UNIVERSITY OF GREATER MANCHESTER

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE

BENG (HONS) MECHANICAL ENGINEERING

SEMESTER TWO RESIT EXAMINATION 2024/2025

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME5012

Date: Thursday, 22 May 2025 **Time: 10:00 am – 12:00 pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE (5) questions on this paper.

Answer ANY FOUR (4) questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleaned prior to the examination.

CANDIDATES REQUIRE:

Formula Sheet (attached)
Graph Paper

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Q1. For the simply supported overhanging beam AC of length 15m which is supported at A and B, shown in **Figure 1**, use Macaulay's method to determine:

a) the slope and deflection equations for the beam **(16 marks)**

b) the slope at A and B **(6 marks)**

c) the deflection at D **(3 marks)**

Take Flexural rigidity, E= 2 x 10^5 N/mm^2 ; I = 10^8mm^4

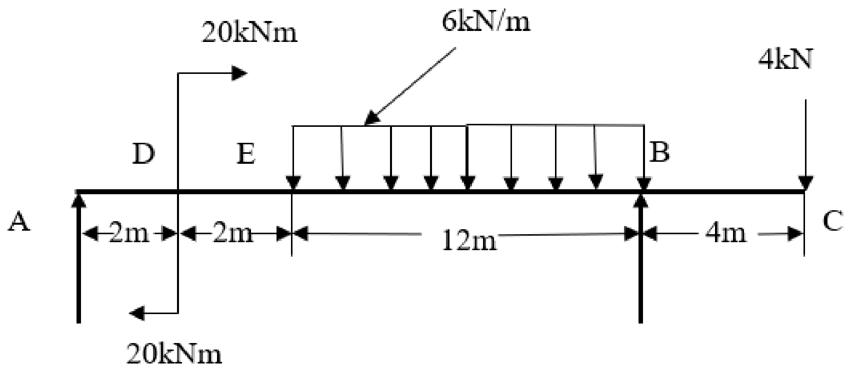

Figure 1: Simply Supported Overhanging Beam

[TOTAL 25 MARKS]

Q2. A machine component of mass m = 20 kg is connected to a spring with stiffness k = 45 kN/m and damping ratio ξ = 0.25. It experiences a harmonic excitation force of F(t) = 60 cos (25t) N.

Determine:

a) Analyse the system response in terms of steady-state amplitude and phase lag of the vibrations. Summarise how damping and frequency of excitation influence the observed amplitude and phase shift in this scenario **(12 marks)**

b) Determine the steady-state amplitude when the frequency of excitation matches the system's natural frequency $\boldsymbol{\omega = \omega_n}$.

(4 marks)

c) Evaluate the optimal frequency of the applied force that results in the maximum amplitude of vibration. Calculate this peak amplitude value and assess its significance in practical applications.

(9 marks)

[TOTAL 25 MARKS]

Q3. A ductile material is used to fabricate a mechanical part. The direct stresses in x and y direction are respectively 150 MPa in tension and 60 MPa in tension. There are also shear stresses present related to xy with a value of 30 MPa.

a) Sketch the elementary square describing the situation. **(2 marks)**

b) Determine via calculation:

(i) The magnitude of the principal stresses. **(4 marks)**

(ii) The angular position of the principal planes in relation to the X-axis **(3 marks)**

(iii) The magnitude of the maximum shear stress. **(3 marks)**

c) Sketch a Mohr's Stress Circle from the information provided in Q3, labelling σ_1, σ_2 the principal stresses and the maximum shear stress τ_{max}. **(8 marks)**

d) Illustrate on a sketch of the element:

(i) The orientation of the principal planes. **(2 marks)**

(ii) The orientation of the plane where the shear stress is maximum. **(3 marks)**

[TOTAL 25 MARKS]

Q4. A thick-walled cylindrical pressure vessel is subjected to internal pressure. The vessel has an internal radius of 10 cm and an external diameter of 36 cm. The cylinder is closed at both ends and is subjected to an internal pressure of 50 MPa. The original length of the vessel is 7 m. Material properties are:

Modulus of Elasticity, E=210 Gpa, Poisson's Ratio, v=0.25

Determine:

a) The circumferential (hoop) stress at both the inner and outer surfaces of the vessel using Lame's theory. Explain how hoop stress and radial stress vary across the cylinder wall. **(8 marks)**

b) Calculate the longitudinal stress acting uniformly through the wall section and explain what factors affect this stress in a closed cylinder. **(3 marks)**

c) Evaluate the effect of internal pressure on the vessel's dimensions by calculating the increase in internal diameter and length due to deformation. **(6 marks)**

d) Sketch the distribution of radial, hoop, and longitudinal stresses across the wall thickness. Clearly label each curve and indicate the inner and outer surfaces. **(8 marks)**

[TOTAL 25 MARKS]

Q5. A straight aluminium alloy bar with a thickness of 8 mm and width of 20 mm is axially loaded until buckling occurs. The aluminium alloy bar is fixed at both ends. Given: The yield stress is 250 MN/m².

a) Calculate the slenderness ratio of the bar when E=69 GN/m^2 and the Euler buckling load is 180 N. **(6 mark)**

b) Calculate the length of the bar. **(4 mark)**

c) Find the maximum central deflection. **(7 mark)**

d) Using Rankine-Gordon Strut theory, determine the maximum load the aluminium alloy bar can support without buckling. **(8 mark)**

[TOTAL 25 MARKS]

END OF QUESTIONS
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FORMULA SHEET

Deflection

$$EI \frac{d^2y}{dx^2} = M$$

Complex Stress

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau \sin 2\theta$$

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta - \tau \cos 2\theta$$

$$\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y}$$

Complex Strain

$$\text{Radius of stress circle} = \frac{(1-\nu)}{(1+\nu)} \text{ x Radius of strain circle}$$

$$\text{Stress circle} = \frac{E}{(1-\nu)} \text{ x strain scale}$$

$$\sigma_1 = \frac{E(\varepsilon_1 + \nu\varepsilon_2)}{1-\nu^2} \qquad \sigma_2 = \frac{E(\varepsilon_2 + \nu\varepsilon_1)}{1-\nu^2}$$

Thick Cylinder

Lame′ Equations

$$\sigma_c = A + \frac{B}{r^2}, \ \sigma_R = A - \frac{B}{r^2}$$

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Strain Format

$$\varepsilon_x = +\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

Strain along any angle

$$\varepsilon_\theta = \varepsilon_x sin2\theta + \varepsilon_y cos2\theta + \gamma_{xy} sin\,\theta cos\,\theta$$

Vibrations

$$f_n = \frac{\varpi_n}{2\pi} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

Damped $$f_d = \frac{\omega_d}{2\pi} \qquad \omega_d = \omega_n\sqrt{1-\xi^2}$$

Log Decrement $$\ell_n \frac{x_1}{x_r} = \frac{2\pi(r-1)\xi}{\sqrt{1-\xi^2}}$$

Critical Damping $$C_c = 2m\,\omega_n \qquad \xi = \frac{C}{C_c}$$

Forced $$X_0 = \frac{F/K}{\sqrt{(2\xi r)^2 + (1-r^2)^2}} \qquad \phi = \tan^{-1}\frac{2\xi r}{1-r^2}, \quad r = \frac{\omega}{\omega_n}$$

$$r_{res} = \sqrt{1-2\xi^2}, \qquad r_{res} = \frac{\omega_{res}}{\omega_n}$$

Transmissibility $$F_T = \sqrt{(kX_0)^2 + (c\omega X_0)^2}$$

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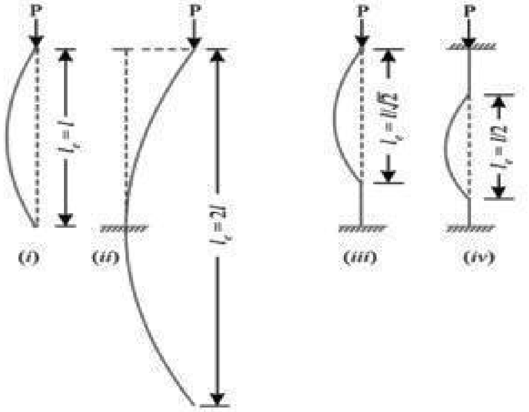
Euler validity

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi\sqrt{\frac{E}{\sigma_{yield}}}$$



(i) *Both ends pin jointed or hinged or rounded or free.*
(ii) *One end fixed and other end free.*
(iii) *One end fixed and the other pin jointed.*
(iv) *Both ends fixed.*

Case	*End conditions*	*Equivalent length, l_e*	*Buckling load, Euler*
1	*Both ends hinged or pin jointed* or *rounded or free*	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	*One end fixed, other end free*	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	*One end fixed, other end pin jointed*	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	*Both ends fixed or encastered*	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ **has thus been introduced to** ***take into account the buckling effect.***

END OF PAPER