ENG13

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) MECHANICAL ENGINEERING

SEMESTER 2 EXAMINATION 2023/2024

FINITE ELEMENT & DIFFERENCE METHODS

MODULE N^{O}: AME6016

Date: Thursday 16th May 2024 **Time: 10:00 – 12:00**

INSTRUCTIONS TO CANDIDATES:

There are FOUR questions

Attempt ANY THREE questions.

All questions carry equal marks.
75 marks equates to 100%

Marks for parts of questions are shown in brackets.

Formula Sheet is attached in the APPENDIX at the end of the paper

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Q1

a)

A loading press can be schematically modeled by three finite elements connected in series as shown in Fig Q1. Each element is assumed to be manufactured from steel, E=205 GPa and L= 200 mm in length**.** The Area A is x 1000mm^2 in cross-section.

Using the FEM and 1D, determine the change in length of the middle block and the stresses in each block if the maximum load F is 200 KN and P is zero.

(15 Marks)

b) Under alternative working conditions a load P is now applied and the displacement at this point is reduced by 50%, therefore determine the value of load P to satisfy this condition.

(10 marks)

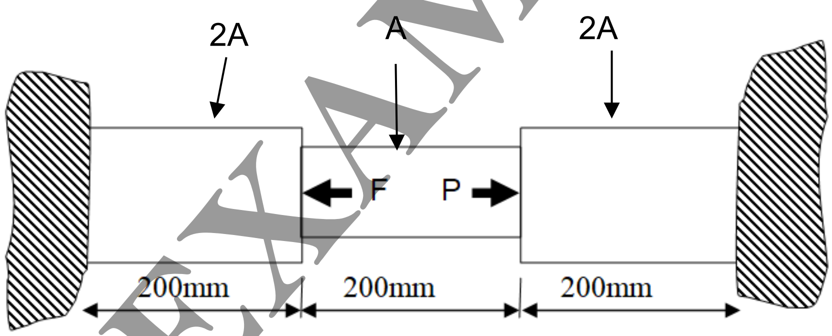

Fig Q1 Schematic set of loading arrangement

Total 25 Marks

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Q2

(a) Fig Q2a shows schematically part an earth quake monitoring system consisting of an end mass (M) and shaft length L which has two springs each with an equivalent stiffness $K = \frac{4EI}{L^3}$ connected to the end mass. One end of the shaft is connected to the mass and the opposite end can be assumed to be fully constrained. All symbols have their usual meaning. The ith natural frequency of the system in rad/s can be approximated by the following expression:

$$\omega_i = \frac{C_i}{L^2}\sqrt{\frac{EI}{\rho A}}$$

Where: C_i is a constant associated with the ith natural frequency.

Using the finite element method and the appropriate element determine C_1 and C_2 for this system assuming the end mass is 1/6th of the mass of the shaft.

(16 marks)

(b) Derive also for the first mode an expression for the mode shape in terms of distance along the beam stating the origin of your coordinate system and show this satisfies the boundary conditions. **(6 marks)**

(c) If the mass of the springs were included state whether the frequencies would increase or decrease and give your reasons. **(3 marks)**

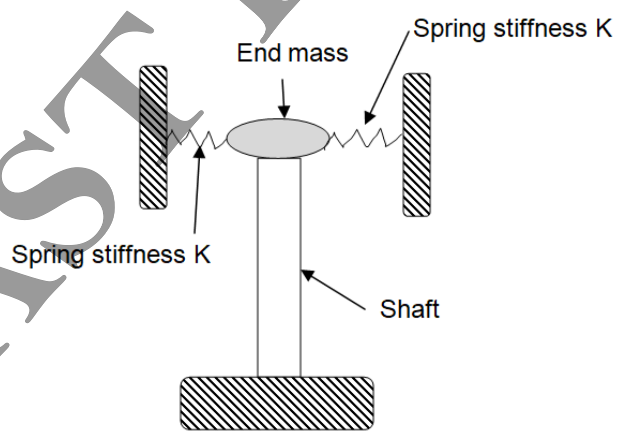

Fig Q2 Schematic view of the idealized monitoring system

Total marks (25)

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Q3

(i) Explain what the difference between explicit and implicit methods used in the finite element solution techniques.

(2 Marks)

(ii) Consider two problems, (a) a pipeline is subjected to low cyclic pressure load and (b) a bullet impacts upon an armour plate. State which method (implicit or explicit) applies to the two problems and explain your answer. **(2 Marks)**

(iii) Describe the difference between H and P methodologies using sketches and what is the HP method?

(6 Marks)

(iv) Explain the difference between averaged and unaveraged stresses when carrying out a static analysis. **(2 marks)**

(v) Why are the stresses normally estimated at the gauss points and not the node positions?

(3 Marks)

(vi) Why is the aspect ratio for a solid 3D finite element important in relation to the solution of the problem? **(3 marks)**

(vii) In a three-dimensional model uses an ten node isoparametric tetrahedral element to estimate the stresses, state what order of stress is associated with this element?

(2 Marks)

(viii) Explain also briefly how the finite element process uses a convergence criterion to evaluate the problem unknowns and how this would work if strain energy is the target function for a static analysis.

(5 Marks)

Total 25 Marks

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Q4: 1D - CFD heat transfer in a high-performance aerospace tube.

The equation provided below, models the temperature behaviour through the wall thickness of a prototype tube for aerospace application and, only a 1D steady-state heat transfer is considered.

$$\frac{dT}{dx} = \sqrt{T + x}$$

The tube, not insulated, faces an ambient temperature of 40°C and a convection coefficient of 110 W/m^2°C. With the tube dimensions of 5m in length and 1m in diameter, and a conductivity of 60 W/m°C. Given σ=5.67x10^{-8} W/m^2 °K^4, we are tasked to:

a) Apply the Finite Difference Method (FDM) to derive the steady-state equation for a tube divided into five segments, assuming an initial temperature of 900°C for the first segment. Also, calculate the resulting temperature distribution. **(12 marks)**

b) Calculate and analyse the radiation and convection heat transfer rate at the hottest segment's surface. **(7 marks)**

c) Determine and discuss the temperature gradient at the hottest segment's surface. **(6 marks)**

Total 25 Marks

END OF QUESTIONS

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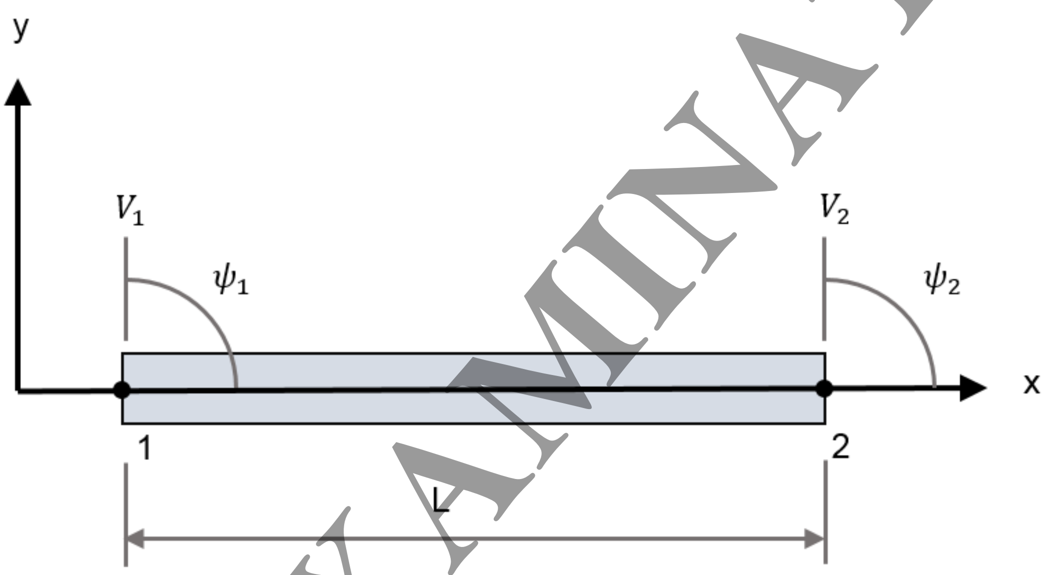
FORMULAE SHEET

Dynamics

$$(-\boldsymbol{\omega}^2\,[\boldsymbol{m}] + [\boldsymbol{K}])\,\{\boldsymbol{u}\} = \mathbf{0}$$

Finite Element Notation for 2D Beams with 2 Nodes and 4 DOF:



Element Consistent Mass Matrix

$$[m]^{\mathrm{e}} = \frac{\rho AL}{420}\begin{bmatrix} \mathbf{156} & \mathbf{22L} & \mathbf{54} & \mathbf{-13L} \\ & \mathbf{4L^2} & \mathbf{13L} & \mathbf{-3L^2} \\ & & \mathbf{156} & \mathbf{-22L} \\ & & & \mathbf{4L^2} \end{bmatrix}$$

Element Stiffness Matrix

$$[K]^{\mathrm{e}} = \frac{EI}{L^3}\begin{bmatrix} \mathbf{12} & \mathbf{6L} & \mathbf{-12} & \mathbf{6L} \\ & \mathbf{4L^2} & \mathbf{-6L} & \mathbf{2L^2} \\ & & \mathbf{12} & \mathbf{-6L} \\ & & & \mathbf{4L^2} \end{bmatrix}$$

Element Displacement Functions

$$v(x) = \left[1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}, x - \frac{2x^2}{L} + \frac{x^3}{L^2}, \frac{3x^2}{L^2} - \frac{2x^3}{L^3}, -\frac{x^2}{L} + \frac{x^3}{L^2}\right]$$

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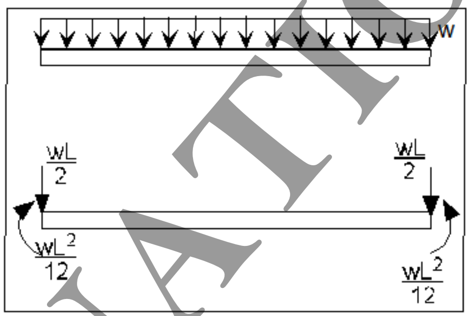
Beam bending stress at x along the beam element is given by, $\sigma(x,y) = -Ey\frac{\partial v^2(x)}{\partial x^2}$

Equivalent nodal force due to Uniformly distributed load w



1- D Bar Element

$$\boldsymbol{U(x) = \left(1 - \frac{x}{L}\right)U_1 + \left(\frac{x}{L}\right)U_2}$$

1-D Beam Deflection Equation

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_i}{\Delta x}$$

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_i} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

Convection:

$\dot{Q} = hA(T_s - T_a)$

Radiation

$$\dot{Q} = \varepsilon\sigma A({T_s}^4 - {T_a}^4)$$

Conduction:

$$\dot{Q} = kA\frac{dT}{dx}$$

END OF FORMULA SHEET

END OF PAPER