[ENG34]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

MSc CIVIL ENGINEERING

SEMESTER TWO EXAMINATION 2023/2024

ADVANCED STRUCTURAL MODELLING, ANALYSIS AND DESIGN

MODULE NO: CIE7002

Date: **Thursday 16th May 2024** **Time: 2:00 – 5:00pm**

INSTRUCTIONS TO CANDIDATES:

There are FOUR questions.

Answer ALL Questions.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

Extracts from EC3 for Question 4 are provided in Appendices A and B.

School of Engineering
MSc Civil Engineering
Semester Two Examination 2023/2024
Advanced Structural Analysis, Modelling, and Design
Module No. CIE7002

Question 1

a) List three types of common mistakes that will cause a singular stiffness matrix.
(3 Marks)

b) For the beam shown in Figure Q1, derive the member stiffness matrices for elements 1-2 and 2-3 in the global axes.
(5 Marks)

c) Assemble the global stiffness matrix and global force vector and write the global system of equations in the global axes.
(3 Marks)

d) Apply the boundary conditions, and write the reduced global stiffness matrix for the beam.
(2 Marks)

e) Solve the reduced global stiffness matrix and find the angular rotations of nodes 2 and 3.
(5 marks)

f) Find the support reactions at nodes 1, 2, and 3. (3 Marks)

g) Draw the bending moment diagram of the beam and show the values of the hogging and sagging moments in the beam.
(4 marks)

The two beams have the same rigidity of $\mathbf{EI = 1\ kN.m^2}$

Total 25 Marks

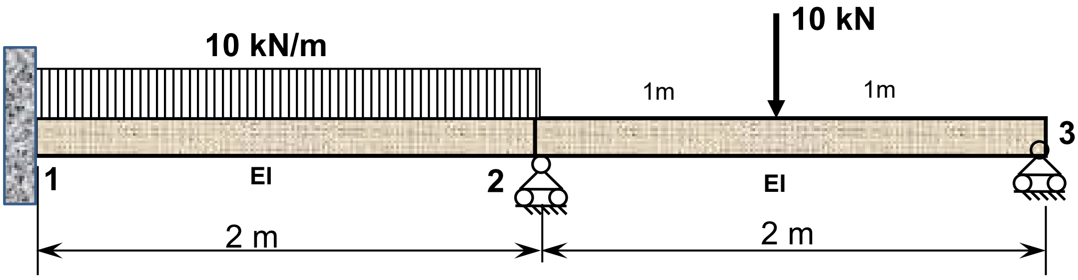


Figure Q1

Formulae to be used with Q1 are included in the next page.

Question 1 continued next page ….

Please turn the page ….

Question 1 continued….

The stiffness matrix for a beam of length L, rigidity EI, and excluding axial effect, is given by:

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

The force-displacement equation for a beam is:

$$\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{\delta}\} + \{\boldsymbol{F_e F}\}$$

Fixed end forces for beam with uniform load ***w***: $\{\boldsymbol{F_e F}\} = \begin{Bmatrix} -\boldsymbol{wl}/\boldsymbol{2} \\ +\boldsymbol{wl^2}/\boldsymbol{12} \\ -\boldsymbol{wl}/\boldsymbol{2} \\ -\boldsymbol{wl^2}/\boldsymbol{12} \end{Bmatrix}$

Fixed end forces for beam with point load ***P*** at mid-span: $\{\boldsymbol{F_e F}\} = \begin{Bmatrix} -\boldsymbol{P}/\boldsymbol{2} \\ +\boldsymbol{PL}/\boldsymbol{8} \\ -\boldsymbol{P}/\boldsymbol{2} \\ -\boldsymbol{PL}/\boldsymbol{8} \end{Bmatrix}$

Inverse matrix (2x2) of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

Please turn the page ….

Question 2

a) Explain how nodal numbering can improve the speed and efficiency of a finite element analysis. (3 marks)

b) Identify the symmetry lines in the two-dimensional truss shown in Figure Q2a and explain your choice.
Discuss the advantages and disadvantages of whether it is possible to reduce the complete truss to one half or one quarter before laying out a finite element model. Draw the reduced finite element model indicating, with rollers or fixed supports, which kind of displacement boundary conditions you would specify on the symmetry line.

(5 Marks)

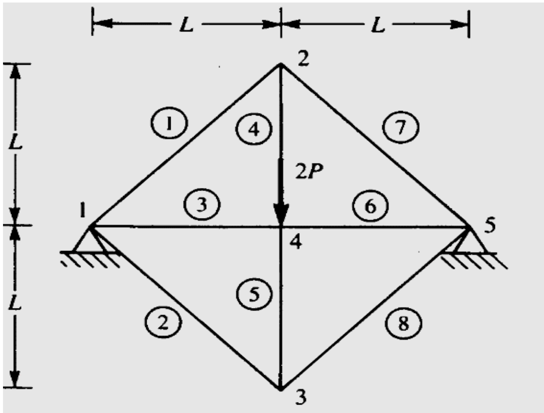


Figure Q2a

c) For the beam shown in Figure Q2b, the nodal displacements have been calculated in meters and radians as:

$$\{u\} = \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.0076 \\ -0.01 \\ 0 \\ 0 \\ 0.0076 \end{Bmatrix}$$

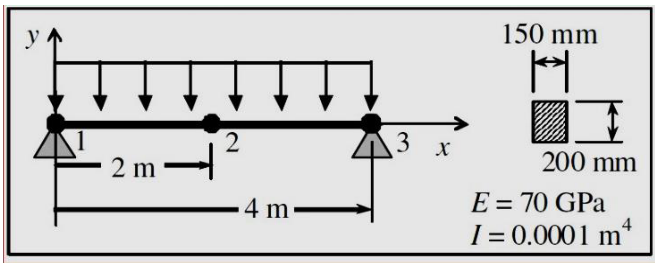


Figure Q2b

i) Calculate the values of the shape functions for the beam for x = 1m and x = 2m (3 marks)

Question 2 continued next page ….
Please turn the page ….

Question 2 continued

ii) Plot (sketch) the vertical displacement $v(x)$ for the beam (both elements). (5 marks)

iii) Plot the rotation $\theta(x)$ of the beam (both elements). (6 marks)

iv) Calculate the value of the bending moment at node 2. (3 marks)

Total 25 Marks

The shape functions for the beam element are given by:

$$N = [N_1 \quad N_2 \quad N_3 \quad N_4]$$

$$N_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \qquad N_2 = x - \frac{2x^2}{L} + \frac{x^3}{L^2} \qquad N_3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \qquad N_4 = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

The vertical deflection and rotation vectors for beam element are given by:

$$v(x) = [N]\{u\} \quad \text{and} \quad \{u\} = \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

$$\theta(x) = \frac{dv}{dx} = \frac{d[N]}{dx}\{u\} = [B]\{u\}$$

The bending moment and stress of a beam element are given by:

$$M = EI\frac{d^2v}{dx^2} = EI\frac{d^2[N]}{dx^2}\{u\}$$

Please turn the page ….

Question 3

A 2-node bar element has length L and Young's modulus E. The cross-sectional area A varies linearly from A1 to A2 as shown in Figure Q3a and is given by:

$$A(x) = \left(1 - {}^{x}/_{L}\right)A_1 + \left({}^{x}/_{L}\right)A_2$$

i) Show that the stiffness matrix of the bar element is:

$$[k] = \frac{E(A_1 + A_2)}{2L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

You will have to integrate:

$$[K] = \int_0^L \mathrm{B^T(x)EA(x)B(x)dx}$$

The shape function of a 2-node bar element is: $\boldsymbol{N} = \left[\left(\mathbf{1} - {}^{\boldsymbol{x}}/_{\boldsymbol{L}}\right), \left({}^{\boldsymbol{x}}/_{\boldsymbol{L}}\right)\right]$

The displacement function vector is $\boldsymbol{u}(\boldsymbol{x}) = [\boldsymbol{N}]\begin{Bmatrix} \boldsymbol{u_1} \\ \boldsymbol{u_2} \end{Bmatrix}$ and $\boldsymbol{B}(\boldsymbol{x}) = \frac{\boldsymbol{dN}}{\boldsymbol{dx}}$

(10 marks)

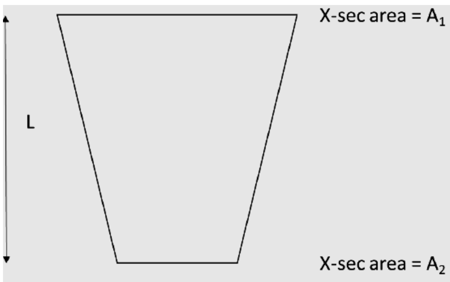


Figure Q3a: Elevation view of the bar

ii) A tapered bar has length 2L and its cross-sectional area varies linearly from **3A** to **2A**, as shown in Figure Q3b. The thick end of the bar is fixed and a weight **W** hangs from the free end. The weight of the bar is negligible.
Using 2-2node bar elements, calculate the displacements and stresses throughout the bar, and the reaction at the ceiling.

(15 marks)

Stress $\sigma = E\varepsilon$

Strain $\varepsilon_1 = (u_2 - u_1)L$, $\varepsilon_2 = (u_3 - u_2)L$

Question 3 continued on next page…

Please turn the page ….

Question 3 continued….

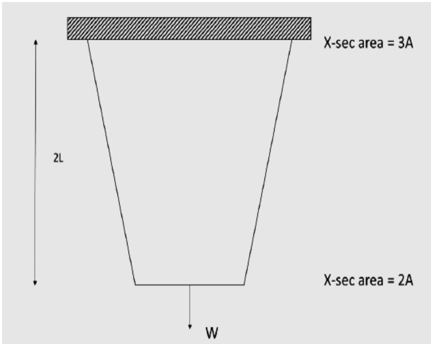


Figure Q3b

Total 25 Marks

Question 4

The frame of an office is shown in Figure Q4a; it consists of steel beams and columns on a 7m x 7m grid. Resistance to sway is provided on each 49 m long side by two 3.5m braced bays, as shown in Figure Q4a; bracing is also provided parallel to the 28 m side, but this is not considered in this question.
Design Data: (loads are factored)
Total design vertical load on roof is 5.88 kN/m^2
Total design vertical load on floor is 13.88 kN/m^2
Design wind loads on one bracing is shown in Figure Q4b.

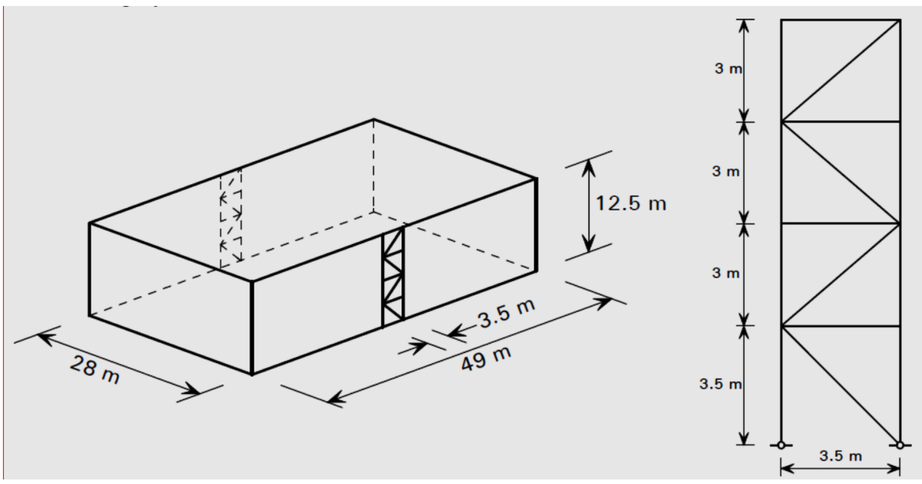


Figure Q4a – Building dimensions and braced frame

Question 4 continued next page ….
Please turn the page ….

Question 4 continued….

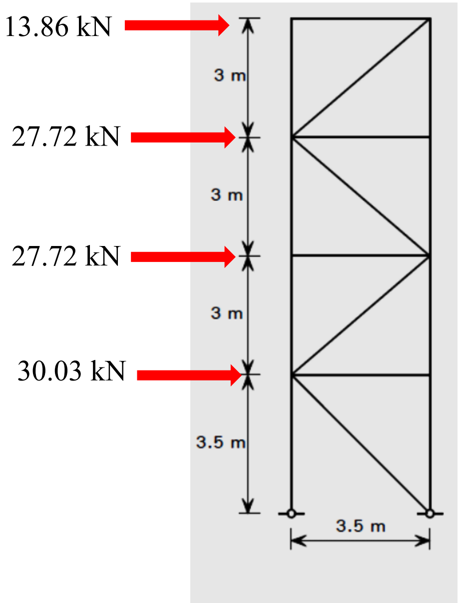


Figure Q4b – Wind loads on bracing

Required tasks:

a) Calculate the global initial sway imperfections factor **ϕ** of the frame. Assume number of columns in a row is 15.

(3 Marks)

b) Calculate the equivalent horizontal forces (EHF) due to the sway imperfections in each floor level.

(5 Marks)

c) By using the Eurocode 3 method, compute the sensitivity factor **α_{cr}**, and check whether second order effects have to be considered or not. Comment on the results.

(10 Marks)

The lateral displacements due to the horizontal loads at each floor level are:

Level	Lateral Displacement (mm)
Roof	16.1
3rd Floor	13.1
2nd Floor	9.3
1st Floor	5.1

Question 4 continues over the page….

Please turn the page ….

Question 4 continued….

d) The middle column at the ground floor is subjected to an axial design load of N_{Ed} = 1835 kN. The column is assumed to be pinned at both ends.

i) Determine the buckling resistance of the column.

(7 Marks)

Column size is a hot finished **200x200x10 SHS** of steel grade S355.

A= 74.9 cm^2
h = 200 mm
t = 10 mm
$I_y = I_z$ = 4470 cm^4
E = 210 kN/mm^2
f_y = 355 N/mm^2 , Partial safety factor $\gamma_{M1} = 1$
Critical load $N_{cr} = \frac{\pi^2 EI}{L_{cr}^2}$

Total 25 Marks

Extracts from EC3 to be used with Question 4a, (b), and (c) are included in Appendix A and Extracts from EC3 to be used with Question 4d are included in Appendix B over the page….

END OF QUESTIONS

Please turn the page for formula sheets ….

School of Engineering
MSc Civil Engineering
Semester Two Examination 2023/2024
Advanced Structural Analysis, Modelling, and Design
Module No. CIE7002

Appendix A

Formulae sheet for instability of steel structures using Eurocode 3 to be used with Question 4(a), (b), and (c).

Global initial sway imperfections

$$\emptyset = \emptyset_0 \alpha_h \alpha_m$$

$$\alpha_0 = \frac{1}{200}$$

$\alpha_h = \frac{2}{\sqrt{h}};$ with $0.66 \leq \alpha_h \leq 1$, *h* is the height of the structure.

$\alpha_m = \sqrt{0.5(1 + \frac{1}{m})}$, m is the number of columns in a row.

Equivalent horizontal force at each floor level, EHF = $\emptyset \times Design\ Vertical\ Load$

Sensitivity to sway, α_{cr}

$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}}\right)\left(\frac{h}{\delta_{H,Ed}}\right)$$

H_{Ed} is the horizontal force.
V_{Ed} is the total deign vertical load on the structure on the bottom of the storey.
$\delta_{H,Ed}$ is the horizontal sway at the top of the storey due to the applied horizontal loads.
h is the storey height.

$\mathbf{3 \leq \alpha_{cr} \leq 10}$

sway effects cannot be ignored, wind and equivalent horizontal loads to be increased by the amplification factor:

$$\frac{\mathbf{1}}{\mathbf{1}-{}^{\mathbf{1}}/_{\boldsymbol{\alpha_{cr}}}}$$

Please turn the page ….

School of Engineering
MSc Civil Engineering
Semester Two Examination 2023/2024
Advanced Structural Analysis, Modelling, and Design
Module No. CIE7002

Appendix B
Extract from EC3 to be used with Question 4(d).

6.3 Buckling resistance of members

6.3.1 Uniform members in compression

6.3.1.1 Buckling resistance

(1) A compression member shall be verified against buckling as follows:

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1,0 \qquad (6.46)$$

where

N_{Ed} is the design value of the compression force
$N_{b,Rd}$ is the design buckling resistance of the compression member.

(3) The design buckling resistance of a compression member should be taken as:

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{for Class 1, 2 and 3 cross-sections} \qquad (6.47)$$

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} \quad \text{for Class 4 cross-sections} \qquad (6.48)$$

where χ is the reduction factor for the relevant buckling mode.

NOTE For determining the buckling resistance of members with tapered sections along the member or for non-uniform distribution of the compression force second-order analysis according to 5.3.4(2) may be performed. For out-of-plane buckling see also 6.3.4.

(4) In determining A and A_{eff} holes for fasteners at the column ends need not to be taken into account.

6.3.1.2 Buckling curves

(1) For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ should be determined from the relevant buckling curve according to:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \leq 1,0 \qquad (6.49)$$

where $\Phi = 0,5\left[1 + \alpha(\bar{\lambda} - 0,2) + \bar{\lambda}^2\right]$

$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}}$ for Class 1, 2 and 3 cross-sections

$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}}$ for Class 4 cross-sections

α is an imperfection factor
N_{cr} is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties.

(2) The imperfection factor α corresponding to the appropriate buckling curve should be obtained from Table 6.1 and Table 6.2.

Table 6.1 — Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

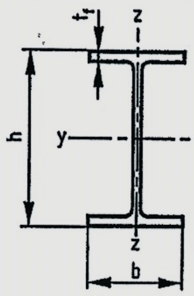
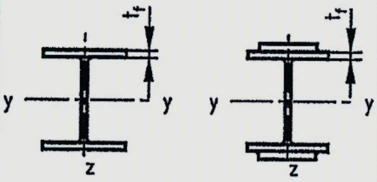
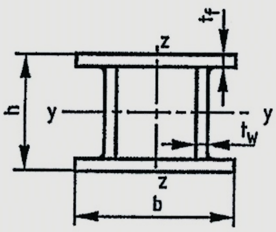
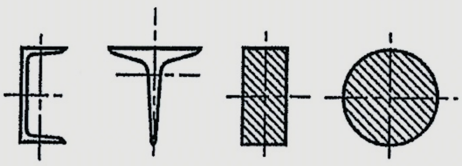
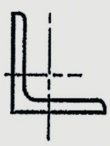
(3) Values of the reduction factor χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ may be obtained from Figure 6.4.

(4) For slenderness $\bar{\lambda} \leq 0,2$ or for $\frac{N_{Ed}}{N_{cr}} \leq 0,04$ the buckling effects may be ignored and only cross-sectional checks apply.

School of Engineering
MSc Civil Engineering
Semester Two Examination 2023/2024
Advanced Structural Analysis, Modelling, and Design
Module No. CIE7002

Appendix B continued …

Table 6.2 — Selection of buckling curve for a cross-section

Cross section		Limits		Buckling about axis	Buckling curve S 235, S 275, S 355, S 420	Buckling curve S 460
Rolled sections		$h/b > 1,2$	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			$40 \text{ mm} < t_f \le 100$	y – y z – z	b c	a a
		$h/b \le 1,2$	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c
Welded I sections		$t_f \le 40$ mm		y – y z – z	b c	b c
		$t_f > 40$ mm		y – y z – z	c d	c d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U, T and solid sections				any	c	c
L sections				any	b	b

END OF PAPER