ENG10

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER 1 EXAMINATION - 2024/2025

ENGINEERING ELECTROMAGNETICS

MODULE NO: EEE6012

Date: Thursday 9th January 2025 **Time: 2:00pm – 4:30pm**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheet (attached).

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Question 1

A. The waveform shown in red in figure Q1.A is given by $v = 5\cos\left(\frac{2\pi t}{8}\right)$. There is a phase difference between the red and the other two waves. Write down the equation describing:

I. The green wave

[4 marks]

II. The blue wave

[4 marks]

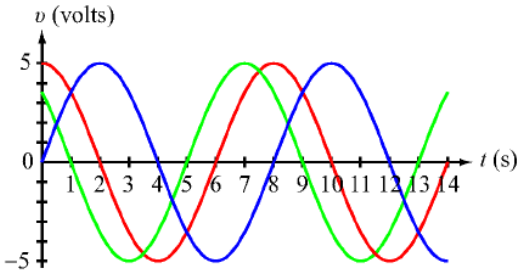

Figure Q1.A

B. Transform vector $\boldsymbol{A} = \hat{\boldsymbol{x}}(x+y) + \hat{\boldsymbol{y}}(y-x) + \hat{\boldsymbol{z}}z$ from Cartesian to cylindrical coordinates.

[8 marks]

C. A series RL circuit is connected to a voltage source given by v_s(t)=150cosωt V. Find:

i. The phasor current

[5 marks]

ii. The instantaneous current i(t) for R=400 Ω , L=3 mH and ω=10^5 rad/s.

[4 marks]

Total 25 marks

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Question 2

A. There are two points in Cartesian coordinates at $P_1(1,2,3)$ and $P_2(-1,-2,3)$, Find:

I. The distance vector $\overrightarrow{P_1P_2}$

[3 marks]

II. The angle between vectors $\overrightarrow{P_1}$ and $\overrightarrow{P_2}$ using the cross product between them

[3 marks]

III. The angle that vector $\overrightarrow{P_2}$ makes with the z-axis

[3 marks]

B. Point P = $(2\sqrt{3}, \frac{\pi}{3}, -2)$ is given in cylindrical coordinates. Express P in spherical coordinates.

[6 marks]

C. Given $V = x^2y + xy^2 + xz^2$, find:

i. The gradient of V

[5 marks]

ii. Evaluate the gradient at (1,-1,2)

[5 marks]

Total 25 marks

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Question 3

A. A square plate in the x-y plane is situated in the space defined by $-3m \leq x \leq 3m \; and - 3m \leq y \leq 3m$. Find the total charge on the plate if the surface charge density is given by $\rho_s = 4y^2 \; \mu C/m^2$.

[5 marks]

B. Four charges of 10 μC each are located in free space at points with Cartesian coordinates (-3,0,0), (3,0,0), (0,-3,0) and (0,3,0). Find the force on a 20 μC charge located at (0,0,4). All distances are in metres.

[10 marks]

C. A horizontal wire with a mass per unit length of 0.2 kg/m carries a current of 4 A in the +x-direction. If the wire is placed in a uniform magnetic flux density B, what should the direction and minimum magnitude of B to magnetically lift the wire vertically upward? The acceleration due to gravity is g=-$\hat{\boldsymbol{z}}\, 9.8 \; \frac{m}{s^2}$.

[6 marks]

D. A square coil of 100 turns and 0.5 m-long sides is in a region with a uniform magnetic flux density of 0.2 Tesla. If the maximum magnetic torque exerted on the coil is 4X10-2 N.m , what is the current flowing in the coil?

[4 marks]

Total 25 marks

Question 4

A. A transmission line of length l connects a load to a sinusoidal voltage source with an oscillation frequency f. Assuming the velocity of wave propagation on the line is c, for which of the following situations it is reasonable to ignore the presence of the transmission line in the solution of the circuit and for which condition(s) it is nonnegligible to ignore.

i. l = 20 cm, f = 20 kHz
ii. l = 50 cm, f = 60 Hz
iii. l = 20 cm, f = 600 MHz
iv. l = 1 mm, f = 100 GHz

[5 marks]

B. A distortionless line has Z_0 = 60 Ω, $\alpha = 20 \; Np/m$, $\mu = 0.6c$ where c is the speed of light in a vacuum, Find R, L, C and G at 100 MHz.

[20 marks]

Total 25 marks

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Question 5

A. The electric field phasor of a uniform plane wave is given by $\tilde{E} = \hat{y}10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5 x 10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find:
   i. Wavelength
   ii. Frequency

[10 marks]

B. A plane wave in air with an electric field amplitude of 20 V/m is incident normally upon the surface of a lossless, nonmagnetic medium with ε_r= 25. Determine the standing wave ratio in the air medium.

[5 marks]

C. A 50-MHz plane wave with electric field amplitude of 50 V/m is normally incident in air onto a semi-infinite, perfect dielectric medium with ε_r = 36. Determine (a) Γ, (b) the average power densities of the incident and reflected waves.

[10 marks]

Total 25 marks

Question 6

A. A transponder with a bandwidth of 400 MHz uses polarization diversity. If the bandwidth allocated to transmit a single telephone channel is 4kHz, how many telephone channels can be carried by the transponder?

[5 marks]

B. A remote sensing satellite is in circular orbit around the earth at an altitude of 1100 km above the earth's surface and the $R_e = 6378\,km$. What is its orbital period?

[5 marks]

C. Calculate the equation 6a to 2.d.p for A = 8.2 and T = 85.9.

$$\int_{-\infty}^{\infty} Arect\,(t)_T dt \qquad \text{6a}$$

[5 marks]

D. A geostationary satellite is at a distance of 40,000 km from a ground receiving station. The satellite transmitting antenna is a circular aperture with a 1-m diameter and the ground station uses a parabolic dish antenna with an effective diameter of 20 cm. If the satellite transmits 1 kW of power at 12 GHz and the ground receiver is characterized by a system noise temperature of 1,000 K, what would be the signal-to-noise ratio of a received TV signal with a bandwidth of 6 MHz? The antennas and the atmosphere may be assumed lossless.

[10 marks]

Total 25 marks

END OF QUESTIONS

PLEASE TURN PAGE FOR FORMULA SHEETS

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Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\widetilde{Z}$, where $z(t) = \mathfrak{Re}\ [\widetilde{Z} e^{j\omega t}]$.

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x + \phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x + \phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0 - \pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\widetilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0 - \pi/2)}$

Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{12}} \;,\; \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3} \;,\; \mathbf{E} = \frac{\mathbf{F}}{Q},\; \mathbf{E} = \int\frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \;,\; \mathbf{E} = \int\frac{\rho_S dS}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \;,\; \mathbf{E} = \int\frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n ,\; \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho \;,\; Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_v \rho_v dv ,\; \nabla\cdot\mathbf{D} = \rho_v ,\; W = -Q\int_A^B \mathbf{E}\cdot d\ell ,\; V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\ell ,\; V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint \mathbf{E}\cdot d\ell = 0 ,\; \nabla\times\mathbf{E} = 0 ,\; \mathbf{E} = -\nabla V \;,\; W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k \;,\; W_E = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E} dv = \frac{1}{2}\int \varepsilon_0 E^2 dv ,\; \mathbf{J} = \rho_v \mathbf{u} ,\; I = \int_S \mathbf{J}\cdot d\mathbf{S} ,\; \mathbf{J} = \sigma\mathbf{E} ,$$

$$R = \frac{\mathrm{V}}{\mathrm{I}} = \frac{\int \mathbf{E}\cdot d\mathbf{I}}{\int \sigma\mathbf{E}\cdot d\mathbf{S}} ,\; \mathbf{D} = \varepsilon\,\mathbf{E},\; \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t},\; E_{1t} = E_{2t} ,\; D_{1n} - D_{2n} = \rho_S ,\; D_{1n} = D_{2n} ,\; \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} ,\; \nabla^2 V = 0 ,\; C = \frac{Q}{V} = \frac{\varepsilon\oint \mathbf{E}\cdot d\mathbf{S}}{\int \mathbf{E}\cdot d\mathbf{I}} ,\; W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C} ,\; C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}} ,\; C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} ,\; RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12} F/m \quad ,\; \mu_o = 4\pi X10^{-7}\; H/m$$

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MAGNETOSTATICS:

$\mathbf{H} = \int_L \frac{Id\mathbf{I} \times \mathbf{a}_R}{4\pi R^2}$, $\mathbf{H} = \int_S \frac{\mathbf{K}dS \times \mathbf{a}_R}{4\pi R^2}$, $\mathbf{H} = \int_v \frac{\mathbf{J}dv \times \mathbf{a}_R}{4\pi R^2}$, $\mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$, $\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$, $\mathbf{a}_\phi = \mathbf{a}_\ell \times \mathbf{a}_\rho$,

$\oint \mathbf{H} \cdot d\mathbf{I} = I_{enc}$, $\nabla \times \mathbf{H} = \mathbf{J}$, $\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$, $\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_n$, $\mathbf{B} = \mu\mathbf{H}$, $\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$, $\oint \mathbf{B} \cdot d\mathbf{S} = 0$, $\nabla \cdot \mathbf{B} = 0$, $\mathbf{H} = -\nabla V_m$,

$\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{A} = \int_L \frac{\mu_0 Id\mathbf{I}}{4\pi R}$, $\mathbf{A} = \int_S \frac{\mu_0 \mathbf{K}dS}{4\pi R}$, $\mathbf{A} = \int_v \frac{\mu_0 \mathbf{J}dv}{4\pi R}$, $\Psi = \oint_L \mathbf{A} \cdot d\mathbf{I}$, $\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$, $d\mathbf{F} = Id\mathbf{I} \times \mathbf{B}$, $\mathbf{B}_{1n} = \mathbf{B}_{2n}$,

$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$, $\mathbf{H}_{1t} = \mathbf{H}_{2t}$, $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$, $L = \frac{\lambda}{I} = \frac{N\psi}{I}$, $M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1\psi_{12}}{I_2}$, $W_m = \frac{1}{2}\int \mathbf{B} \cdot \mathbf{H}dv = \frac{1}{2}\int \mu H^2 dv$

WAVES AND APPLICATIONS:

$V_{emf} = -\frac{d\psi}{dt}$, $V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{I} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, $V_{emf} = \oint_L \mathbf{E}_m \cdot d\mathbf{I} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{I}$

$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{I} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{I}$, $\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt}$, $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{dt}$, $\beta = \frac{2\pi}{\lambda}$, $\underline{\gamma} = \alpha + j\beta$

$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$, $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$, $\mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$

$|\underline{\eta}| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$, $\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$, $\mathbf{H} = \frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\mathbf{a}_y$, $\tan\theta = \frac{\sigma}{\omega\varepsilon}$, $\mathbf{a}_E \times \mathbf{a}_H = \mathbf{a}_k$

$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377\Omega$, $p(t) = \mathbf{E} \times \mathbf{H}$, $p_{ave}(z) = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s \times \mathbf{H}^*_s)$, $p_{ave}(z) = \frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta \mathbf{a}_z$, $P_{ave} = \int_S p_{ave} \cdot d\mathbf{S}$,

$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, $\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$, $s = \frac{|\mathbf{E}_1|_{max}}{|\mathbf{E}_1|_{min}} = \frac{|\mathbf{H}_1|_{max}}{|\mathbf{H}_1|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$, $k_i \sin\theta_i = k_t \sin\theta_t$,

$\Gamma_\parallel = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$, $\tau_\parallel = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$, $\sin^2\theta_{B\parallel} = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2}$,

$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$, $\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$, $\sin^2\theta_{B\perp} = \frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - (\mu_1/\mu_2)^2}$

$\omega = \beta c$

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

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ENGINEERING ELECTROMAGNETISM
MODULE NO. EEE6012

Antenna and Radar formula

Dipole

Solid angle:

$$\Omega_p = \iint_{4\pi} F(\theta,\phi)\, d\Omega$$

Directivity:

$$D = \frac{4\pi}{\Omega_p} \quad \text{or} \quad D = \frac{4\pi A_e}{\lambda^2}$$

Shorted dipole

$$S_0 = \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 .$$

Hertzian monopole

$$R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

Half –wave dipole

$$\widetilde{E}_\theta = j\,60 I_0 \left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}\left(\frac{e^{-jkR}}{R}\right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0} .$$

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

For Transmission line

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Distortionless line

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

Open-circuited line

$$\tilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\tilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0}\sin\beta d,$$

$$Z_{in}^{oc} = \frac{\tilde{V}_{oc}(l)}{\tilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

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Short-circuited line

$$\widetilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin\beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0}\cos\beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan\beta d.$$

$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right]$$

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}\right]$$

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g}V_g \quad , \quad I_o = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt}G_{dr}}{4\pi}\left[\frac{\lambda}{4\pi r_1 r_2}\right]^2 \sigma P_{rad}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

$$P_{rec} = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

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