UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE

BENG(HONS) ELECTRICAL AND ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATION 2023/24

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Saturday 6 January 2024 **Time: 10:00 AM – 12:30 PM**

INSTRUCTIONS TO CANDIDATES: **There are FIVE questions on this paper.**

Answer ANY FOUR questions.

All questions carry equal marks.

Question 1

a) Find the electric field resulting from a given electric potential.

$$V = 6xy - 2xz + z$$

(7 marks)

b) Given that electric flux density

$$D = z\rho Cos^2 Ø\, a_z C/m^2$$

Calculate the charge density at (1, П/4,3) and the total charge enclosed by the cylinder of radius 1m with $-2 \leq z \leq 2m$.

(8 marks)

c) If $J = \frac{1}{r^3}(2\ \cos\cos\theta\ a_r + \sin\sin\theta\ a_\theta)A/m^2$, calculate the current passing through a hemispherical shell of radius 20cm, 0$\leq \Pi \leq \frac{\Pi}{2}$, 0$< Ø < 2\Pi$.

(5 marks)

d) Given the magnetic vector potential

$$A = \frac{-\rho^2}{4}\, a_z Wb/m$$

Calculate the total magnetic flux crossing the surface

$$Ø = \Pi/2, 1 \leq \rho \leq 2m, 0 \leq z \leq 5m$$

(5 marks)

Total 25 marks

Please turn the page

Question 2

Find the amplitude of the displacement current density.

i) adjacent to an automobile antenna where the magnetic field intensity of an FM signal is

$$H_x = 0.15\, Cos\,[3.12\,(3 *\ 10^8 t - y)] A/m$$

(4 marks)

ii) in the air space at a point within a large power distribution transformer, when

$$B = 0.8\, Cos\,[1.257 *\ 10^{-6}(3 * 10^8 t - x)]\overrightarrow{y} T$$

(6 marks)

iii) within a large, oil filled power capacitor where $\epsilon_r = 5$ and

$$E = 0.9\, Cos\left[1.257 *\ 10^{-6}\left(3 *\ 10^8 t - z\sqrt{5}\right)\right]\overrightarrow{x}\ MV/m$$

(7 marks)

iv) in a metallic conductor at 60Hz, if $\epsilon = \epsilon_0\,, \mu = \mu_0, \sigma = 5.8 *\ 10^7 S/m$ and

$$J = sin\, sin\,(377t - 117.1z)\,\overrightarrow{x}\, MA/m^2$$

(8 marks)

Total 25 marks

Question 3

a) The electric field of an electromagnetic wave is given as

$$\vec{E} = E_0 \hat{j}\, sin\, \frac{\Pi z}{z_0}\, cos\, cos\,(kx - \omega t)$$

Question 3 continued over…
Please turn the page

Question 3 continued…

Evaluate the field given above.

(4 marks)

b) An electromagnetic wave propagates along the x direction while the magnetic field oscillates at a frequency of 10^{10} Hz and has an amplitude of 10-5T, acting along the y direction. Compute the wavelength of the electromagnetic wave and provide the expression for electric field in this case.

(6 marks)

c) The magnetic field of a plane electromagnetic wave is described as follows

$$\vec{B} = B_O \, sin \, sin \, (kx - \omega t \,)\hat{j}$$

i) Calculate the wave's wavelength λ.

(2 marks)

ii) Compute the electric field E corresponding to the magnetic field. Illustrate the direction of the unit vector.

(4 marks)

iii) Determine the magnitude and direction of the Poynting vector related to this wave.

(3 marks)

Question 3 continued over…

Please turn the page

Question 3 continued…

d) Compute the energy intensity of the standing electromagnetic wave given by

$E_y(x,t) = 2E_0\ cos\ cos\ kx\ cos\omega t$ and $B_Z(x,t) = 2B_0\ sin\ sin\ kx\ sin\omega t$

(6 marks)

Total 25 marks

Question 4.

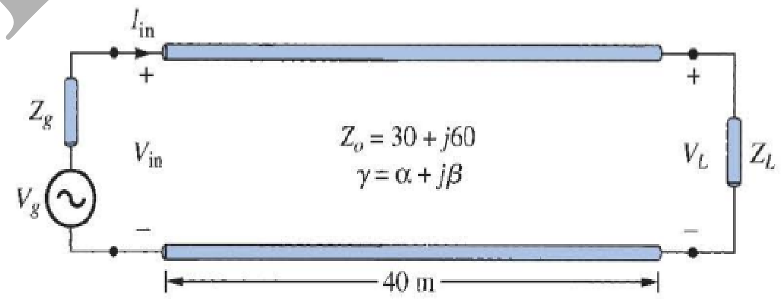
a) Consider a lossless transmission line with a characteristic impedance of 75 Ω. The line is terminated with a load impedance of 100+j150 Ω.

i. Calculate the reflection coefficient at the load, the standing wave ratio on the transmission line, and the input impedance at a distance of 0.4 λ from the load. (7 marks)

ii. Evaluate the values obtained in part (i) using smith chart provided in **page 7**. Determine the locations of the first minimum voltage and the first maximum voltage from the load. (10 marks)

iii.

b)



Question 4 continued over…

Please turn the page

Question 4 continued…

The transmission line shown in Figure is 40 m long operating at 500MHz and has V_g = 15< 0° vrms, Z_0 = 30 + j60Ω, and V_L= 5 <-48° Vrms, Z_g = 0. If the line is matched to the load and Z_g = 0. Calculate the propogation constant and the sending-end current I_{in} and voltage V_{in}.

(8 marks)

Total 25 marks

Question 5

a) An antenna is designed with operating frequency 7 GHz, featuring a circular aperture with a diameter of 3 m. The antenna exhibits a radiation resistance of 70 Ω and a loss resistance of 6 Ω.

i. Discuss the factors influencing the radiated power of the antenna. Given a current draw is 10 A, calculate the power radiated by the antenna.

(5 marks)

ii. Calculate key antenna characteristics, including capture area, gain in decibels, and directivity. Analyse how these metrics collectively contribute to defining the performance of the antenna.

(7 marks)

iii. Discuss the relationship between Q factor and the antenna's bandwidth and calculate the Q factor, taking into account the specified bandwidth of 4MHz.

(3 marks)

Question 5 continued over…

Please turn the page

Question 5 continued…

b) A radar operating in the S-band transmits at a frequency of 5 GHz with a power output of 400 kW.

i. Calculate the signal power density at distances of 100 and 400 nautical miles, considering the effective area of the radar antenna to be 12 m²

(7 marks)

ii. Consider a target with an effective area of 20 m² located at a range of 300 nautical miles. Evaluate the power of the reflected signal received by the radar from this target.

(3 marks)

Total 25 marks

END OF QUESTIONS

Please turn the page for Equations

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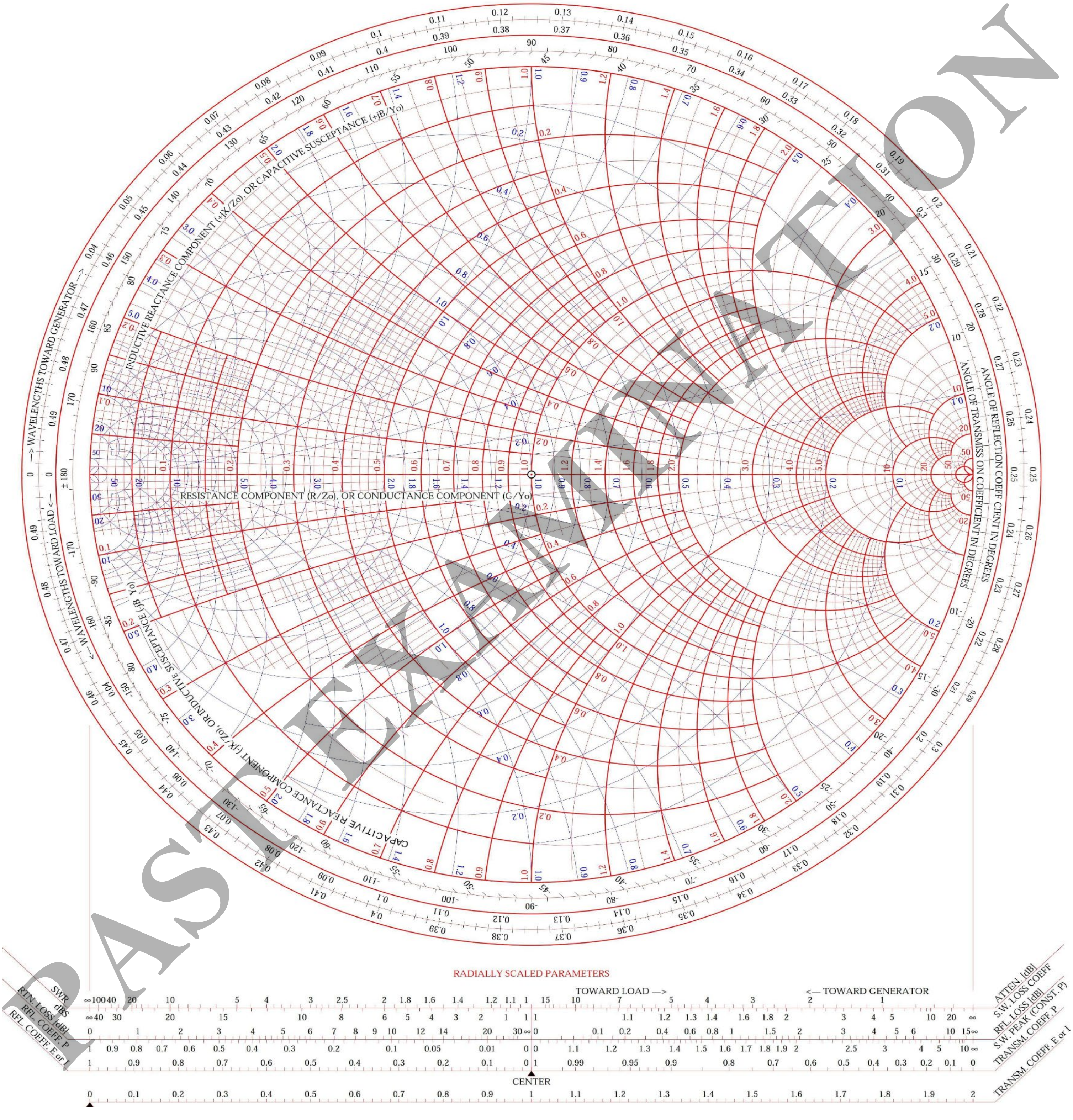


EQUATION SHEET

CIRCULAR CYLINDRICAL COORDINATES (ρ, ϕ, z)

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

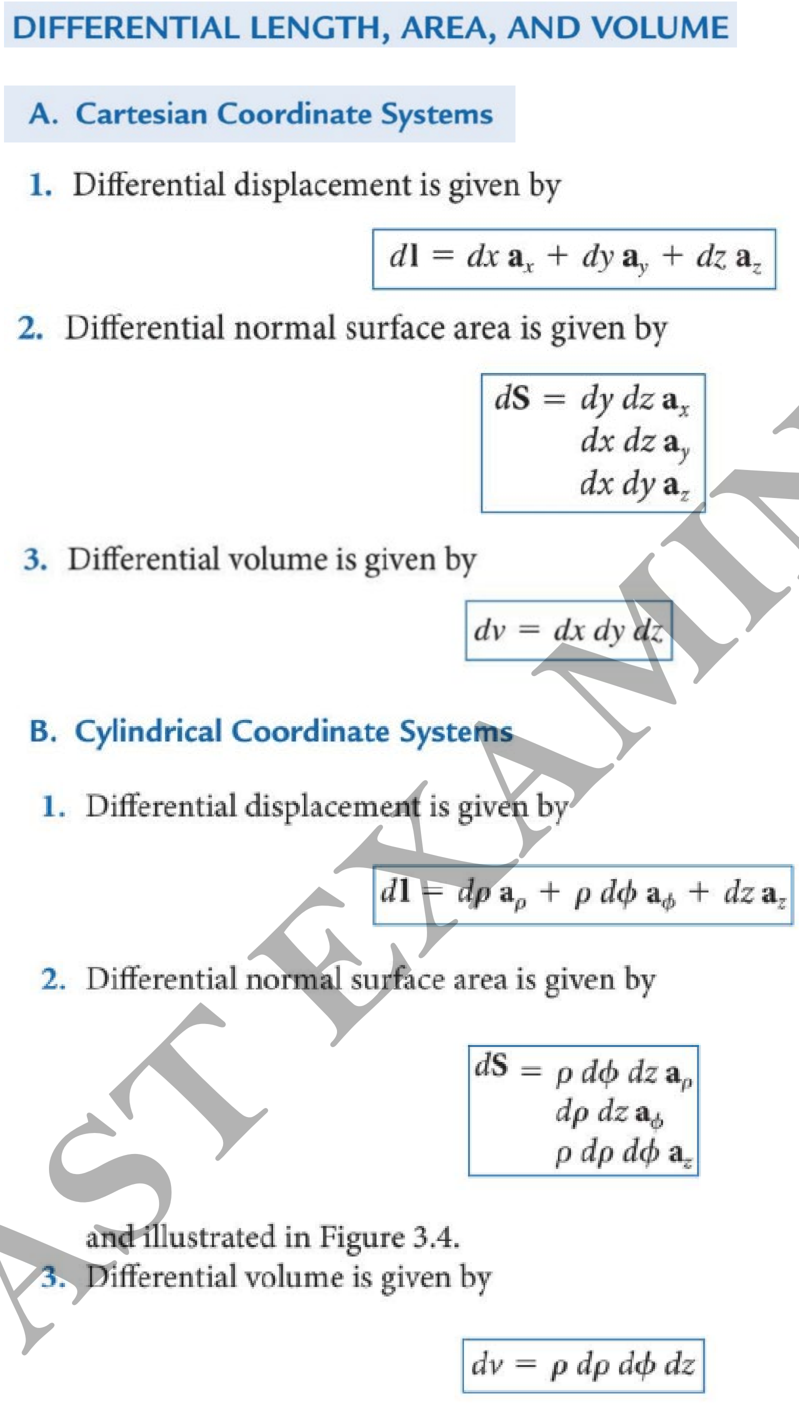
SPHERICAL COORDINATES (r, θ, ϕ)

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \quad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$



DIFFERENTIAL LENGTH, AREA, AND VOLUME

A. Cartesian Coordinate Systems

1. Differential displacement is given by

$$\boxed{d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z}$$

2. Differential normal surface area is given by

$$\boxed{\begin{aligned} d\mathbf{S} = {} & dy\,dz\,\mathbf{a}_x \\ & dx\,dz\,\mathbf{a}_y \\ & dx\,dy\,\mathbf{a}_z \end{aligned}}$$

3. Differential volume is given by

$$\boxed{dv = dx\,dy\,dz}$$

B. Cylindrical Coordinate Systems

1. Differential displacement is given by

$$\boxed{d\mathbf{l} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z}$$

2. Differential normal surface area is given by

$$\boxed{\begin{aligned} d\mathbf{S} = {} & \rho\,d\phi\,dz\,\mathbf{a}_\rho \\ & d\rho\,dz\,\mathbf{a}_\phi \\ & \rho\,d\rho\,d\phi\,\mathbf{a}_z \end{aligned}}$$

and illustrated in Figure 3.4.

3. Differential volume is given by

$$\boxed{dv = \rho\,d\rho\,d\phi\,dz}$$

C. Spherical Coordinate Systems

2. The differential normal surface area is

$$\begin{aligned} d\mathbf{S} = {} & r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r \\ & r \sin\theta \, dr \, d\phi \, \mathbf{a}_\theta \\ & r \, dr \, d\theta \, \mathbf{a}_\phi \end{aligned}$$

3. The differential volume is

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

DEL OPERATOR

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

$$\nabla = \mathbf{a}_\rho \frac{\partial}{\partial \rho} + \mathbf{a}_\phi \frac{1}{\rho}\frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z}$$

$$\nabla = \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \mathbf{a}_\phi \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}$$

GRADIENT OF A SCALAR

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

DIVERGENCE OF A VECTOR

$$\boxed{\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

$$\boxed{\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}}$$

$$\boxed{\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \mathbf{A}_\phi}{\partial \phi}}$$

CURL OF A VECTOR

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\,\mathbf{a}_\phi & \mathbf{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\,\mathbf{a}_\theta & r\sin\theta\,\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A}\, dv = 0$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

$$\boxed{\mathbf{E} = \frac{\mathbf{F}}{Q}}$$

$$\boxed{\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r}$$

$$\mathbf{E} = \int_S \frac{\rho_S \, dS \, \mathbf{a}_R}{4\pi\varepsilon_0 r^2} = \int_S \frac{\rho_S \, dS \, (\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

$$Q = \int_L \rho_L \, dl \qquad \text{for line charge}$$

$$Q = \int_S \rho_S \, dS \qquad \text{for surface charge}$$

$$Q = \int_v \rho_v \, dv \qquad \text{for volume charge}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E}$$

ELECTRIC FLUX DENSITY

$$\boxed{\mathbf{D} = \varepsilon_0 \mathbf{E}}$$

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\boxed{\rho_v = \nabla \cdot \mathbf{D}}$$

$$\boxed{\mathbf{E} = -\nabla V}$$

electric flux through a surface S is

$$\Psi = \int_S \mathbf{D} \cdot d\mathbf{S}$$

$$I = \oint \mathbf{J} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{J}\, dv$$

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

$$\rho_v = ne$$

$$J = \sigma E$$

$$\mathbf{D} = \varepsilon_o(1 + \chi_e)\,\mathbf{E} = \varepsilon_o \varepsilon_r \mathbf{E}$$

$$\boxed{\mathbf{D} = \varepsilon \mathbf{E}}$$

$$\boxed{\varepsilon = \varepsilon_o \varepsilon_r}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

$$\mathbf{F} = \int_v \rho_v \mathbf{E}\, dv$$

$$\boxed{\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}}}$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

$$\boxed{\mathbf{B} = \mu_o \mathbf{H}}$$

$$\boxed{\mu_o = 4\pi \times 10^{-7}\ \text{H/m}}$$

$$\boxed{\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}}$$

$$\boxed{\oint_S \mathbf{B} \cdot d\mathbf{S} = 0}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B} \, dv = 0$$

$$\nabla \cdot \mathbf{D} = \rho_v \qquad \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\nabla \times \mathbf{E} = \mathbf{0} \qquad \oint_L \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\boxed{\nabla \cdot \mathbf{A} = 0}$$

$$\boxed{\mathbf{F} = \oint_L I \, d\mathbf{l} \times \mathbf{B}}$$

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

$$F_m = I\ell B$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\boxed{\mathcal{P} = \mathbf{E} \times \mathbf{H}}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$k = \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\text{Re}(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{E_o^2}{2\eta}\mathbf{a}_k$$

$\lambda = 2\Pi/k$

$C = f\lambda$

$C = E_0/B_0$

$K = \omega/c$

$\omega = 2\Pi$

$$\underline{S} = \frac{\underline{E}x\,\underline{B}}{\mu_0}$$

TRANSMISSION LINES

1 Np= 8.686db

Propagation constant
$\gamma = \alpha + j\beta$

Wave velocity, $u = \frac{\omega\omega}{\beta\beta} = f\lambda$

Wavelength, $\lambda = \frac{2\pi 2\pi}{\beta\ \beta}$

Input impedance

$$Z_{in} = Z_o \left[\frac{Z_L + Z_0 \tanh \gamma\ell}{Z_0 + Z_L \tanh \gamma\ell} \right]$$

$$\tanh(x \pm jy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} \pm j \frac{\sin 2y}{\cosh 2x + \cos 2y}$$

$$Z_{in} = \frac{V_s\ (z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

Voltage and current at any point z

$$V_sV_s(z) = V_0^+ e^{-\gamma z}V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}V_0^- e^{\gamma z}$$

$$I_sI_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z}\frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}\frac{V_0^-}{Z_0} e^{\gamma z}$$

$$V_0^+ = \frac{1}{2}V_0^+ = \frac{1}{2}(V_0 + Z_0 I_o)V_0 + Z_0 I_o)$$

$$V_0^- = \frac{1}{2}V_0^- = \frac{1}{2}(V_0 - Z_0 I_o)V_0 - Z_0 I_o)$$

Sending end current and voltage

$$I_0 = \frac{V_g}{Z_{in} + Z_g}$$

$$V_0 = Z_{in} I_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g$$

Reflection coefficient

$$\Gamma_L\Gamma_L = \frac{Z_L - Z_0 Z_L - Z_0}{Z_L + Z_0 Z_L + Z_0}$$

Standing wave ratio

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Antenna

Wavelength

$$\lambda = \frac{c}{f}$$

Power radiated,

$$P_{rad} or\ W = I_{rms}^2 P_{rad} or\ W = I_{rms}^2 \text{ x } R_{rad} R_{rad}$$

Effective area,

$$A_e A_e = \frac{\lambda^2 \lambda^2}{4\pi 4\pi} \text{ D}$$

Capture area of a circular aperture,

$$A_e = \frac{\pi D^2}{4}$$

Radiation Efficiency

$$\eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_\ell} \eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_\ell}$$

$$\eta_r = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_\ell}$$

Directivity

D = $\frac{4\pi\, U_{max} 4\pi\, U_{max}}{P_{rad} \quad P_{rad}}$

$U_{max} U_{max}$ – Radiation intensity

$$D = \frac{4\pi}{\lambda^2} A_e$$

Gain of an Antenna

G= ηD

η – Radiation Efficiency

G = $KD\, KD$

$$G = K \frac{4\pi}{\lambda^2} G = K \frac{4\pi}{\lambda^2} A_e A_e$$

K- antenna factor , 1 if no losses present

Gain in db, $G_{db} G_{db}$*= 10* $\log_{10} G \log_{10} G$

Q factor

Q= $\frac{f_r f_r}{\Delta f \Delta f}$

$\Delta f \Delta f$ *- Bandwidth*

1 nautical mile(nm) =1852m

Radar power density

$$P = \frac{G_{dt}\ P_{rad} G_{dt}\ P_{rad}}{4\pi r^2 \quad 4\pi r^2}$$

Power of the reflected signal at the radar

$$P_r = \frac{A_e\ \sigma G_d P_{rad}}{[4\pi r^2]^2}$$

END OF PAPER