OCD008

UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE

BENG (HONS) ELECTRICAL AND ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATION 2024/25

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Saturday, 4 January 2025 **Time: 10:00 am – 12:30 pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE (5) questions on this paper.

Answer any FOUR (4) questions.

All questions carry equal marks.

QUESTION 1

To verify the performance of a new cylindrical sensor design, it's essential to calculate key properties of the electric and magnetic fields within the device's operational region.

a) The electric potential in the sensor's operating region is defined as

$$V = \frac{10}{\rho^2} \sin\varphi$$

(i) Find the electric flux density D at $(2, \frac{\pi}{2}, 0)$. **(7 marks)**

(ii) Determine the volume charge density ρ_v at any point in space and find the total charge enclosed by a cylindrical region with radius ρ=1 and height −1 ≤ z ≤ 1 m.

(7 marks)

(iii) Calculate the work done in moving a 10 μC charge from point A (3, 30°, 2) to B (4, 90°,5).

(5 marks)

b) In the device's magnetic field, the magnetic vector potential is given by

$$A = \frac{-\rho z}{4} a_z \ Wb/m.$$

Calculate the total magnetic flux crossing a surface at ϕ= $\frac{\pi}{2}$, within a radial range 1≤ ρ≤ 21 metres and height 0≤ z≤ 50 m.

(6 marks)

[TOTAL 25 MARKS]

QUESTION 2

a) Given the magnetic vector potential in a certain region in a space as

$$A = (3y - z)a_x + 2xza_z \text{ Wb/m}$$

Question 2 continued over the page…

Question 2 continued…

Find the value of magnetic vector potential (A), magnetic flux density (B), magnetic field intensity (H) and current density (J) at point P (2, -1,3).

(10 marks)

b) A toroid has 5000 turns wound upon it and carries a current of 15 A. What is the magnetic flux density inside the toroid at point 25 cm from the centre of the toroidal circle and outside the toroid?

(5 marks)

c) An infinitely long solid conductor of radius **a** is placed along the z-axis. If the conductor carries current I in the + z direction, show that

$$H = \frac{I_\rho}{2\pi a^2}\, a_\varphi$$

within the conductor. if I = 3 A and a = 2 cm, find H at (0, 1 cm,0) and (0, 4 cm, 0).

(10 marks)

[TOTAL 25 MARKS]

QUESTION 3

a) The electric field of an EM wave in free space with the intrinsic impedance of 377 Ω is given as.

$$E = 50\, Cos\,(10^8 t + \beta x)\, a_y \text{ V/m}$$

(i) Calculate β and the time it takes to travel a distance of λ/2 **(5 marks)**
(ii) Calculate displacement current density J_d . **(4 marks)**
(iii) The magnetic field H. **(2 marks)**

b) A plane wave in a nonmagnetic medium has

$$E = 4\, Sin\,(2\pi * 10^7\, t - 0.8x)\, a_z \text{ V/m.}$$

(i) Find Relative Permittivity ε_r and Refractive Index (η)

(6 marks)

Question 3 continued over the page...

Question 3 continued…

(ii) The time average power carried by the wave (poynting vector) and the total power crossing $100\ cm^2$ of plane 2x + y = 5 **(8 marks)**

[TOTAL 25 MARKS]

QUESTION 4

a) A certain transmission line operating at $\omega = 10^6$ rad/s has α = 8 dB/m, β = 1 rad/m, and $Z_o = 60 + j40\Omega$, and is 2m long. If the line is connected to a source of $10\angle 0^\circ$V, $Z_g = 40\Omega$ and terminated by a load of 20 + j50 Ω, determine

(i) The input impedance

(4 marks)

(ii) The sending-end current

(3 marks)

(iii) The current at the middle of the line

(6 marks)

b) For a general transmission line at an operating radian frequency of 600 Mrad/s, the parameters of the line are resistance per unit length as 17 Ω/m, inductance per unit length as 0.35 μH/m, conductance per unit length as 75 μS/m and capacitance per unit length as 40 pF/m. Evaluate,

(i) The attenuation, phase and propagation constants and wavelength of the transmission line.

(9 marks)

(ii) The characteristic impedance of the transmission line

(3 marks)

[TOTAL 25 MARKS]

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QUESTION 5

a) Antenna with impedance 40 +j 30 Ω is to be matched to a 100 Ω lossless line with a shorted stub. Determine

(i) The required stub admittance **(4 marks)**

(ii) The distance between the stub and the antenna **(4 marks)**

(iii) The stub length **(4 marks)**

(iv) The standing wave ratio on each ratio of the system **(4 marks)**

b) A 30-m-long lossless transmission line with Z_o = 50 Ω operating at 2 MHz is terminated with a load Z_L = 60 + j40 Ω. If u = 0.6c on the line, find

(i) The reflection coefficient **Γ** **(4 marks)**

(ii) The standing wave ratio s **(2 marks)**

(iii) The input impedance **(3 marks)**

[TOTAL 25 MARKS]

END OF QUESTIONS

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EQUATION SHEET

CIRCULAR CYLINDRICAL COORDINATES (ρ, ϕ, z)

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

SPHERICAL COORDINATES (r, θ, ϕ)

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \quad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r \sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi, \quad z = r\cos\theta$$

DIFFERENTIAL LENGTH, AREA, AND VOLUME

A. Cartesian Coordinate Systems

1. Differential displacement is given by

$$d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$$

2. Differential normal surface area is given by

$$d\mathbf{S} = \begin{matrix} dy\,dz\,\mathbf{a}_x \\ dx\,dz\,\mathbf{a}_y \\ dx\,dy\,\mathbf{a}_z \end{matrix}$$

3. Differential volume is given by

$$dv = dx\,dy\,dz$$

B. Cylindrical Coordinate Systems

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1. Differential displacement is given by

$$d\mathbf{l} = d\rho\, \mathbf{a}_\rho + \rho\, d\phi\, \mathbf{a}_\phi + dz\, \mathbf{a}_z$$

2. Differential normal surface area is given by

$$d\mathbf{S} = \begin{array}{l} \rho\, d\phi\, dz\, \mathbf{a}_\rho \\ d\rho\, dz\, \mathbf{a}_\phi \\ \rho\, d\rho\, d\phi\, \mathbf{a}_z \end{array}$$

and illustrated in Figure 3.4.

3. Differential volume is given by

$$dv = \rho\, d\rho\, d\phi\, dz$$

DEL OPERATOR

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

$$\nabla = \mathbf{a}_\rho \frac{\partial}{\partial \rho} + \mathbf{a}_\phi \frac{1}{\rho}\frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z}$$

$$\nabla = \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \mathbf{a}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

GRADIENT OF A SCALAR

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

DIVERGENCE OF A VECTOR

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial \mathbf{A}_\phi}{\partial \phi}$$

CURL OF A VECTOR

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\,\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \mathbf{a}_r & r\,\mathbf{a}_\theta & r\sin\theta\,\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A}\, dv = 0$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}$$

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\mathbf{a}_r$$

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$$Q = \int_L \rho_L \, dl \qquad \text{for line charge}$$

$$Q = \int_S \rho_S \, dS \qquad \text{for surface charge}$$

$$Q = \int_v \rho_v \, dv \qquad \text{for volume charge}$$

ELECTRIC FLUX DENSITY

$$\boxed{\mathbf{D} = \varepsilon_0 \mathbf{E}}$$

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\boxed{\rho_v = \nabla \cdot \mathbf{D}}$$

$$\boxed{\mathbf{E} = -\nabla V}$$

electric flux through a surface S is

$$\Psi = \int_S \mathbf{D} \cdot d\mathbf{S}$$

$$I = \oint \mathbf{J} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{J} \, dv$$

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

$$\rho_v = ne$$

$$J = \sigma E$$

$$W = -Q \int_A^B \mathbf{E} \cdot d\mathbf{l} = QV_{AB}$$
$$= Q(V_B - V_A)$$

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$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

$$\mathbf{F} = \int_v \rho_v \mathbf{E}\, dv$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\mathbf{B} = \mu_o \mathbf{H}$$

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B}\, dv = 0$$

$$B = \mu_r \mu_0 (N/2\pi r)\, i$$

$$H_\phi = \begin{cases} \dfrac{I\rho}{2\pi a^2}, & \rho < a \\ \dfrac{I}{2\pi\rho}, & \rho > a \end{cases}$$

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$$\nabla \cdot \mathbf{D} = \rho_v \qquad \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\nabla \times \mathbf{E} = \mathbf{0} \qquad \oint_L \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\boxed{\nabla \cdot \mathbf{A} = 0}$$

$$\boxed{\mathbf{F} = \oint_L I \, d\mathbf{l} \times \mathbf{B}}$$

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\boxed{\mathcal{P} = \mathbf{E} \times \mathbf{H}}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$k = \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\text{Re}(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{E_o^2}{2\eta}\mathbf{a}_k$$

$$P_{\text{ave}} = \int \mathcal{P}_{\text{ave}} \cdot d\mathbf{S} = \mathcal{P}_{\text{ave}} \cdot S\,\mathbf{a}_n$$

TRANSMISSION LINES

1 Np= 8.686db

Propagation constant

$$\gamma = \alpha + j\beta$$

Wave velocity, $u = \frac{\omega}{\beta} = f\lambda$

Wavelength, $\lambda = \frac{2\pi}{\beta}$

Input impedance

$$Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh tanh\, \gamma l}{Z_0 + Z_L \tanh tanh\, \gamma l} \right]$$

$$tanh\ \tanh (x \pm jy) = \frac{sinh\ \sinh 2x}{cosh\ \cosh 2x + cos\ \cos 2y} \pm j \frac{sinh\ \sinh 2y}{cosh\ \cosh 2x + cos\ \cos 2y}$$

$$Z_{in} = \frac{V_S\ (z)}{I_S(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

Voltage and current at any point z

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_o)$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_o)$$

Sending end current and voltage

$$I_0 = \frac{V_g}{Z_{in} + Z_g}$$

$$V_0 = Z_{in} I_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g$$

Reflection coefficient

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing wave ratio

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

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University of Bolton
Off Campus Division - Western International College
BEng (Hons) Electrical & Electronic Engineering
Semester One Examination 2024/25
Engineering Electromagnetism
Module No: EEE6012

Antenna

Wavelength

$$\lambda = \frac{c}{f}$$

Power radiated,

$P_{rad} or\ W = I_{rms}^2 \text{ x } R_{rad}$

Effective area,

$A_e = \frac{\lambda^2}{4\pi} \text{D}$

Capture area of a circular aperture,

$$A_e = \frac{\pi D^2}{4}$$

Radiation Efficiency

$\eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_l}$

$\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_l}$

Directivity

$\text{D} = \frac{4\pi\ U_{max}}{P_{rad}}$

U_{max} – Radiation intensity

$$D = \frac{4\pi}{\lambda^2} A_e$$

Gain of an Antenna

$G = \boldsymbol{\eta} D$

η – Radiation Efficiency

$G = KD$

$G = K \frac{4\pi}{\lambda^2} A_e$

K- antenna factor , 1 if no losses present

Gain in db, $G_{db} = 10\, log_{10} G$

Q factor

$Q = \frac{f_r}{\Delta f}$

Δf *- Bandwidth*

END OF PAPER