UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE FZE

BENG (HONS) MECHANICAL ENGINEERING

SEMESTER TWO EXAMINATION 2023/24

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

MODULE NO. AME6016

Date: 9th May 2024 **Time: 10:00am-12:00**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions on the paper.

Answer any FOUR questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is erased or cleared prior to the examination.

CANDIDATES REQUIRE: **A Formula Sheet (attached)**

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Semester 2 Examination 2023/24
Finite Element and Difference Solutions.
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Question 1

a) Explain the fundamental steps involved in the Finite Element Method (FEM) and identify five key engineering fields where FEM is extensively utilized. Additionally, elaborate on the advantages and disadvantages of employing FEM. **(10 Marks)**

b) For the spring assemblages shown in **Figure Q1** below, the springs are arranged in series with node 1 and 4 as fixed. Use the direct stiffness method for problems.

Given the following k_1 = 100 N/mm, k_2 = 200 N/mm, k_3 = 100 N/mm, P=500N.

Determine the following,

1) The global stiffness matrix **(5 Marks)**
2) Displacements of nodes 2 and 3 **(5 Marks)**
3) The reaction force at nodes 1 and 4 **(5 Marks)**

Total 25 Marks

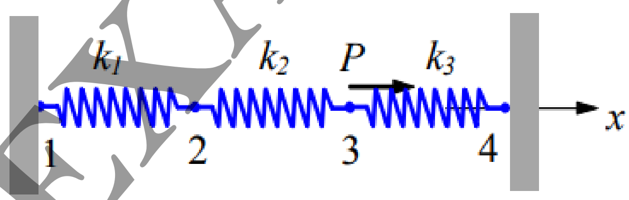

Figure Q1: Spring assemblages

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Question 2

For the Beam Element shown in the **figure Q2**.

- **a.** Draw the finite element model under the given load condition. **(05 Marks)**
- **b.** Determine the Global Stiffness matrix. **(10 Marks)**
- **c.** Determine deflection under the given load. **(10 Marks)**

Total 25 Marks

Take $E = 2 \times 10^8$ kN/m^2 and $I = 4 \times 10^{-6}$ m^4.

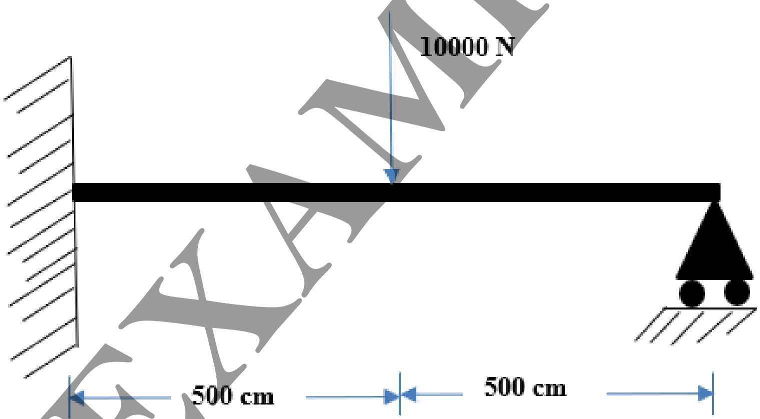

Figure Q2: Beam Element

Question 3

A compound bar is loaded as shown in **Figure Q3**, E_{steel} = 200GPa, E_{cu} = 100GPa, using penalty method of boundary condition,

- **a)** Develop the individual stiffness matrix for Steel and Aluminium **(5 Marks)**
- **b)** Develop the global stiffness matrix using the method of superposition **(5 Marks)**
- **c)** Determine the displacement at nodes **(5 Marks)**
- **d)** Determine the stresses in each element **(5 Marks)**
- **e)** Determine the reaction at the support **(5 marks)**

Total 25 marks

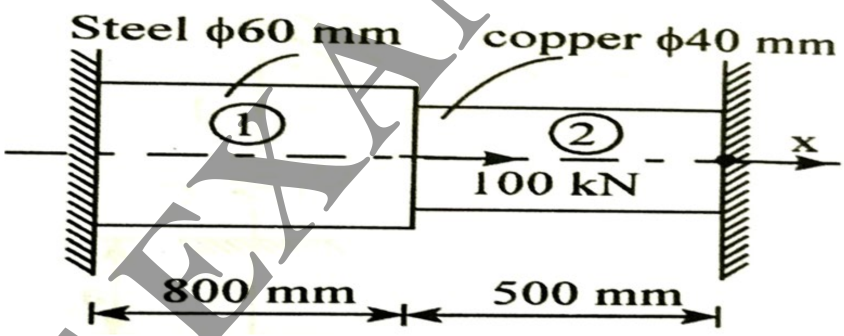

Figure Q3: Assembly of steel and copper pipes

Question 4

An induction furnace wall is made up of three layers which include inside, middle and outer layer with thermal conductivity K_1= 8.5W/mK, K_2= 0.25W/mK, K_3= 0.088W/mK, convective heat transfer coefficient, h=45W/m^{2}K, outside temperature T_∞=30^{0}C, as shown in **Figure Q4** below, where T_1 is the internal temperature of the furnace, T_2, T_3 are the intermediate temperature of the furnace walls and T_4 is the temperature at the last node in Celsius.

Determine the following.

a) Individual stiffness matrix for the heat conduction. **(9 Marks)**

b) Global stiffness matrix for the system. **(6 Marks)**

c) Nodal Temperatures. **(10 Marks)**

Total 25 Marks

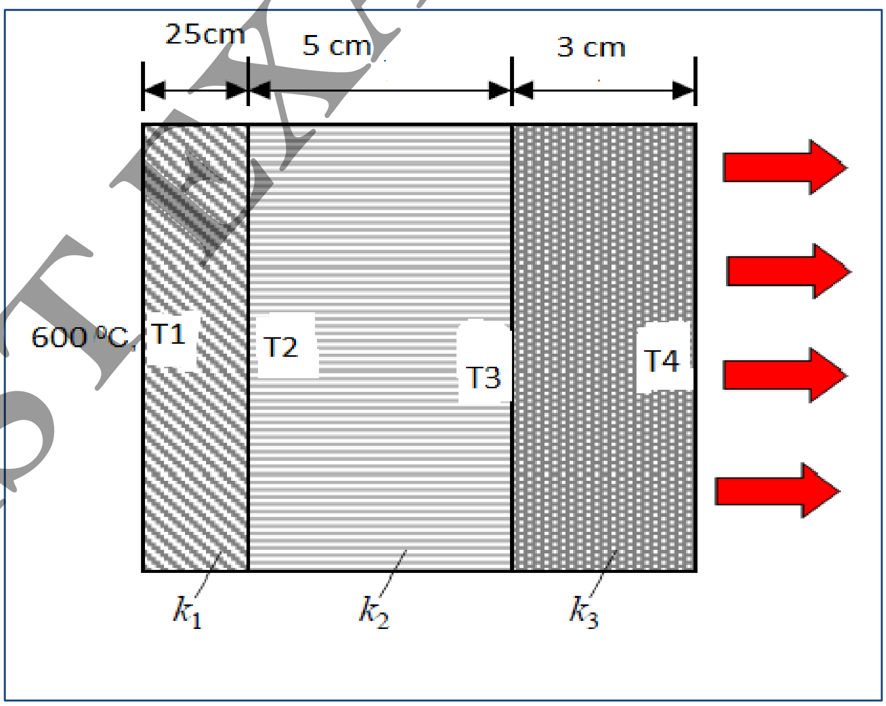

Figure Q4: Induction furnace with three layers.

Question 5

Figure Q5 represents a two-dimensional Truss element subjected to point loads. The truss is simply supported at the ends and all the joints are pin joints (node 1 & 3) and is subjected to forces X, Y & Z kN that are co-planar (at node 2 and 4). Distances A= 4m, B= 4m and C =4m and Load X=10 kN, Y = 12 kN, Z= 10 kN can be prescribed as per standard truss problem.

Find the displacement at each node.

Take E= 2.1×10^{11} N/m2, Diameter D = 3cm.

Total 25 Marks

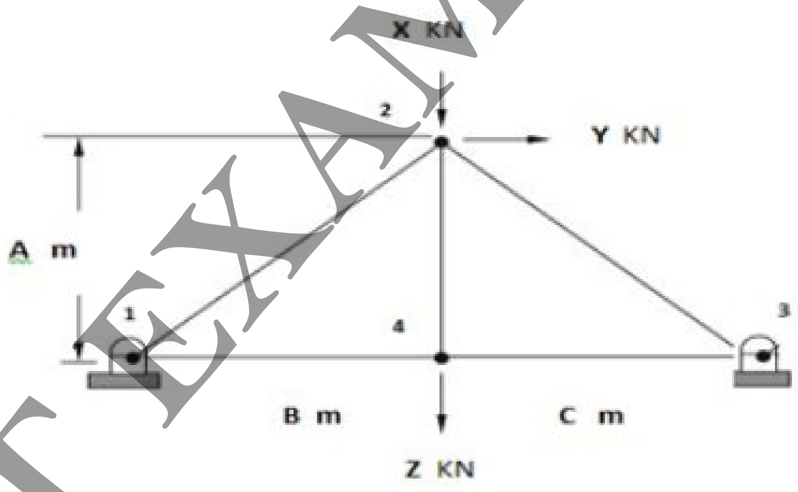

Figure Q5: Two-Dimensional Truss Element

END OF QUESTIONS

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FORMULA SHEET

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

Stiffness matrix for Bar/Truss Element

$$[\boldsymbol{K_C}] = \frac{\boldsymbol{AE}}{\boldsymbol{L}}\begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}$$

Reaction at the support

$R_n = -C[q_n - a_n]$

Stiffness matrix for heat conduction.

$$[\boldsymbol{K_C}] = \frac{\boldsymbol{AK}}{\boldsymbol{L}}\begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}$$

Stiffness Matrix for Beams:

$$\mathbf{K} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Stiffness matrix due to convection.

$$[K_h] = hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Force vector due to free end convection

$$[F_h] = hA\,T_\infty \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Element conduction matrix.

$$[k] = \frac{AK_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Elemental Force Matrix.

$$\{f\} = \frac{QAL + q^*PL + hT_\infty PL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Elemental stress.

$$\underline{\sigma} = \underline{C}'\underline{d} \qquad \underline{C}' = \frac{E}{L}[-C \quad -S \quad C \quad S]$$

Lumped Mass Matrix

$$[\hat{m}] = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Frequency.

$$f_1 = \omega_1 / 2\pi$$

$$|\underline{K} - \omega^2 \underline{M}| = 0$$

END OF FORMULA SHEET