A Formula Sheet (attached)**

University of Greater Manchester
Off Campus Division - Western International College
BEng (Hons) Mechanical Engineering
Semester Two Examination 2024/2025
Finite Element and Difference Solutions
Module No: AME6016

QUESTION 1

a) **Figure Q1** illustrates a spring system consisting of linear elastic springs arranged in a combination of series and parallel configurations. The system includes five springs with the following stiffness values:

- k_1 = 500N/mm (between nodes 1 and 2)
- $k_2 = k_3$ = 300N/mm (two parallel springs between nodes 2 and 3)
- $k_4 = k_5$ = 400N/mm (two parallel springs between nodes 3 and 4)

Nodes 1 and 4 are fixed and an external force of 1 kN is applied at node 3 in the positive x-direction. Nodes 2 and 3 are free to displace along the x-axis. Using the Direct Stiffness Method and the **Elimination Technique**:

i. Construct the connectivity table and develop the global stiffness matrix for the system. **(10 marks)**

ii. Determine the displacements at nodes 2 and 3. **(10 marks)**

Assume all displacements occur in the horizontal direction only, and all springs behave linearly.

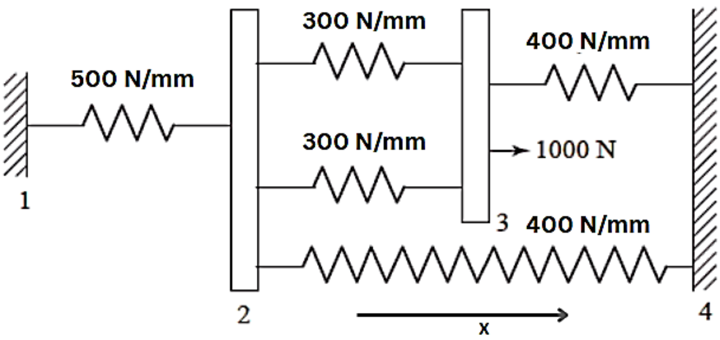

Figure Q1: Spring assemblages

b) Critically evaluate the applicability of the Finite Element Method (FEM) in engineering analysis. In your discussion, justify its advantages and acknowledge its limitations when applied to real-world mechanical or structural problems.

(5 marks)

[TOTAL 25 MARKS]

University of Greater Manchester
Off Campus Division - Western International College
BEng (Hons) Mechanical Engineering
Semester Two Examination 2024/2025
Finite Element and Difference Solutions
Module No: AME6016

QUESTION 2

a) The **Figure Q2** illustrates a simply supported beam of total length 10 m, subjected to a central point load of 10,000 N. The beam is discretised into two equal-length Euler-Bernoulli beam elements, each of length 5 m. The left end is fixed, and the right end is a roller support.

The beam has the following properties:

- Young's Modulus (E) = 200 GPa
- Second Moment of Area (I) = 4×10^{-6} m^4

Using the Finite Element Method (FEM) and the **Elimination Method** for applying boundary conditions:

i. Draw the finite element model under the given load condition. **(5 marks)**

ii. Determine the Global Stiffness matrix. **(7 marks)**

iii. Determine deflection under the given load. **(8 marks)**

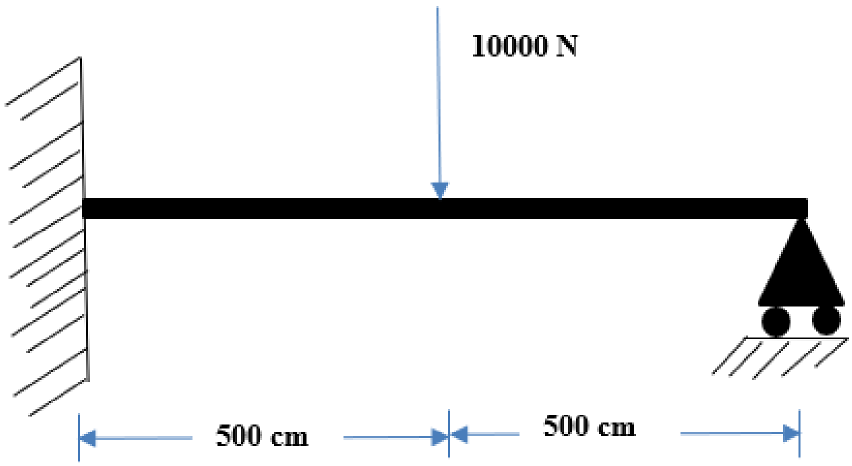

Figure Q2: Beam Element

b) Compare and contrast the Penalty Method and Elimination Method used to apply boundary conditions in FEM.

(5 marks)

[TOTAL 25 MARKS]

Please turn the page

QUESTION 3

The **Figure Q3** below represents a compound axial bar composed of three connected segments, each differing in cross-sectional area and material properties. Axial loads of 20 kN and 10 kN are applied at intermediate nodes, as shown.

Segment	Length (mm)	Area (mm²)	Young's Modulus (GPa)
1	80	900	70
2	90	400	105
3	70	200	200

Both ends of the bar are fixed, resulting in zero displacement at the boundaries.

Using the Penalty Method to implement boundary conditions:

a) Develop the individual stiffness matrix for each individual bar segment. **(5 marks)**

b) Develop the global stiffness matrix using the method of superposition. **(5 marks)**

c) Determine the displacement at the internal nodes. **(5 marks)**

d) Determine the stress in the thickest section. **(5 marks)**

e) Determine the reaction at the support **(5 marks)**

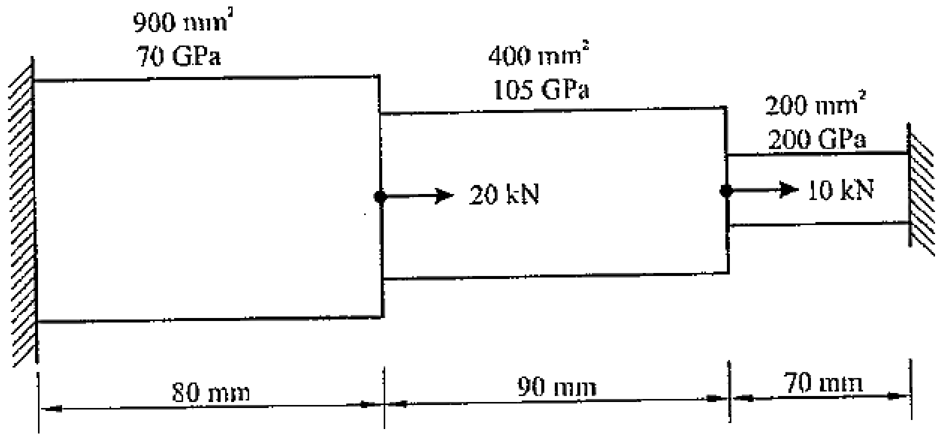

Figure Q3: Assembly of bar segments

[TOTAL 25 MARKS]

QUESTION 4

An induction furnace wall is made up of three layers which include inside, middle and outer layer with thermal conductivity K_1= 8.5 W/m-K, K_2= 0.25 W/m-K, K_3= 0.088 W/m-K, convective heat transfer coefficient, h=45 W/m^2-K, outside temperature T_∞=30^{0}C, as shown in **Figure Q4** below, where T_1 is the internal temperature of the furnace, T_2, T_3 are the intermediate temperature of the furnace walls and T_4 is the temperature at the last node in Celsius.

Determine the following.

a) Elemental stiffness matrix for the heat conduction. **(9 marks)**

b) Global stiffness matrix for the system. **(6 marks)**

c) Nodal Temperatures. **(10 marks)**

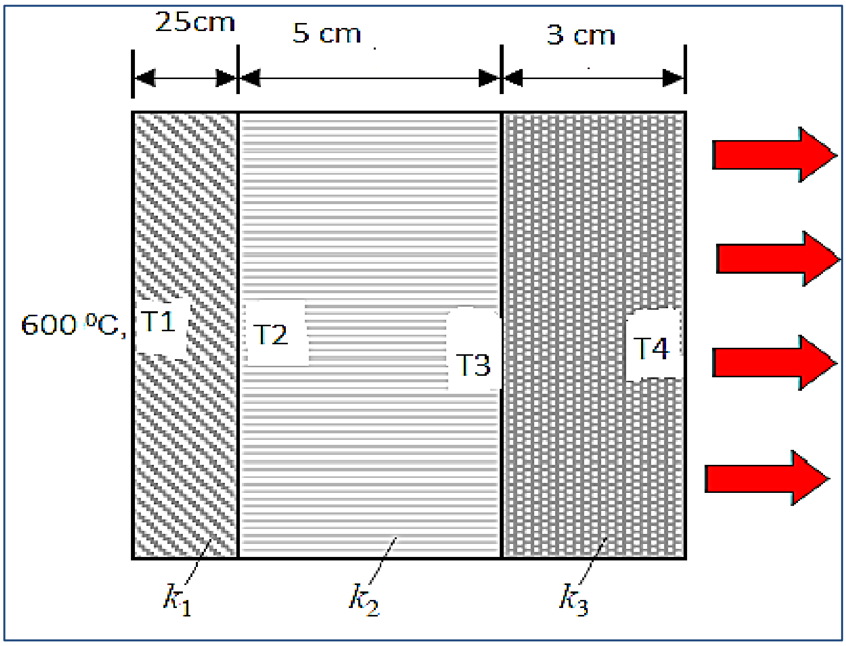

Figure Q4: Induction furnace with three layers

[TOTAL 25 MARKS]

Please turn the page

QUESTION 5

The **Figure Q5** below shows a simply supported beam of total length 8 metres, subjected to a linearly varying distributed load. The intensity of the load increases from 12 kN/m at the fixed support to 24 kN/m at the roller support. A hinge is located at the mid-span of the beam, dividing it into two equal elements.

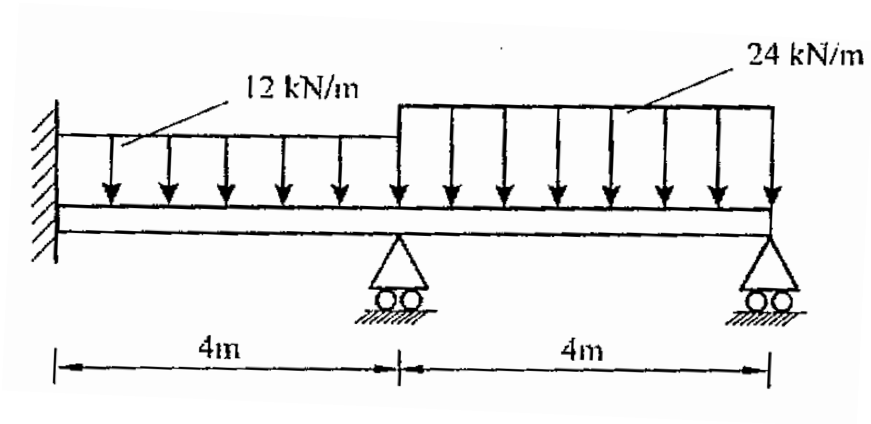

Figure Q5: Simply supported beam

Material and geometric properties:

- Young's Modulus: E=200 GPa
- Moment of Inertia: I=4×10^6 mm^4

Use the Finite Element Method (FEM) with appropriate boundary conditions and Elimination Method for solving the system.

a) Develop the individual stiffness matrix for each beam element. **(5 marks)**

b) Assemble the global stiffness matrix using the superposition principle **(5 marks)**

c) Using the Elimination Method, solve the system to determine the maximum deflection at the mid-span of the beam. **(8 marks)**

d) Using the displacement results, compute the end reaction forces at the left and right supports of the beam. **(7 marks)**

[TOTAL 25 MARKS]

END OF QUESTIONS
PLEASE TURN THE PAGE FOR FORMULA SHEET
FORMULA SHEET

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

1. GENERAL FEM FORMULATION:

- **Assembly of Global Stiffness Matrix: Superposition of individual element matrices based on DOF mapping.**

- **Displacement Solution:**

$$[K]\{u\} = \{F\} \Rightarrow \{u\} = [K]^{-1}\{F\}$$

- **Reaction Forces:**

$$\{R\} = [K]\{u\} - \{F\}$$

2. SPRING SYSTEM:

$$[k] = k[1 \ -1 \ -1 \ 1]$$

Force-Displacement Relation:

$$[K]\{u\} = \{F\}$$

3. BAR/TRUSS ELEMENT:

- **Elemental Stiffness Matrix:**

$$[K_C] = \frac{AE}{L}[1 \ -1 \ -1 \ 1]$$

- **Stress in Bar Element:**

$$\sigma = \frac{E}{L}(u_2 - u_1)$$

- **Force in Bar Element:**

$$F = \sigma A = \frac{AE}{L}(u_2 - u_1)$$

Please turn the page

- **Reaction at the support**

$R_n = -C[q_n - a_n]$

University of Greater Manchester
Off Campus Division - Western International College
BEng (Hons) Mechanical Engineering
Semester Two Examination 2024/2025
Finite Element and Difference Solutions
Module No: AME6016

4. STIFFNESS MATRIX FOR BEAMS:

$$[k_e] = \frac{EI}{L^3}[12\ 6L\ -12\ 6L\ 6L\ 4L^2\ -6L\ 2L^2\ -12\ -6L\ 12\ -6L\ 6L\ 2L^2\ -6L\ 4L^2\ ]$$

- **Equivalent Nodal Load Vector (for Uniform Load www)**:

$$\{F\} = \left[\frac{wL}{2}\ \frac{wL^2}{12}\ \frac{wL}{2}\ -\frac{wL^2}{12}\right]$$

- **Bending Stress**:

$$\sigma = \frac{My}{I}$$

- **Moment from FEM result**:

$$M(x) = EI \cdot \frac{d^2v}{dx^2}$$

5. HEAT TRANSFER:

- **STIFFNESS MATRIX DUE TO CONVECTION.**

$$[K_h] = hA\ [0\ 0\ 0\ 1\ ]$$

- **Force vector due to free end convection**

$$[F_h] = hA\ T_\infty\{\ 0\ \ 1\ \ \}$$

- **Stiffness matrix for heat conduction.**

$$[K_C] = \frac{AK}{L}\ [1\ -1\ -1\ 1\ ]$$

Please turn the page

- **Element conduction matrix.**

$$[k] = \frac{AK_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

- **Elemental Force Matrix.**

$$\{f\} = \frac{QAL + q^*PL + hT_\infty PL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

- **Elemental stress.**

$$\underline{\sigma} = \underline{C}'\underline{d}$$

$$\underline{C}' = \frac{E}{L}[-C \quad -S \quad C \quad S]$$

SHAPE FUNCTIONS IN MATRIX FORM:

$$[H_1(\xi)\ H_2(\xi)\ H_3(\xi)\ H_4(\xi)\ ] = \frac{1}{4}[2\ -3\ 0\ 1\ 1\ -1\ -1\ 1\ 2\ \ 3\ 0\ -1\ -1\ -1\ 1\ 1\ ][1\ \xi\ \xi^2\ \xi^3\ ].$$

$$[H]\ =\ \left[H_1(\xi),\ \left(\frac{L}{2}\right) H_2(\xi),\ \ H_3(\xi),\ \left(\frac{L}{2}\right) H_4(\xi)\right].$$

END OF PAPER