UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) ELECTRICAL & ELECTRONICS ENGINEERING

SEMESTER ONE EXAMINATION 2023/2024

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Thursday 11th January 2024 **Time: 2:00 – 4:30**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

You are required to answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

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Question 1

(a) A series RLC circuit is connected to a voltage source given by $v_S(t)=150\cos \omega t$ V.

Find

(i) the phasor current **I** and

(ii) (ii) the instantaneous current $i(t)$ for $R=400\ \Omega$, $L= 3$ mH, $C=16.67$ nF,and $\omega=10^5$ rad/s.

[7 marks]

(b) Two points in a cartesian coordinates are P1(2,3,3) and P2(-1,-5,-1). Find (i) the distance vector between P1 and P2. (ii) the angle between vectors $\overrightarrow{(OP1)}$ and $\overrightarrow{(OP2)}$ using the cross product between them. (iii) the angle between vector $\overrightarrow{(OP2)}$ and the y-axis.

[12 marks]

(c) Transform vector **A**$=\hat{\mathbf{x}}(x+y)+\hat{\mathbf{y}}(y-x)+\hat{\mathbf{z}}\,z$ from Cartesian to Cylindrical coordinates.

[6 marks]

Total [25 marks]

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Question 2

a) A scalar quantity of $V = rz^2cos2\phi$. Find its directional derivative along the direction **A**=$\hat{\boldsymbol{r}}\,\mathbf{2} - \hat{\boldsymbol{z}}\mathbf{3}$ and evaluate it at $(1, 0.5\pi, 2)$. **[9 marks]**

b) Find the divergence and the curl of the given vector A=$e^{-7y}(\hat{\boldsymbol{x}}\,sin3x$+$\hat{\boldsymbol{y}}$cos3x) at x=10 and y=1.0. **[4 marks]**

c) Four charges of 100 μC each are located in free space at points with Cartesian coordinates (-3,0,0), (3,0,0), (0, -3,0) and (0,3,0). Find the force on a 200 μC charge located at (0,0,4). All distances are in metres. **[12 marks]**

Total [25 marks]

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Question 3

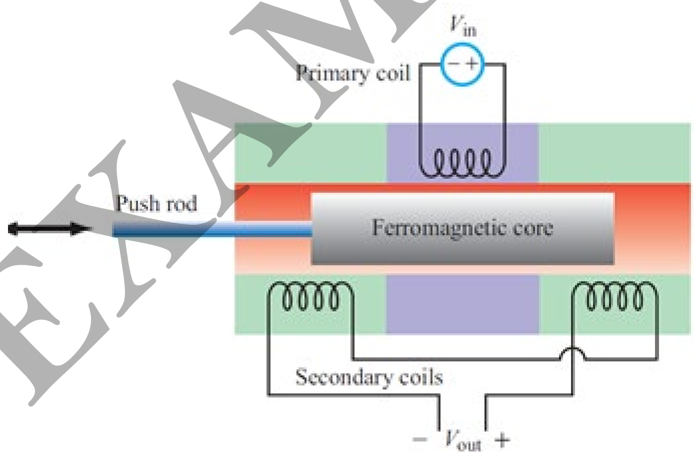
(a) The potential difference between two points in volts is numerically equal to the work in joules per coulomb necessary to move a coulomb of charge between the two points. A two-wire airline (single-phase system) has conductors of straight cylindrical bare wires with identical radius of 25 mm and spacing of 0.544 m. .

(i) What is the charge on each conductor?; **[1 mark]**

(ii) What is the voltage drop between the two conductor and **[4 marks]**

(iii)Find the capacitance of a two-wire airline (single-phase system). Then calculate the capacitance of each wire to ground. **[5 marks]**

(b) Briefly explain the operation of the Linear Variable Differential Transformer LVDT sensor shown in figure Q3c. **[8 marks]**



(c) A square coil of 200 turns and 0.5 m long sides is in a region with a uniform magnetic flux density of 0.2 T. If the maximum magnetic torque exerted on the coil is $4X10^{-2}$ N.m what is the current flowing in the coil? And what is the MMF produced by the coil? **[7 marks]**

Total [25 marks]

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Question 4

a) An 11.7 GHz satellite downlink operates from geosynchronous orbit with 25 W of transmitter output power connected to a 20 dB gain antenna with 2 dB feeder losses. The earth station is at a range of 38000 km from the satellite and uses a 15 m diameter receive antenna, with 55% efficiency, feeding a low noise (cooled) amplifier, which results in a receiver system noise temperature of 100 K. If an E_b/N_0 of 20 dB is required for adequate BER performance, what maximum bit rate can be accommodated using BPSK modulation, assuming performance is limited by the downlink, and atmospheric attenuation can be neglected.

[14 marks]

b) For a distortionless line with $Z_0 = 50\Omega$, $\alpha = 20$ (mNp/m), $u_p = 2.5 \times 10^8$ (m/s), determine the line parameters and λ at 100 MHz.

[11 marks]

Total [25 marks]

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Question 5

a) Consider a typical satellite communication system that has dual-conversion down converters with an intermediate frequency (IF) tuning, as shown in Figure Q5.

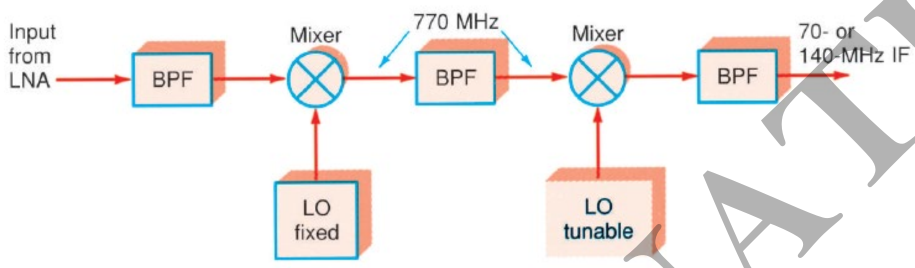

Figure Q5: Dual-conversion down converters with IF tuning

If the earth station downlink signal received is at $f_s = 4080$ GHz, what local-oscillator frequencies f_{LO} are needed to achieve intermediate frequencies (IFs) of 770 and 140 MHz? Assume the local oscillator frequencies are lower than the received signals. **[5 marks]**

b) A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 + j50)\ \Omega$. The wavelength is 8 cm. Find the:

i. reflection coefficient at the load, **[5 marks]**

ii. standing-wave ratio on the line, **[5 marks]**

iii. position of the voltage maximum nearest the load, **[5 marks]**

iv. position of the current maximum nearest the load. **[5 marks]**

Total [25 marks]

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Question 6

a) A 2-kHz sound wave travelling in the x-direction in air has a differential pressure $p(x,t) = 10$ N/m^2 at $x = 0$ and $t = 50\,\mu s$. If the reference phase of $p(x,t)$ is 36^0, find a complete expression for $p(x,t)$. The velocity of sound in air is 330 m/s. **[13 marks]**

b) A certain electromagnetic wave travelling in seawater was observed to have an amplitude of 98.02 (V/m) at a depth of 10 m and an amplitude of 81.87 (V/m) at a depth of 100 m. What is the attenuation constant of seawater?

[12 marks]

Total [25 marks]

END OF QUESTIONS

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Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\widetilde{Z}$, where $z(t) = \mathfrak{Re}\ [\widetilde{Z}e^{j\omega t}]$.

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\widetilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation A =	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A} \cdot \mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{12}}\ ,\ \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3}\ ,\ \mathbf{E} = \frac{\mathbf{F}}{Q},\ \mathbf{E} = \int\frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\mathbf{a}_R\ ,\ \mathbf{E} = \int\frac{\rho_S dS}{4\pi\varepsilon_0 R^2}\mathbf{a}_R\ ,\ \mathbf{E} = \int\frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n\ ,\ \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho\ ,\ Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_v \rho_v dv\ ,\ \nabla\cdot\mathbf{D} = \rho_v\ ,\ W = -Q\int_A^B \mathbf{E}\cdot d\ell\ ,\ V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\ell\ ,\ V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint \mathbf{E}\cdot d\ell = 0,\ \nabla\times\mathbf{E} = 0,\ \mathbf{E} = -\nabla V\ ,\ W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k\ ,\ W_E = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int \varepsilon_0 E^2 dv\ ,\ \mathbf{J} = \rho_v\mathbf{u}\ ,\ I = \int_S \mathbf{J}\cdot d\mathbf{S}\ ,\ \mathbf{J} = \sigma\mathbf{E},$$

$$R = \frac{\mathrm{V}}{\mathrm{I}} = \frac{\int \mathbf{E}\cdot d\mathbf{l}}{\int \sigma\mathbf{E}\cdot d\mathrm{S}}\ ,\ \mathbf{D} = \varepsilon\,\mathbf{E},\ \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t},\ E_{1t} = E_{2t}\ ,\ D_{1n} - D_{2n} = \rho_S\ ,\ D_{1n} = D_{2n}\ ,\ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}\ ,\ \nabla^2 V = 0\ ,\ C = \frac{Q}{V} = \frac{\varepsilon\oint \mathbf{E}\cdot d\mathbf{S}}{\int \mathbf{E}\cdot d\mathbf{l}}\ ,\ W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}\ ,\ C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}\ ,\ C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}\ ,\ RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12}F/m\quad ,\ \mu_o = 4\pi X10^{-7}\ H/m$$

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MAGNETOSTATICS:

$$\mathbf{H} = \int_L \frac{Id\mathbf{l} \times \mathbf{a}_R}{4\pi R^2},\ \mathbf{H} = \int_S \frac{\mathbf{K}dS \times \mathbf{a}_R}{4\pi R^2},\ \mathbf{H} = \int_v \frac{\mathbf{J}dv \times \mathbf{a}_R}{4\pi R^2},\ \mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi,\ \mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{a}_\phi = \mathbf{a}_\ell \times \mathbf{a}_\rho,$$

$$\oint \mathbf{H}\cdot d\mathbf{l} = I_{enc},\ \nabla\times\mathbf{H} = \mathbf{J},\ \mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{H} = \frac{1}{2}\mathbf{K}\times\mathbf{a}_n,\ \mathbf{B} = \mu\mathbf{H},\ \Psi = \int_S \mathbf{B}\cdot d\mathbf{S},\ \oint \mathbf{B}\cdot d\mathbf{S} = 0,\ \nabla\cdot\mathbf{B} = 0,\ \mathbf{H} = -\nabla V_m,$$

$$\mathbf{B} = \nabla\times\mathbf{A},\ \mathbf{A} = \int_L \frac{\mu_0 Id\mathbf{l}}{4\pi R},\ \mathbf{A} = \int_S \frac{\mu_0 \mathbf{K}dS}{4\pi R},\ \mathbf{A} = \int_v \frac{\mu_0 \mathbf{J}dv}{4\pi R},\ \Psi = \oint_L \mathbf{A}\cdot d\mathbf{l},\ \mathbf{F} = Q(\mathbf{E} + \mathbf{u}\times\mathbf{B}),\ d\mathbf{F} = Id\mathbf{l}\times\mathbf{B},\ \mathbf{B}_{1n} = \mathbf{B}_{2n},$$

$$(\mathbf{H}_1 - \mathbf{H}_2)\times\mathbf{a}_{n12} = \mathbf{K},\ \mathbf{H}_{1t} = \mathbf{H}_{2t},\ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2},\ L = \frac{\lambda}{I} = \frac{N\psi}{I},\ M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1\psi_{12}}{I_2},\ W_m = \frac{1}{2}\int \mathbf{B}\cdot\mathbf{H}dv = \frac{1}{2}\int \mu H^2 dv$$

WAVES AND APPLICATIONS:

$$V_{emf} = -\frac{d\psi}{dt},\ V_{emf} = \oint_L \mathbf{E}\cdot d\mathbf{l} = -\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S},\ \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},\ V_{emf} = \oint_L \mathbf{E}_m\cdot d\mathbf{l} = \oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$V_{emf} = \oint_L \mathbf{E}\cdot d\mathbf{l} = -\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S} + \oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l},\ \mathbf{J}_d = \frac{\partial\mathbf{D}}{dt},\ \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{dt},\ \beta = \frac{2\pi}{\lambda},\ \underline{\gamma} = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]},\quad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]},\ \mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$

$$|\underline{\eta}| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}},\quad \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon},\ \mathbf{H} = \frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\mathbf{a}_y,\ \tan\theta = \frac{\sigma}{\omega\varepsilon},\ \mathbf{a}_E\times\mathbf{a}_H = \mathbf{a}_k$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377\Omega,\ p(t) = \mathbf{E}\times\mathbf{H},\ p_{ave}(z) = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}^*{}_s),\ p_{ave}(z) = \frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta\mathbf{a}_z,\ P_{ave} = \int_S p_{ave}\cdot d\mathbf{S},$$

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1},\ \tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1},\ s = \frac{|\mathbf{E}_1|_{\max}}{|\mathbf{E}_1|_{\min}} = \frac{|\mathbf{H}_1|_{\max}}{|\mathbf{H}_1|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|},\quad k_i\sin\theta_i = k_t\sin\theta_t,$$

$$\Gamma_\parallel = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i},\ \tau_\parallel = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i},\ \sin^2\theta_{B\parallel} = \frac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2},$$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t},\ \tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t},\ \sin^2\theta_{B\perp} = \frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - (\mu_1/\mu_2)^2}$$

$\boldsymbol{\omega = \beta c}$

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

University of Bolton
School of Engineering
B.Eng (Hons) Electrical & Electronics Engineering
Semester 1 Examination 2023/2024
Engineering Electromagnetism
Module No. EEE6012

Antenna and Radar formula

Dipole

Solid angle:

$$\Omega_p = \iint_{4\pi} F(\theta, \phi)\, d\Omega$$

Directivity:

$$D = \frac{4\pi}{\Omega_p} \text{ or } D = \frac{4\pi A_e}{\lambda^2}$$

Shorted dipole

$$S_0 = \frac{15\pi I_0^2}{R^2} \left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda}\right)^2.$$

Hertzian monopole

$$R_{rad} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

Half –wave dipole

$$\widetilde{E}_\theta = j\,60 I_0 \left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\} \left(\frac{e^{-jkR}}{R}\right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}.$$

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

For Transmission line

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Distortionless line

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

Open-circuited line

$$\widetilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\widetilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0} \sin\beta d,$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

Short-circuited line

$$\widetilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin\beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0} \cos\beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan\beta d.$$

$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right]$$

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}\right]$$

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

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For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt}G_{dr}}{4\pi}\left[\frac{\lambda}{4\pi r_1 r_2}\right]^2 \sigma P_{\text{rad}}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

$$P_{\text{rec}} = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

END OF PAPER