[ENG19]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) AUTOMOTIVE PERFORMANCE ENGINEERING (MOTORSPORT)

SEMESTER TWO EXAMINATION 2023/2024

ENGINEERING SCIENCE II

MODULE NO: MSP5016

Date: 17th May 2024 **Time: 10:00 – 12:00**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer FOUR questions.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached after questions).

Q1. Figure 1a shows the symmetrical cross section of a hollow cylindrical beam. Figure 1b shows the loading conditions for the simply supported beam. The beam is made from steel with a modulus of elasticity, E = 210 GPa.

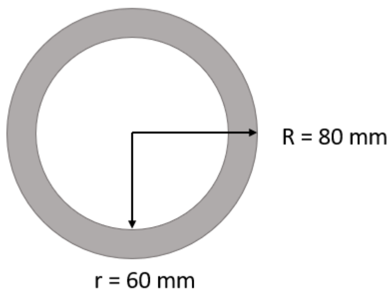

Figure 1a- Cross section of beam

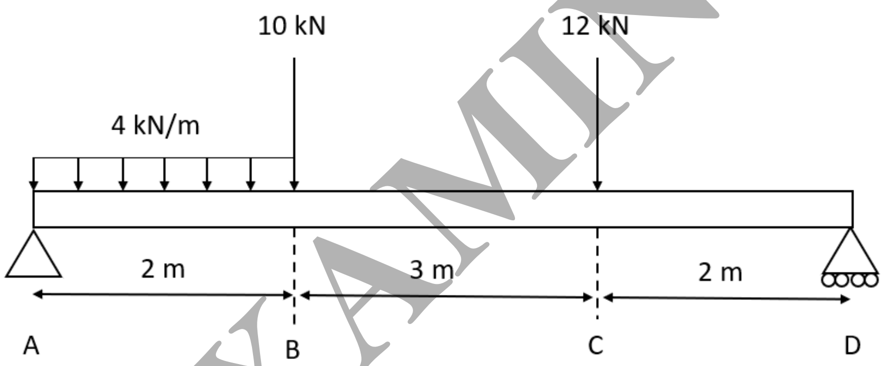

Figure 1b- Beam loading

a) Find the support reactions at A and D.

(3marks)

b) Calculate and plot the bending moment distribution across the beam. Show that the maximum bending moment is 26.9 kNm.

(5marks)

c) Find the maximum bending stress in the beam.

(4marks)

d) Use Macauley's method to find the deflection at the centre of the beam.

(13marks)

Total for Q1 - 25 marks

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Q2. Figure 2 shows the cross section of an L beam. The beam is not symmetrical and exhibits unsymmetrical bending when a load is applied.

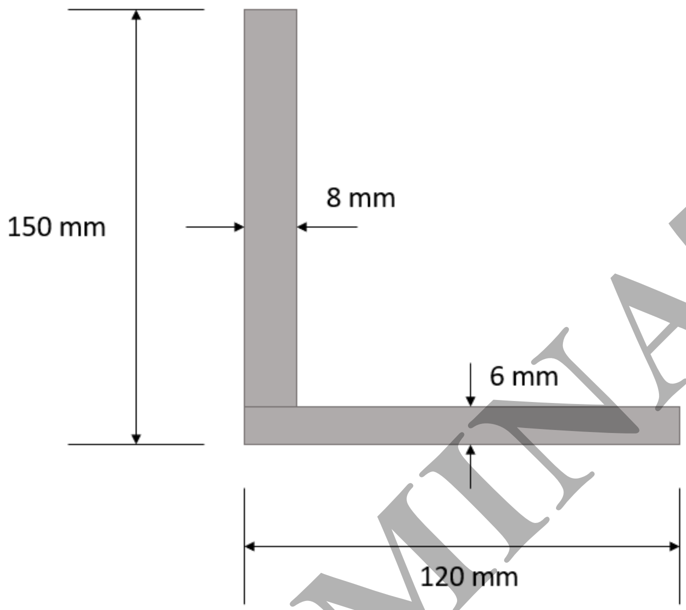

Figure 2- cross section of L beam

a) Find the position of the centroid for the section in Figure 2.

(6marks)

b) Use the parallel axis theorem to find the inertia about the x axis, y axis and the product inertia $(I_{xx}\ , I_{yy}\ and\ I_{xy})$

(12marks)

c) Find the angle of inclination of the principal axis to the x, y plane.

(3marks)

d) Find the maximum and minimum inertias about the principal axis.

(4marks)

Total for Q2 – 25 marks

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Q3. Consider the spherical pressure vessel displayed in Figure 3.

a) Show that the tangential stress is given by: $\boldsymbol{\sigma_T = \frac{Pr}{2t}}$

(4marks)

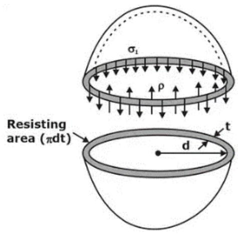

Figure 3- Spherical pressure vessel

The pressure vessel in figure 3 carries a pressurised gas. The internal pressure will reach 6 MPa. The vessel has a radius of 300mm and thickness of 6mm. It is constructed from a titanium alloy with a yield strength of 275 MPa.

b) Calculate the factor of safety of the sphere when the internal pressure load of 6 Mpa is applied to the vessel.

(4marks)

The pressure vessel is now required to carry gas at an internal pressure of 80 MPa in a pressurised environment. The design has been changed to a thick-walled cylinder. The vessel is to be made from steel and has an internal diameter of 400mm and an outer diameter of 500mm. The external pressure is assumed to be 20 MPa. Take E = 200 GPa. And Poisson's ratio, v = 0.3 for the steel.

c) Roughly sketch the distribution of hoop stress and radial stress across the thickness of the new cylindrical pressure vessel.

(4marks)

d) Calculate the hoop stress and radial stress at both the inner and outer radius.

(10marks)

e) Calculate the longitudinal stress in the cylindrical vessel.

(3marks)

Total for Q3 – 25 marks

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Q4) Struts are structural components that have a compressive load acting upon them. Two methods for calculating the critical loads for struts are Euler's formula and Rankine-Gordon's formula.

a) Provide a sketch of showing how the approximate critical load varies with slenderness ratio for each of the methods.

(3marks)

b) What are the limitations of using Euler's method for shorter struts?

(2marks)

A solid cylindrical column of cast iron is placed vertically and has one end fully fixed with the other end pinned, the column has a diameter of 75 mm, and it is 4.2 m in height.

Take modulus of elasticity, E=80 GPa and material constant c=1/1600, elastic limit stress as σ_c= 550 Mpa.

c) Find the radius of gyration of the strut.

(4marks)

d) Determine the Euler critical load and critical stress of the hollow cylindrical column.

(8marks)

e) Compare this stress with the critical stress given by Rankine-Gordon's formula.

(8marks)

Total for Q4 – 25 marks

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Q5) A coil spring that forms part of a suspension system undergoes a testing process. A mass is added to the spring and the outcomes observed. A mass of 50 kg is placed on its lower end. the spring constant, k, is 19.6 kN/m. The system undergoes some initial displacement.

a) Find the periodic time and natural frequency of vibration.

(4marks)

b) Sketch a displacement time graph that models the oscillations for the system when subjected to an initial displacement if a damper were to be added. Show a response for a scenario when the system is over damped, under damped and critically damped.

(3marks)

A damper is installed on the system with a damping coefficient of 750Ns/m.

c) Calculate the damping ratio of the system and state the nature of the damping.

(3marks)

d) Calculate the damped frequency and periodic time.

(3marks)

e) Calculate the second positive amplitude of oscillation given that the first positive amplitude is 0.25 m.

(4marks)

The mass, spring and damper system is now subjected to a forced vibration, which consists of a force with magnitude 300N and a forcing frequency of 10 Hz.

f) Calculate the amplitude of the resulting vibration and the phase angle that exists between the forcing frequency and that of the resulting system.

(8marks)

Total for Q5 – 25 marks

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Q6. A pressure vessel is made of a ductile material is subjected to a two-dimensional stress system as in Figure 4a below. The stress in x direction is 60 MPa in tension, 30 MPa compressive in y direction, and shear stress of 20 MPa in clockwise and anticlockwise direction.

a) Determine, by calculation, the direct and shear stress on the plane AB which is inclined 30 degrees from the y direction, as shown in figure 4a.
(5marks)

b) Find the inclination of the principal plain for which maximum normal stress occurs and the inclination of the principal plain for which shear stress occurs.
(6marks)

c) Hence calculate the principal normal stresses and maximum shear stress.
(6marks)

A rosette strain gauge is now connected to the surface of the vessel. The angles between gauges are 45 degrees as shown in Figure 4b. The pressure vessel undergoes new loading and the readings on gauges a, b and c are:
250μstrain, -120μstrain and 375μstrain respectively.

d) Determine the orientation of the plain of principal strain with respect to the x axis and then find the principal strains.
(8marks)

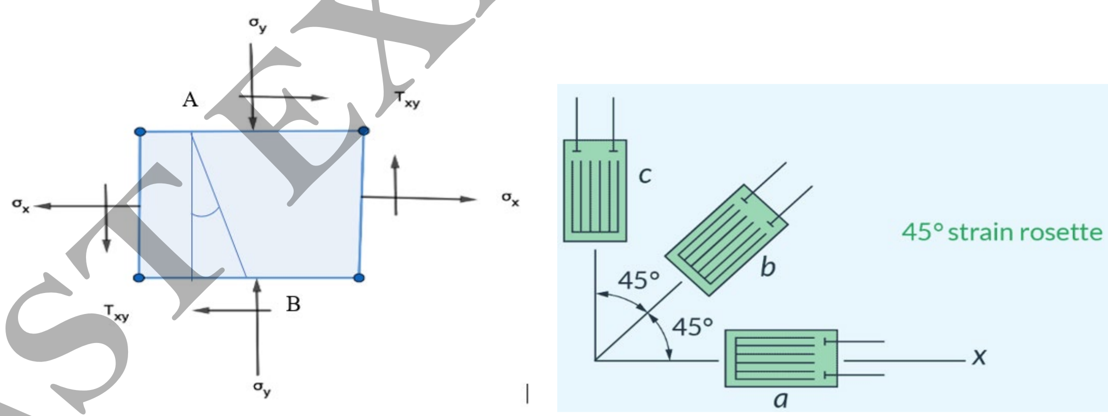

Figure 4a, b -2D Stress system and 45-degree strain rosette

Total for Q6 – 25 marks

END OF QUESTIONS
Formula Sheet follows over the page

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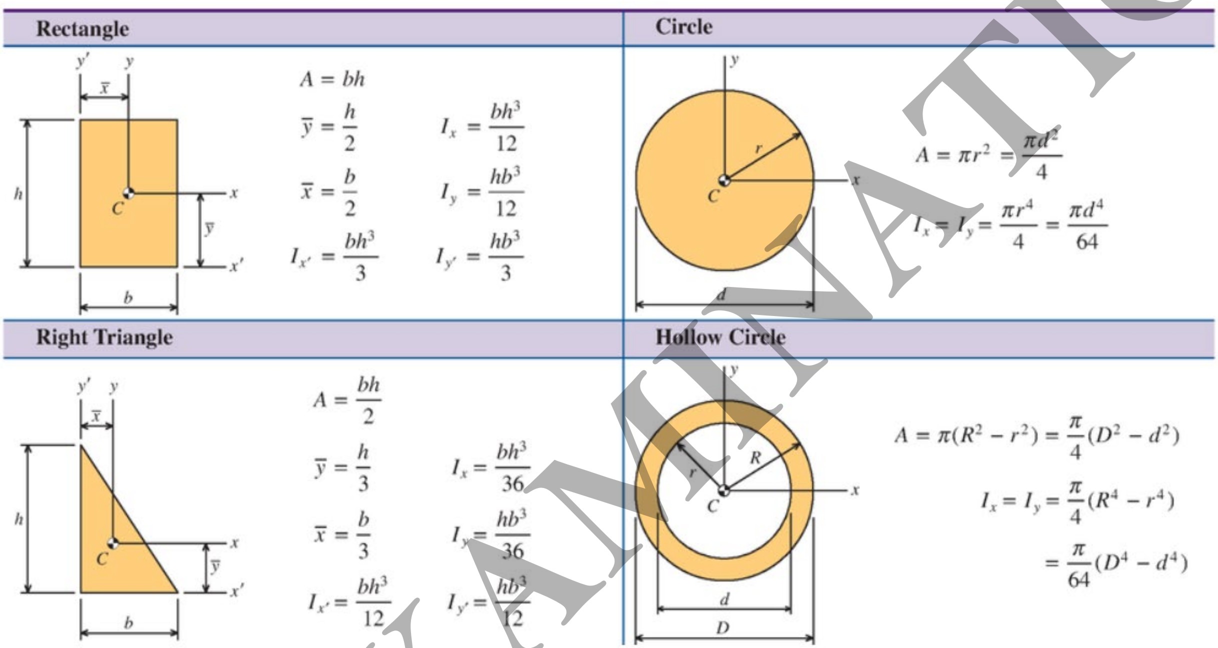
BEng (Hons) Automotive Performance Engineering (Motorsport)
Semester Two Examination 2023/24
Engineering Science II
Module No. MSP5016

FORMULA SHEET

Bending And Deflection

2nd moment of area for basic shapes:



Bending equation: $\sigma = \frac{My}{I} = \frac{E}{\rho}$

Beam deflection (v): $\frac{d^2 v}{dx^2} = \frac{M(x)}{EI}$

Method of moments for centroid: $\bar{x} = \frac{\sum a_i x_i}{\sum a_i}$ $\bar{y} = \frac{\sum a_i y_i}{\sum a_i}$

Parallel axis theorem: $I_{xx} = I_{\overline{xx}} + Ay^2$ $I_{yy} = I_{\overline{yy}} + Ax^2$

Product Inertia: $I_{xy} = \sum I_{\overline{xy}} + \sum A_{xy}$

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For principal Inertias: $I_{u,v} = \frac{1}{2}(I_{xx} + I_{yy}) \pm \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$

$$I_u + I_v = I_{xx} + I_{yy}$$

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

Pressure Vessels

Thin-Walled Pressure Vessels (sphere): $\sigma_T = \frac{Pr}{2t}$

Thin-Walled Pressure Vessels (cylinder): $\sigma_h = \frac{Pr}{t}$ $\sigma_L = \frac{Pr}{2t}$

For Thick-Walled vessels stresses and strains:

$$\sigma_h = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2}$$

$$\sigma_l = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

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$$\varepsilon_L = \frac{1}{E}[\sigma_L - v(\sigma_R + \sigma_h)]$$

$$\varepsilon_h = \frac{1}{E}[\sigma_h - v(\sigma_L + \sigma_r)]$$

$$\varepsilon_r = \frac{1}{E}[\sigma_r - v(\sigma_h + \sigma_L)]$$

Vibrations

$$f = \frac{1}{T}$$

$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}}$$

$$c = \zeta 2m\omega_n \qquad c_c = 2m\omega_n$$

$$\zeta = \frac{c}{c_c}$$

$$f_{damp} = f_n\sqrt{1-\zeta^2}$$

$$\Delta = ln\frac{x_1}{x_r} = \frac{2\pi(r-1)\zeta}{\sqrt{1-\zeta^2}}$$

$\left(\frac{x_1}{x_2}\right) = e^{\zeta\omega_n(r-1)T}$ where; $r = position\ of\ maxima$

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Forced Vibration

$$X_0 = \frac{\frac{F}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$\varphi = tan^{-1}\left(\frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right)$$

Struts

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

$$Slenderness\ ratio(SR) = \frac{L_e}{k}$$

Case	End conditions	Equivalent length, l_e	Buckling load, Euler
1	Both ends hinged or pin jointed or rounded or free	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	One end fixed, other end free	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	One end fixed, other end pin jointed	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	Both ends fixed or encastered	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

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$$\text{Critical Euler stress } = \frac{\pi^2 E}{\left(\frac{l_e}{k}\right)^2}$$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect.*

$$a = \frac{\sigma_c}{\pi^2 E}$$

Complex Stresses and Strains

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) cos(2\theta) - \tau sin(2\theta)$$

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) sin(2\theta) + \tau cos(2\theta)$$

$$tan(2\theta_p) = \frac{-2\tau}{\sigma_x - \sigma_y}$$

$$tan(2\theta_s) = \frac{\sigma_x - \sigma_y}{2\tau}$$

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$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Strain Rosette 45-degree

$$\varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \varepsilon_c$$

$$\gamma_{xy} = 2\varepsilon_b - (\varepsilon_a + \varepsilon_c)$$

$$tan(2\theta_p) = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$tan(2\theta_s) = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}$$

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\sigma_1 = \frac{E}{1 - v^2}(\varepsilon_1 + v\varepsilon_2)$$

$$\sigma_2 = \frac{E}{1 - v^2}(\varepsilon_2 + v\varepsilon_1)$$

END OF PAPER