ENG20

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MECHANICAL ENGINEERING

SEMESTER 1 EXAMINATION 2024/25

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME5012

Date: Thursday 9th January 2025 **Time: 2:00pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Answer ANY FOUR questions.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached after questions).

Q1. Thick Cylinder

A thick-walled cylindrical pressure vessel is used in hydrogen fuel cell vehicles to store compressed hydrogen gas at an internal pressure of $7\ MPa$. The external and internal diameters of the vessel are $950\ mm$ and $475\ mm$, respectively. The vessel is made of high-strength steel with a Young's modulus $E = 210\ GPa$ and Poisson's ratio $\nu = 0.3$.

In this application, the integrity of the storage vessel is critical to prevent leaks and ensure safe, long-term operation of hydrogen fuel cell systems in electric vehicles.

To assess the vessel's structural performance and redesign potential:

a) **Calculate the inner and outer hoop stresses**. (8 marks)

b) **Determine the longitudinal stress and strain** in the cylindrical vessel. (5 marks)

c) **Find the inner and outer hoop strains**. (5 marks)

d) The vessel is proposed to be redesigned as a **thin-walled pressure vessel**, with a pressure difference of $7\ MPa$. The maximum allowable stress for the material is $120\ MPa$, and the design factor is 2. Propose **a ratio of radius and thickness** for the new thin-walled pressure vessel design, taking into consideration that the vessel will be used in hydrogen fuel cell vehicles where weight optimisation is essential.

(7 marks)

Total 25 Marks

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Q2: Eigenvalues and Principal Stresses

A drone propeller hub, made from a high-strength aluminum alloy, was redesigned under the following stress conditions: Direct stresses $\sigma_{xx} = -170\ MPa$ and $\sigma_{yy} = -170\ MPa$, accompanied by a shear stress $\tau_{xy} = 75\ MPa$. Using this information:

a) **Sketch the elemental square representing the state of stress** for the given stress components. (4 marks)

b) **Derive the characteristic equation representing the state of stress** at the given point using the **matrix method** and **calculate the corresponding principal stresses**. (8 marks)

c) **Verify the values of the principal stresses** using the **general stress transformation equations**. Also, **calculate the direction of the maximum principal stress** and illustrate it through a simple sketch. (8 marks)

d) If the yield stress of the aluminum alloy is $320\ MPa$ and the principal stresses are $\sigma_1 = 160\ MPa$, $\sigma_2 = 0\ MPa,$ and $\sigma_3 = -190\ MPa$, **determine the factor of safety using the von Mises criterion**. (5 marks)

Total 25 Marks

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Q3. Struts

A steel pin-ended strut, which has a length of $2.8\ m$ and a rectangular cross-section of $15\ mm$ by $8\ mm$, is axially loaded in a robotic arm used in an automated manufacturing system until buckling occurs. The strut is made from high-strength steel with a Young's modulus of $210\ GPa$.

Given: The yield stress is $450\ MPa$.

a) **Determine the slenderness ratio of the strut**. (8 marks)

b) **Calculate the maximum critical load using Euler's formula.** (5 marks)

c) **Find the maximum central deflection** under the critical load condition.

(5 marks)

d) **Calculate the Rankine-Gordon maximum load** if the Rankine constant $a = 1/5000$ (7 marks)

Total 25 Marks

Q4: Application of Beam theory

A section of a crane boom can be modeled as a cantilever beam (**Figure Q4**). The beam has a length of $4\ m$, a width of $120\ mm$, and a height of $60\ mm$. The beam is made from a steel alloy with a Young's modulus of 200 GPa. A point load F of $2000\ N$ is applied at the free end of the cantilever.

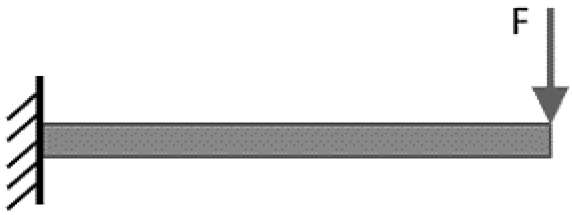

Figure Q4: Cantiliver beam-crane boom model.

Determine:

a) **The maximum shear force** in the beam. (4 Marks)

b) **The maximum bending moment** at the fixed end of the beam. (4 Marks)

c) **The maximum bending stress** in the beam cross-section. (6 Marks)

d) **The maximum deflection** at the free end of the cantilever. (7 Marks)

e) **If the yield stress of the material is $300\ MPa$, what will be the factor of safety?**

(4 Marks)

Total 25 Marks

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Q5: Application of Beam theory

A logistics company is designing shelving systems to store heavy pallets. Each shelf is supported by aluminum beams (Young's modulus: $70\ GPa$) with a span of $2\ m$, width of $50\ mm$, and height of $25\ mm$. The shelves must support a uniformly distributed load of $4000\ N/m$ from fully loaded pallets. The beams need to be analysed to ensure they can safely support the load without excessive deflection or failure.

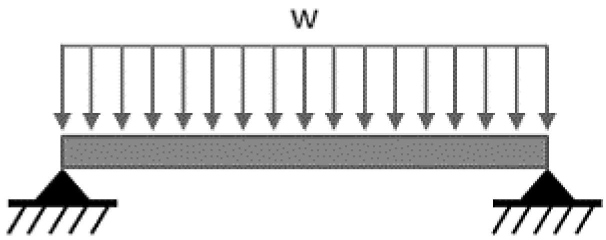

Figure Q5: Simply supported beam-shelving system.

Given the aluminum beam in the warehouse shelving system, determine the following:

a) **The maximum shear force in the beam due to the distributed load**. (4 Marks)

b) **The maximum bending moment in the beam.** (4 Marks)

c) **The maximum bending stress at the critical section of the beam.** (6 Marks)

d) **The maximum deflection of the beam under full load.** (7 Marks)

e) **If a factor of safety of 2.5 is used, what is the minimum required yield stress for the aluminum to ensure safety?** (4 Marks)

Total 25 Marks

END OF QUESTIONS

FORMULA SHEETS OVER THE PAGE….

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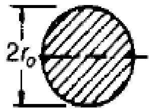
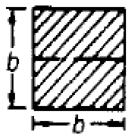
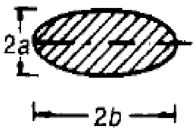
FORMULA SHEET

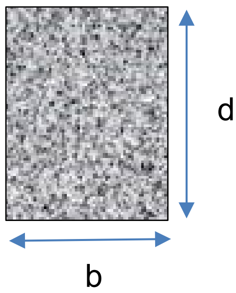
Deflection:

$$M_{xx} = EI \frac{d^2y}{dx^2}$$

Section Shape	$A(m^2)$	$I_{xx}(m^4)$
	πr^2	$\frac{\pi}{4} r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4} a^3 b$

For solid rectangular Cross-section

$$I_{xx} = \frac{bd^3}{12}$$



Plane Stress:

a) Stresses in function of the angle ϴ:

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2} sin(2\theta) + \tau_{xy} cos(2\theta)$$

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b) Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Hoop Stress of a thin Pressure Vessel:

$$\sigma_{h=\frac{\Delta Pr}{t}}$$

Lame's equation

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall. Note: R_1 = inner cylinder radius, R_2 = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - \nu(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - \nu(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - \nu(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1R_1^2 - P_2R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

Stress

σ = Force/Area = F/A

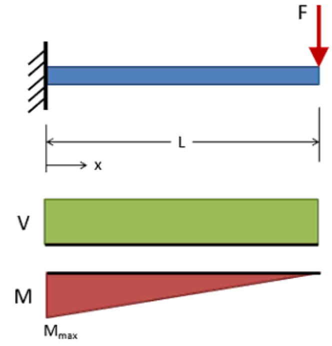
Hook's law

σ = E·ε

ε = ∆L/L

Cantilever Beam with End-Point-Load:

Cantilever, End Load



Deflection:

$$\delta = -\frac{Fx^2}{6EI}(3L - x)$$

$$\delta_{max} = \frac{FL^3}{3EI} \quad @\, x = L$$

Slope:

$$\theta = -\frac{Fx}{2EI}(2L - x)$$

$$\theta_{max} = \frac{FL^2}{2EI} \quad @\, x = L$$

Shear:

$$V = +F$$

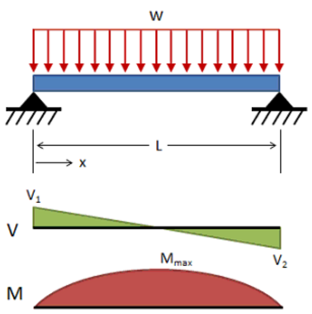
Moment:

$$M = -F\,(L - x)$$

$$M_{max} = -FL \quad @\, x = 0$$

Simply Supported Beam with UDL:

Simply Supported, Uniform Distributed Load



Deflection:

$$\delta = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$\delta_{max} = \frac{5wL^4}{384EI} \quad @\, x = L/2$$

Slope:

$$\theta = -\frac{w}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\theta_1 = -\frac{wL^3}{24EI} \quad @\, x = 0$$

$$\theta_2 = +\frac{wL^3}{24EI} \quad @\, x = L$$

Shear:

$$V = w\,(L/2 - x)$$

$$V_1 = +wL/2 \quad @\, x = 0$$

$$V_2 = -wL/2 \quad @\, x = L$$

Moment:

$$M_{max} = wL^2/8 \quad @\, x = L/2$$

Maximum Bending Stress

$$\sigma_{max} = \frac{M_{max}\,y}{I}$$

Yield Criterion

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Quadratic equation: $ax^2+bx+c=0$

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Safety factor and Design factor:

$$\boldsymbol{Factor\ Of\ Safety} = \frac{\boldsymbol{\sigma_{yield}}}{\boldsymbol{\sigma_{applied}}}$$

$$\boldsymbol{Design\ Factor} = \frac{\boldsymbol{Design\ Stress}}{\boldsymbol{Allowable\ Stress}}$$

Struts:

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

Eigenvalues

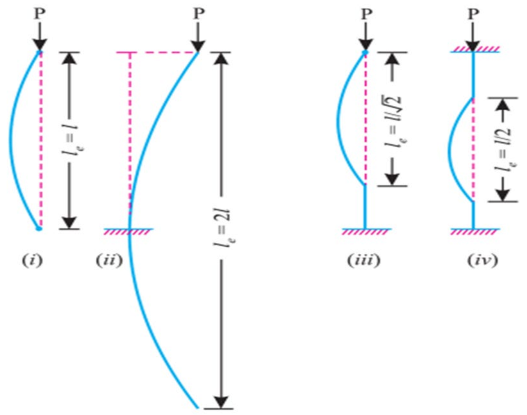
$$|A - \lambda I| = 0$$

Eigenvectors

$$(A - \lambda_r I)x_r = 0$$

<u>Euler validity</u>

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi\sqrt{\frac{E}{\sigma_{yield}}}$$



(i) *Both ends pin jointed or hinged or rounded or free.*
(ii) *One end fixed and other end free.*
(iii) *One end fixed and the other pin jointed.*
(iv) *Both ends fixed.*

Case	End conditions	Equivalent length, l_e	Buckling load, Euler
1	*Both ends hinged or pin jointed* or *rounded or free*	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	*One end fixed, other end free*	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	*One end fixed, other end pin jointed*	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	*Both ends fixed or encastered*	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect*.

$$\boldsymbol{a = \frac{\sigma_c}{\pi^2 \cdot E}}$$

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