[ENG26]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) CIVIL ENGINEERING

SEMESTER 1 EXAM 2024/2025

MATHEMATICAL METHODS FOR CIVIL ENGINEERING

MODULE NO: CIE4022

Date: Wednesday 8th January 2025 **Time: 10:00 - 12:00**

INSTRUCTIONS TO CANDIDATES:

This is an OPEN book examination

There are FOUR questions

Answer ALL Questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of

100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Question 1

(a) Find $\frac{dy}{dx}$ when: $y = x^3 + x + 1 + \frac{1}{x} + \frac{1}{x^3}$ (2 marks)

Evaluate your answer at $x = 2$ and $x = -2$ (2 marks)

(b) Given that $y = sin(2x) + cos(2x)$ show that $\frac{d^4y}{dx^4} = 16y$ (4 marks)

(c) Given that: $y = e^x - \frac{1}{x}$ show that: $x^2\frac{dy}{dx} - 1 = x^2e^x$ (4 marks)

(d)

(i) Expand the right hand side of: $y = (x - 1)^2(x + 2)$. (2 marks)

(ii) Sketch the graph. (2 marks)

(iii) Write down the co-ordinates any points of intersection with the axes. (3 marks)

(iv) Find the co-ordinates of any turning points and points of inflection. (6 marks)

Total 25 Marks

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Question 2

(a) Find $\frac{dy}{dx}$ when:

(i) $y = x^2 cos(x)$ (3 marks)

(ii) $y = \frac{cos(x)}{x^2}$ (3 marks)

(iii) $y = cos(x^2)$ (3 marks)

(b) Show that: $\frac{dy}{dx} + (x + m)\, sin(x) = cos(x)$ given that: $y = (x + m)\, cos(x)$ (5 marks)

(c) Differentiate $\frac{e^x cos(x)}{x^3}$ (5 marks)

(d) The volume of a cylinder is given by: $V = \pi r^2 h$

(i) Suppose that $h = r - 1, \quad (r > 0)$ then find $\frac{dV}{dr}$ in terms of r. (2 marks)

(ii) Assuming that radius r can be measured with error $\Delta r = 0.05$, find an approximation to the % error in the volume when $r = 6$. (4 marks)

Total 25 Marks

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Question 3

(a) Integrate:

(i) $\int x^3 + x + 1 + \frac{1}{x} + \frac{1}{x^3}\ dx$ (3 marks)

(ii) $\int \frac{1}{\sqrt[5]{x^2}}\ dx$ (3 marks)

(iii) $\int \frac{120x-10}{6x^2-x+10}\ dx$ (2 marks)

(v) $\int cos(a) - sin(3a) + e^{9a} da$ (3 marks)

(b) Find the exact value and an approximation to the definite integral:

$$\int_{-3}^{2} (x - 1)^2(x + 2)dx$$

using the Trapezium Rule with 5 strips (6 marks)

(c)

(i) Evaluate the definite integrals:

$\int_{-1}^{1} x - x^2\ dx$, $\int_{-1}^{0} x - x^2\ dx$ and $\int_{0}^{1} x - x^2\ dx$ (3 marks)

(ii) Sketch the graph: $y = x - x^2$, for $-2 \leq x \leq 2$ (2 marks)

(d) Integrate:

$$\int (5x - 1)^{1/2} dx$$ (3 marks)

Giving your answer in the form:

$$\frac{(5x-1)^n}{m}$$

where n and m are rational numbers.

Total 25 Marks

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Question 4

(a) (i) Sketch the graph of $y = \sqrt{(4 - x^2)}$ $\quad 0 \leq x \leq 2$ (2 marks)

(ii) Integrate:

$$\int_0^2 \sqrt{(4 - x^2)}\ dx$$

using the substitution: $x = 2\,sin(\theta)$ and the identity: $cos^2(\theta) = \frac{1}{2}[1 + cos(2\theta)]$.

(6 marks)

Hence, find the area of a circle with radius $r = 2$. (2 marks)

(b) Intrgrate:

(i) $\int x^2 cos(x)\ dx$ (6 marks)

(ii) $\int 3x^2\ (x^3 - 2)^{-1/3}\ dx$ (4 marks)

(c) A function that passes through the point (0,2) is differentiated to produce $\frac{dy}{dx} = 2e^{4x} + 2$.
Find the original function. (2 marks)

(d) Evaluate: $\int_{0.01}^{1} \frac{1}{x} dx$ (2 marks)

Comment on the value of $\int_a^1 \frac{1}{x} dx$ as 'a' tends to zero. (1 marks)

Total 25 Marks

END OF QUESTIONS

FORMULA SHEET FOLLOWS OVER THE PAGE

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FORMULA SHEET

Function f(x) or y	Differentiation f '(x) or $\frac{dy}{dx}$
x^n	nx^{n-1}
e^x	e^x
e^{ax}	ae^{ax}
ln (x)	$\frac{1}{x}$
sin (x)	cos (x)
sin (ax)	a cos (ax)
cos (ax)	-a sin (ax)

	Chain rule	**Product rule**	**Quotient rule**
Differentiation	$y = f(g(x))$ $\quad u = g(x)$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	$y = u\,v$ $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	$y = \frac{u}{v}$ $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
	By parts		
Integration	$y = u\,\frac{dv}{dx}$ $\int u\,\frac{dv}{dx} = uv - \int v\,\frac{du}{dx}$		

END OF PAPER