ENG22

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATION 2024/25

ADVANCED MATERIALS & STRUCTURES

MODULE NO: AME6012

Date: Tuesday 7th January 2025 **Time: (10:00 am - 1:00pm)**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Attempt ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

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Q1.

a) A finite element analysis calculated the following stresses at one of the Gauss points: Direct stresses: xx= 145 MPa compressive, yy= 155 MPa tensile and zz= 105 MPa tensile accompanied by two shear stresses: xy= 54 MPa and yz= 62 MPa. Using this information:

(i) Sketch the elemental cube representing the state of stress at this point.

(3 marks)

(ii) Show that the characteristic equation representing the state of stress at this point is given as: $\sigma^3 - 115\sigma^2 - 28185\sigma + 2108675 = 0$ and show the largest stress acting at this point is 202.72 MPa to within ±5%.

(7 marks)

(iii) Calculate direction of the largest stress and show this by a simple sketch.

(6 Marks)

b) If the yield stress of the material is 525 MPa determine the factor of safety at this point based upon the von Mises criterion assuming the other principal stress at this point are 154.88 MPa in compression and 67.16 MPa in tension.

(5 Marks)

c) The component was manufactured by initially rolling the stamping along the z direction. Explain how this would influence the choice of yield criteria and how this would change the von Mises criterion currently used.

(4 Marks)

Total 25 Marks

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Q2.

a) Part of a north sea rig leg is immersed in the water and is fabricated from a coated high strength steel with the properties given in Table Q2. The leg is subjected to cyclic stresses ranging from 320MPa tensile to 120 MPa compressive approximately every 15 minutes. The leg however is susceptible to cracks on the outside edge if damaged and therefore is monitored regularly; however, the equipment used can only detect cracks larger than 3mm.

Using the above information and the material data in table Q2, determine the time taken for the crack to grow to 6mm. **(9 Marks)**

Table Q2	
Yield Strength	810 MPa
Young's Modulus	208 GPa
Poisson's Ratio	0.32
Fracture toughness	87 $MPa.m^{0.5}$
Paris coefficients M & C	3.3 & $1.1x10^{-12}$
Shape factor Y	1.15

b) Also estimate how much longer life the leg has under these conditions.

(7 Marks)

c) If the material could potentially yield at the crack tip describe briefly how you could modify the crack growth expressions to take aspects of ductility into account?

(4 marks)

d) Explain briefly why this estimate is conservative and what other factors could be considered to improve the life predictions **(5 Marks)**

Total 25 Marks

Q3.

a) Figure Q3 shows schematically a portal frame representing a funfair safety barrier with a worst case scenario load case consisting of a horizontal load of 20 KN and a vertical load of 15 KN. Joints A, B and D can be assumed to be welded whilst joint C is a safety pin. Use this information to determine a suitable tubular section manufactured from steel with a yield stress of 420 MPa and a factor of safety of 3.

Assume for the analysis the material is rigid-perfectly plastic.

Take Z_p as D^2t where: D is the nominal bore and t the thickness of a tubular section.

(12 Marks)

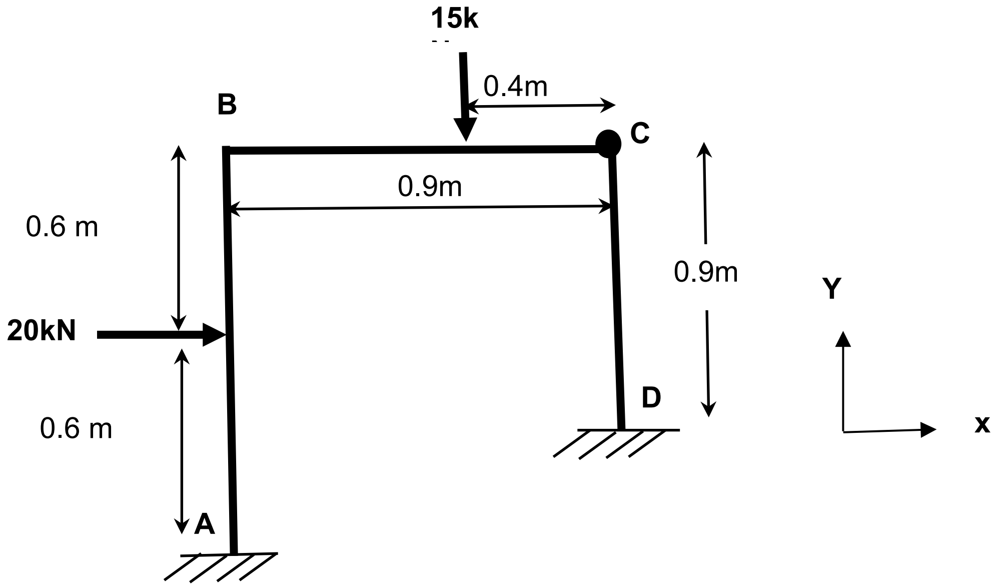

Fig Q3 frame set up

b) An alternative proposal is also considered with the same size tubing, but this time the 20KN load is acting 0.8m from A. Determine the new factor of safety.

(9 Marks)

c) Describe two other material models that could be used in place of the rigid perfectly plastic one stating in each case whether they would produce a higher or lower factor of safety. **(4 Marks)**

Total 25 Marks

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Q4.

a) A high performance automotive component is to be manufactured from high modulus carbon fibre reinforcement and an epoxy matrix with a density of 1.3Tonnes/m^3 in the form of a prepreg skin bonded to a 25mm thick ROHACELL® core; this is to replace an existing aluminum square hollow section structure. The foam has a density of 75Kg/m^3 and a failure strain of 0.4%.The component is subject to both flexure and torsion; these loads are shown in Fig. Q4. Using this information determine a suitable layup and wall thickness for the composite and illustrate this by a sketch.

(20 marks)

Fibre Modulus GPa	**Fibre Density Kg/m^3**	**Volume fraction %**	**Safe working strain %**	**Bond strength of skin MPa**	**Lamina Thickness mm**
320	1800	60	0.4	12	0.125

Table Q4

b) If the original aluminum hollow section was 25mm X25mm x 6mm wall thickness and has a density of 2.7 Tonnes/m^3; determine the mass saving on the component.

(5 marks)

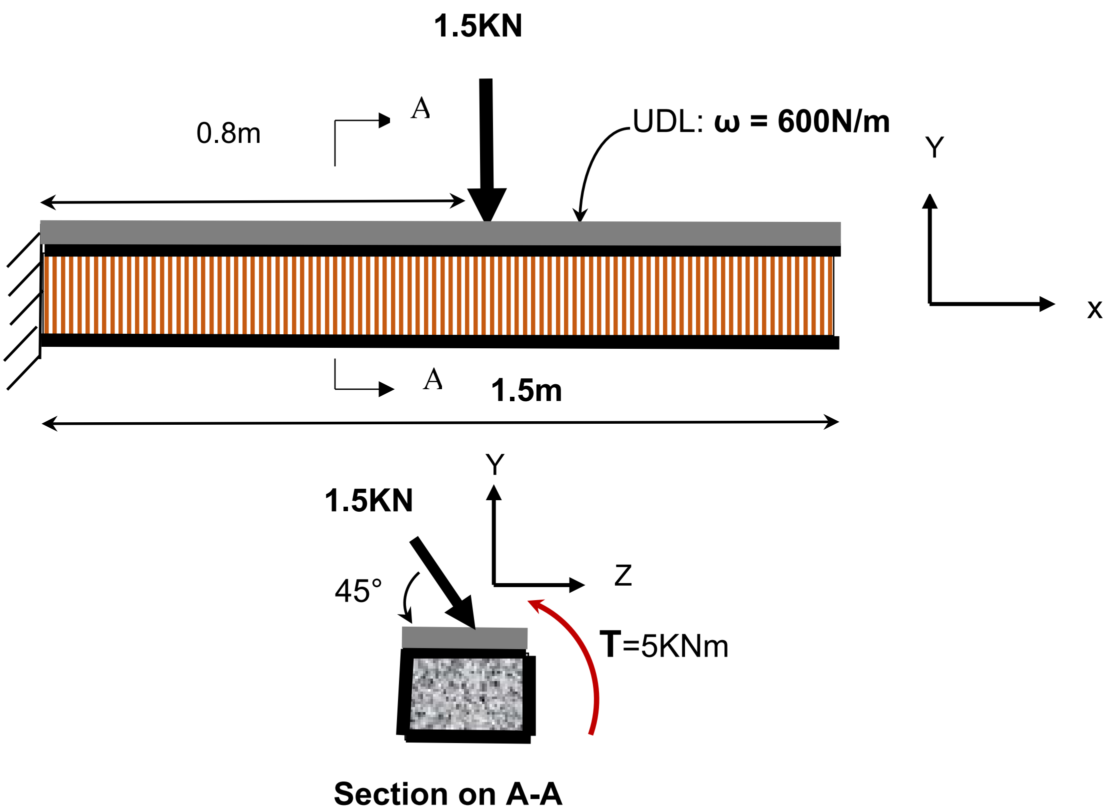

Fig Q4 schematic of the component

Total 25 Marks

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Q5.

a) A 3mm diameter biomedical valve is to be potentially manufactured from CoCrMo as shown in Fig Q5a. The valve has a Young's modulus of 104 GPa and ν = 0.3 is to be evaluated for future use.

It is also expected that the component under its normal usage would be under repeated cyclic loading with a maximum bending moment of 1.3Nm along with a lower load of 0.5Nm. Assuming at the position of largest stress for this geometry, $K_t = 1.4$ based on historic photoelastic test data and the notch sensitivity factor $q = 0.65$, hence, estimate the maximum stress and predict the life of the component under this condition.

(10 marks)

Fig Q5a Schematic of the valve

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Q5 continued

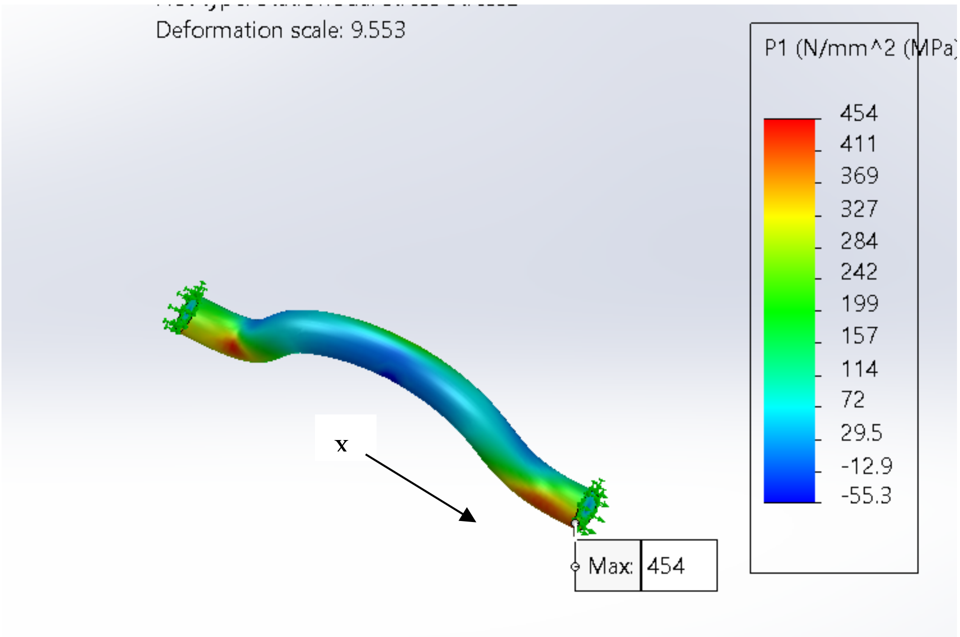

Fig Q5b FEA plot of the Principal stress under an inplane moment of 1.3Nm

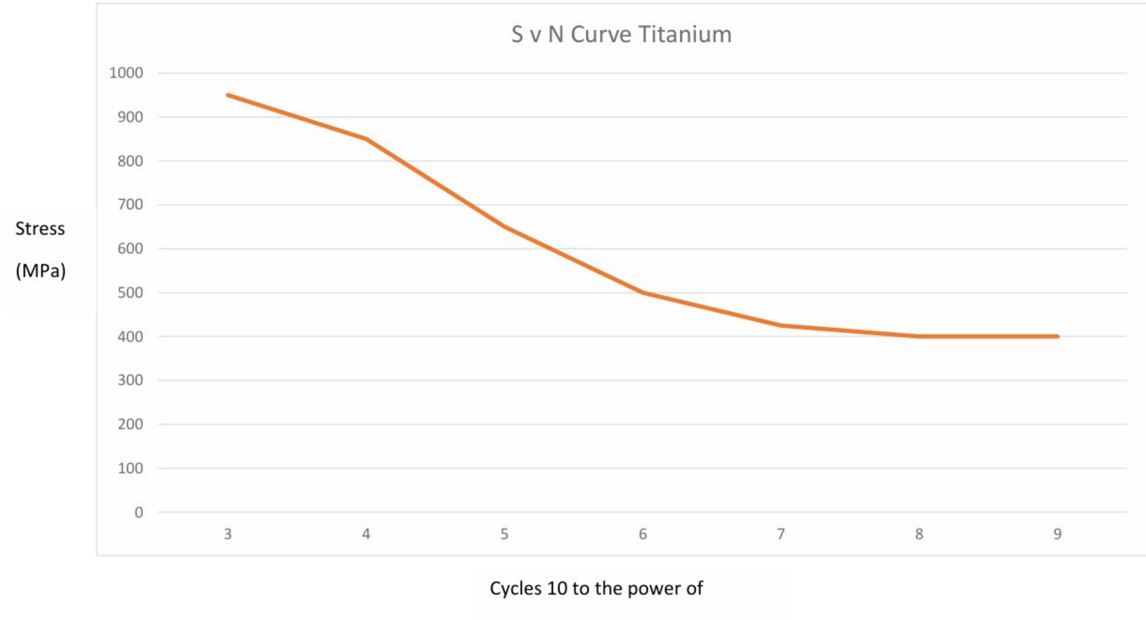

Fig Q5c- S-N Curve for CoCrMo material used.

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Q5 continued

b) In order to verify the behaviour both finite element analysis and strain gauge techniques were used to evaluate the design. The output from the finite element model is shown in figure Q5c indicating the principal stress values at the position of interest.

Further confirmation was achieved using a specialist micro-strain gauge set-up consisting of three gauges in the pattern shown in figure Q5d bonded to the surface at an angle of 10^o to the axis of symmetry. The gauges had a gauge length of 0.4mm and bonded using an adhesive. The output results under the maximum load condition for the three gauges are given below

ε_0 = 4550 x 10^{-6} mm/mm (0^o)

ε_{45} = 2663 x 10^{-6} mm/mm (45^o)

ε_{90} = -3655 x 10^{-6} mm/mm (90^o)

Using this data calculate the maximum strain obtained and compare with the predicted experimental stress that was obtained using the finite element method. Explain also why there is a difference between the two results and where the main source of error is likely to occur.

(15 Marks)

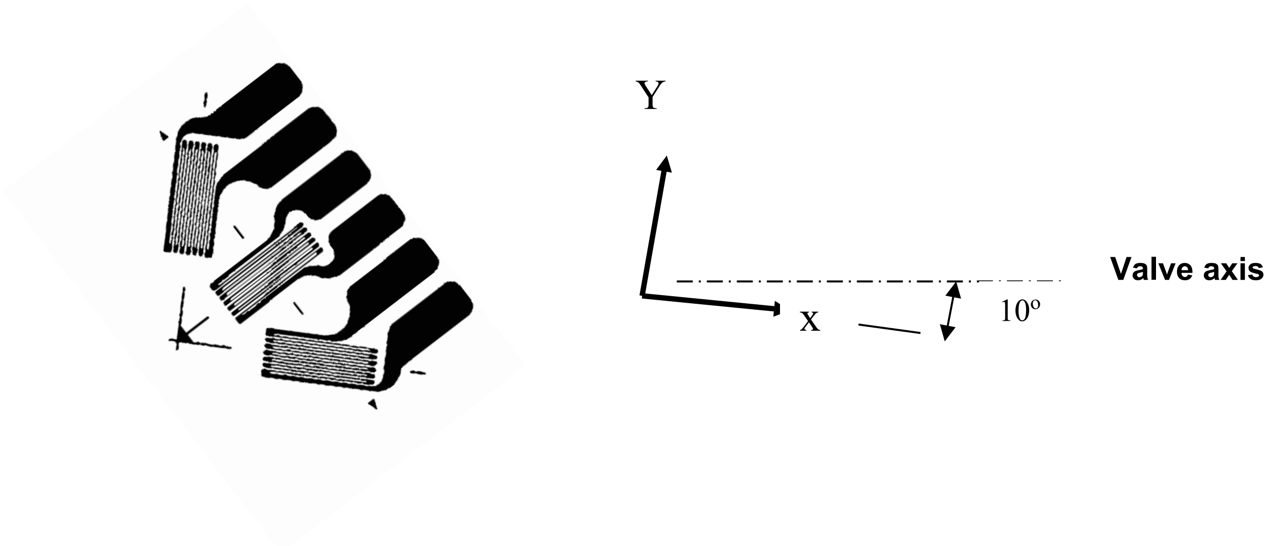

Fig Q5d Strain Gauge set up

END OF QUESTIONS

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FORMULA SHEET

Formulae used in Structures and Materials Module

Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = (Stress\ Tensor)\begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Where a, b and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \qquad l = \cos\alpha,$$

$$m = bk \qquad m = \cos\theta,$$

$$n = ck \qquad n = \cos\varphi.$$

Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

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Yield Criterion

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Tresca

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

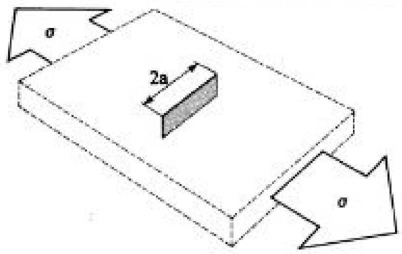
$$\sigma_{tresca} = 2 \cdot \tau_{\max}$$

$$\tau_{\max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}; \frac{|\sigma_1 - \sigma_3|}{2}; \frac{|\sigma_3 - \sigma_2|}{2}\right)$$

$$\frac{\sigma_{von\ Mises}}{\sigma_{Tresca}} = \frac{\sqrt{3}}{2}$$

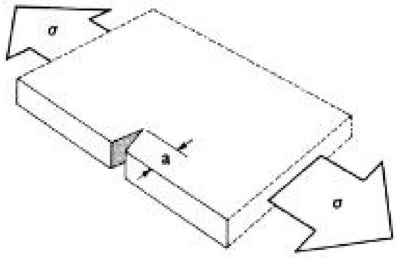
Fracture mechanics

Table: Y values for plates loaded in tension



(1) Through crack of length $2a$ in an *infinite* plate

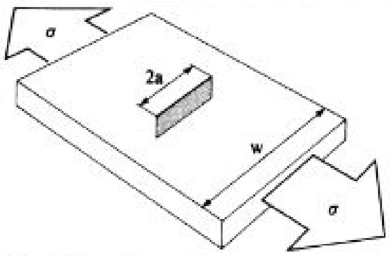
$Y = 1$



(2) Edge crack of length a in an *infinite* plate

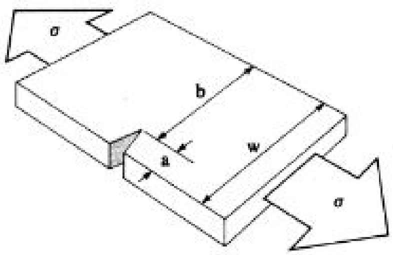
$Y = 1.12$

Because plane strain and plane stress have identical stress fields, this calibration is also for an edge scratch of depth a on a large body carrying tensile stress σ.



(3) Through crack of length $2a$ in a plate of width w.

$$Y = \left(\sec\frac{\pi a}{w}\right)^{1/2}, \quad \frac{2a}{w} \le 0.7$$



(4) Edge crack of length a in a plate of width w.

$$Y = 0.265\left(\frac{b}{w}\right)^{4} + \frac{0.875 + 0.265a/w}{(b/w)^{3/2}}$$

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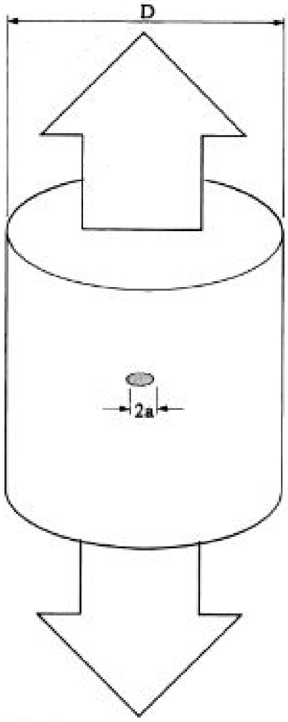
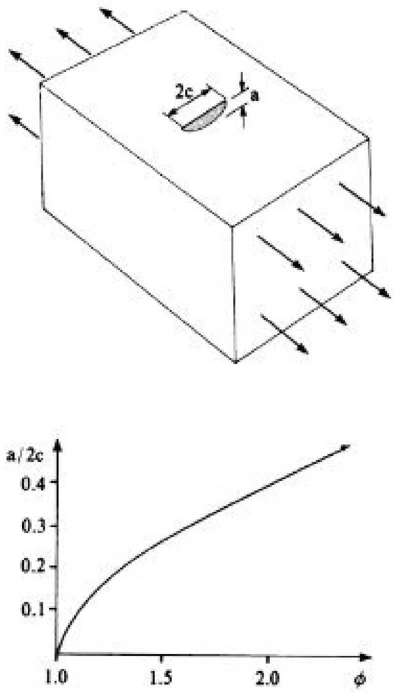


(5) Penny-shaped internal crack of radius a.

$Y = \frac{2}{\pi}, \quad a \ll D$



(6) Semi-elliptical surface flaw

$Y = \frac{1.12}{\phi^{1/2}}$

Life Calculations

$$\frac{da}{dN} = C(\Delta K)^m$$

$$N = \frac{1}{CY^m \sigma_a^{\,m} \pi^{\frac{m}{2}}} \int_{a_0}^{a_1} \frac{da}{a^{\frac{m}{2}}}$$

$$K_f = 1 + q(K_t - 1) \qquad 1 \le K_f \le K_t$$

Composite materials

$$E_{composite} = E_{fibre}V_{fibre} + E_{matrix}(1 - V_{fibre})$$

Fracture Toughness

Table: Fracture toughness of some engineering materials

Material	K_{IC} ($MNm^{-3/2}$)	E (GN/m^2)	$\mathcal{G}_{IC}$ (kJ/m^2)
Plain carbon steels	140 - 200	200	100 - 200
High strength steels	30 - 150	200	5 - 110
Low to medium strength steels	10 - 100	200	0.5 - 50
Titanium alloys	30 – 120	120	7 – 120
Aluminium alloys	22 – 33	70	7 - 16
Glass	0.3 – 0.6	70	0.002 – 0.008
Polycrystalline alumina	5	300	0.08
Teak – crack moves across the grain	8	10	6
Concrete	0.4	16	1
PMMA (Perspex)	1.2	4	0.4
Polystyrene	1.7	3	0.01
Polycarbonate (ductile)	1.1	0.02	54
Polycarbonate (brittle)	0.4	0.02	6.7
Epoxy resin	0.8	3	0.2
Fibreglass laminate	10	20	5
Aligned glass fibre composite – crack across fibres	10	35	3
Aligned glass fibre composite – crack down fibres	0.03	10	0.0001
Aligned carbon fibre composite – crack across fibres	20	185	2

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Strain relationships

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

where ε_x = normal strain at a point in x-direction

ε_y = normal strain at a point in y- direction

γ_{xy} = shear strain at a point on x face in y direction

$$\text{2D Strain tensor} = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_y \end{bmatrix}$$

Hooke's Law in 2D

$$\sigma_1 = \frac{E}{(1-\upsilon^2)}(\varepsilon_1 + \upsilon\varepsilon_2)$$

$$\sigma_2 = \frac{E}{(1-\upsilon^2)}(\varepsilon_2 + \upsilon\varepsilon_1)$$

Related Mathematics

Cubic Equations-General form

,

$\sigma^3 + F_1\,\sigma^2 + F_2\sigma + F_3 = 0$ where: F_1, F_2, & F_3 are constants then the solution has three roots, say a, b & c, giving: $(\sigma-a).(\sigma-b).(\sigma-c) = 0$,

hence,

$$\sigma^3 - \sigma^2\,(a+b+c) + \sigma\,(a+c)b - abc = 0$$

as a general form.

If either a, b or c is known a simple quadratic equation based upon the other two unknowns can derived and solved.

Finding determinants using cofactors

Sign of cofactor

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

Find determinants

$$2\begin{vmatrix} 0 & 4 \\ -1 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$$

$$2[(0\times2)-(-1\times4)]-4[(1\times2)-(2\times4)]-3[(1\times-1)-(0\times2)]$$

$$8+24+3=35$$

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