UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BSc (HONS) MOTORSPORT TECHNOLOGY ENGINEERING MATHEMATICS 1

SEMESTER 1 EXAMINATION 2024/2025

MODULE NO: MSP4022

Date: Wednesday8th January 2025 Time: 2:00pm – 4:00pm

<u>INSTRUCTIONS TO CANDIDATES:</u> There are <u>EIGHT</u> questions

Answer **ANY FIVE** questions.

All questions carry equal marks.

Marks for parts of questions are shown

in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the

examination.

<u>CANDIDATES REQUIRE:</u> Formula Sheet (provided).

Question 1.

Solve the following quadratic equations by factorisation:

(a)
$$x^2 + 7x - 30 = 0$$
 (6 marks)

(b)
$$6x^2 - 7x - 3 = 0$$
 (6 marks)

Solve the following quadratic equation by using the formula:

(c)
$$2x^2 + 6x + 3 = 0$$
 (8 marks)

Total 20 marks

Question 2

(a) Solve the following systems of simultaneous linear equations:

(i)
$$4x + 3y = 13$$

$$3x+2y=11$$

(7 marks)

(ii)
$$p + q + r = 6 2p - q + 2r = 6 3p - 3q + 2r = 3$$

(13 marks)

Total 20 marks

Question 3

1. Simplify the following

i.
$$\sqrt[3]{y^2} \div \sqrt{y}$$
 (5 marks)

ii.
$$K^{\frac{1}{3}} \times 4K^{\frac{1}{2}}$$
 (5 marks)

iii.
$$\log_3(3x - 1) = 3 \tag{5 marks}$$

iv.
$$2\log x = 3\log 4$$
 (5 marks)

Total 20 marks

Question 4

- (a) The first four terms of an arithmetic sequence are 4, 9, 14, 19.
 - i. Write down a formula for the nth term.
 - ii. Find the sum of the first thirty terms of the sequence.

(8 marks)

(b) A geometric sequence is given by the formula

$$X_n = 8 \times \left(\frac{2}{3}\right)^n$$
 for $n = 0,1,2,3,...$

- i. Simplify and write down the first four terms of the sequence.
- ii. Calculate the sum of **all** of the terms.

(12 marks)

Total 20 marks

Question 5

(a) Using Pascal's triangle, expand and simplify

i.
$$(a+b)^4$$
 (6 marks)

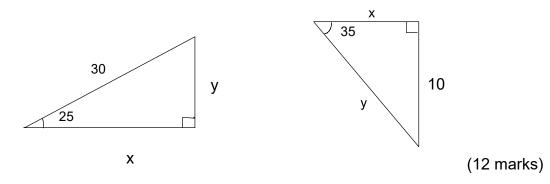
ii.
$$(2a - 3b)^4$$
 (10 marks)

(b) Simplify
$$\left(\frac{64}{125}\right)^{\frac{1}{3}}$$
 (4 marks)

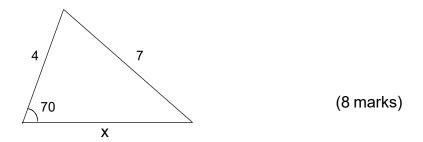
Total 20 marks

Question 6

(a) For each of the right-angled triangles below, use trigonometry to calculate the lengths *x* and *y* of the sides indicated:



(b) Find the length *x* of the side indicated in the following triangle:



Total 20 marks

Question 7

Differentiate the following functions;

i.
$$y = 3t^3 - 2t^2 + 5t - 3$$
 (5 marks)
ii. $y = x^4 e^{2x}$ (5 marks)
iii. $y = \frac{\cos x}{x^2}$ (5 marks)
iv. $y = (\cos x)^7$ (5 marks)

Total 20 marks

Question 8

Evaluate the following definite integral:

i.
$$\int_2^3 (12x^3 - 6x^2 + 8x) dx$$
 (10 marks)

ii.
$$\int t^{-2} dt$$
 (5 marks)

iii.
$$\int (x^2 + x^5) dx$$
 (5marks)

END OF QUESTIONS

PLEASE TURN PAGE FOR FORMULA SHEETS

FORMULA SHEET

Cosine rule

Sine Formula

$$a^{2} = b^{2} + c^{2} - 2bc \cos(A)$$
 or $\cos(A) = \frac{b^{2} + c^{2} - a^{2}}{2bc}$
 $b^{2} = a^{2} + c^{2} - 2ac \cos(B)$ or $\cos(B) = \frac{a^{2} + c^{2} - b^{2}}{2ac}$ $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

$$a^{2} + c^{2} - 2ac\cos(B)$$
 or $\cos(B) = \frac{a^{2} + c^{2} - B^{2}}{2ac}$

$$c^{2} = b^{2} + a^{2} - 2bc\cos(C)$$
 or $\cos(C) = \frac{a^{2} + b^{2} - c^{2}}{2ab}$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Sum of Arithmetic series with last term	$S_n = \frac{n}{2}(a+l)$
Arithmetic series	a + (n-1)d
Sum of Geometric series (infinite series)	$S_{\infty} = \frac{a}{(1-r)}$
Sum of Arithmetic series	$S_n = \frac{n}{2}(2a + (n-1)d)$
Geometric series	$\mathbf{a}_{\mathbf{n}} = \mathbf{ar}^{\mathbf{n}-1}$
Sum of Geometric series	$S_n = \frac{a(r^n - 1)}{(r - 1)}$

Derivatives or Differentiation Formulas $\frac{d}{dx}[\sin x] = \cos x$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\frac{\mathrm{d}(uv)}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$$

Quotient Rule of differentiation

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Function	Indefinite Integral
] a dx	ax + C
∫ x dx	x ² / 2 + C
∫1/x dx	In x + C
∫ x² dx	x³ / 3 + C
∫ sin (x) dx	- cos (x) + C
cos (x) dx	sin (x) + C
sec ² (x) dx	tan (x) + C
∫ e ^x dx	ex + C

$$\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\log_a(xy) = \log_a x + \log_a y$$
$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a(x)^n = n\log_a(x)$$

$$\log_a(x) = \frac{\log_b(x)}{\log_b a}$$
$$\log_a a = 1$$
$$\log_a 1 = 0$$