UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER 1 EXAMINATIONS 2024/25

INTRODUCTORY ENGINEERING MATHEMATICS

MODULE NO: EEE4011

Date: Thursday 9th January 2025 **Time: 2:00pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **A formula sheet is included.**

Question 1.

(a) A force in Newtons is given by the vector

$$\mathrm{F} = \begin{pmatrix} 3 \\ 9 \\ -5 \end{pmatrix}$$

The force propels a body along the displacement vector

$$\mathrm{d} = \begin{pmatrix} -1 \\ 8 \\ 2 \end{pmatrix}$$

where the values are given in millimetres.

(i) Find the magnitude of the force, in newtons. **[2 marks]**

(ii) Find the distance that the body moves, in millimetres. **[2 marks]**

(iii) Find the work done by the force in displacing the body, stating the units. **[2 marks]**

(iv) Find the angle between the force vector and the displacement vector. **[3 marks]**

(b) Let A and B be the following matrices:

$$\mathrm{A} = \begin{pmatrix} 3 & -1 & 5 \\ 2 & 0 & -4 \end{pmatrix} \qquad \mathrm{B} = \begin{pmatrix} 4 & -2 \\ 1 & 6 \\ 0 & 3 \end{pmatrix}$$

Calculate the products AB and BA. **[8 marks]**

(c) Write the following system of simultaneous linear equations as an equation of matrices:

$$3\mathrm{x} + 4\mathrm{y} = 8$$

$$7\mathrm{x} + 10\mathrm{y} = 5$$

[2 marks]

By finding the inverse of the square matrix, solve the system of equations. **[6 marks]**

[Total 25 marks]

Question 2

(a) Find the complex solutions of the following quadratic equation:

$$x^2 + 12x + 100 = 0$$

Indicate the position of these on a sketch of the Argand diagram.

[6 marks]

(b) A resistor of resistance $R = 3k\Omega$ is in series with a capacitor of value $C = 6.25nF$.

A sine wave voltage source with a frequency of 20000 radians per second is connected across the series pair of components.

Represent the combined impedance as a complex number.

Find the absolute value of the impedance, and the phase shift between the voltage source and the current.

[9 marks]

(c) Let $z_1 = 3 + 4j$ and $z_2 = 8 - 6j$.

Convert each of z_1 and z_2 into polar form.

Hence find each of the following in polar form:

(i) ${z_1}^3$ (ii) $\sqrt{z_2}$

[10 marks]

[Total 25 marks]

Question 3

(a) Differentiate each of the following functions to find $\frac{dy}{dt}$:

(i) $y = 3t^5 + 4t^3 - 5t^2 + 6$

(ii) $y = t^4 \cos 3t$

(iii) $y = (e^{4t} + 2)^{10}$ **[11 marks]**

(b) Find the turning points of the function

$$y = t^4 - 8t^2 + 2$$

Determine whether each turning point is a local maximum or a local minimum.

[10 marks]

(c) The current through an inductor of inductance 40mH is given by

$$i(t) = 0.1\sin(500t).$$

Find an expression for the value of the back emf across the inductor

You may use the formula $v = L\frac{di}{dt}$. **[4 marks]**

[Total 25 marks]

Question 4

(a) Evaluate each of the following definite integrals:

(i) $\int_1^2 \frac{4dt}{t^3}$ **[6 marks]**

(ii) $\int_0^{\frac{\pi}{4}} 6\cos 2t - 12\sin 6t\,.\,dt$ **[6 marks]**

(b) Find each of the following integrals:

(i) $\int t^3 \sin(t^4)dt$ **[5 marks]**

(ii) $\int t^2 e^{3t}.\,dt$ **[8 marks]**

[Total 25 marks]

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Question 5

(a) Find the solution of the following differential equation:

$$\frac{dy}{dt} = 10t^4$$

The boundary condition is $y = 9$ at $t = 1$.

[5 marks]

(b) Consider the following linear differential equation:

$$\frac{dy}{dt} + 4y = 20t$$

(i) Find the complementary function. **[2 marks]**

(ii) Find the particular integral. **[5 marks]**

(iii) Hence find the solution given that $y = 2$ when $t = 0$. **[3 marks]**

(c) Find the general solution of the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 35y = 0$$

[6 marks]

For the particular solution that satisfies the following initial condition:

$$y = 1, \frac{dy}{dt} = -3 \text{ at } t = 0$$

[4 marks]

[Total 25 marks]

Question 6

(a) The resistances eight resistors in ohms are as follows:

1040 1005 890 1102 1025 974 954 1058

Find the median value of the resistances.

Find the interquartile range.

Calculate the mean and standard deviation of the resistances.

[8 marks]

Please give answers to parts (b) and (c) correct to three decimal places.

(b) In a digital communication system operating under noisy conditions, the probability that there is an error in a single bit is 0.06.

In a block of seven bits, find the probability that a single error occurs.

In a block of seven bits, find the probability that exactly two errors occur.

In a block of eight bits, find the probability that seven or more bits are received correctly.

[8 marks]

(c) A plant requires maintenance on average 0.8 times per week.

Calculate the probability that the plant requires maintenance exactly twice in a week.

Calculate the probability that the plant does not require maintenance over a period of two weeks.

Calculate the probability that the plant requires maintenance exactly five times over a period of four weeks.

[9 marks]

[Total 25 marks]

END OF QUESTIONS

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FORMULA SHEET

Derivatives and integrals

Integral	Function	Derivative
$\int y\,dt$	y	$\frac{dy}{dt}$
t	1	0
$\frac{1}{n+1}t^{n+1}$	t^n	nt^{n-1}
$-\frac{1}{a}\cos at$	$\sin at$	$a\cos at$
$\frac{1}{a}\sin at$	$\cos at$	$-a\sin at$
$\frac{1}{a}e^{at}$	e^{at}	ae^{at}

Integration by parts

$$\int \mathrm{u}\frac{\mathrm{dv}}{\mathrm{dt}}\mathrm{dt} = \mathrm{uv} - \int \frac{\mathrm{du}}{\mathrm{dt}}\mathrm{vdt}$$

Binomial Distribution:

The probability of *r* successes in *n* trials is $\binom{n}{r}p^r q^{n-r}$

where *p* is the probability of success in a single trial and $p + q = 1$.

Poisson Distribution:

The probability of *r* successes is $\frac{m^r}{r!}e^{-m}$

where *m* is the expected number of successes.

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