

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

**BENG (HONS) ELECTRICAL & ELECTRONIC
ENGINEERING**

SEMESTER 1 EXAMINATIONS 2023/24

INTRODUCTORY ENGINEERING MATHEMATICS

MODULE NO: EEE4011

Date: Thursday 11th January 2024

Time: 2:00 – 4:00

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

A formula sheet is included.

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Question 1.

- (a) A uniform electric field is given by the vector

$$E = \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}$$

where the values are given in newtons per coulomb.

- (i) If a particle with charge of 2mC is placed in the field, find the vector representing the force on the particle. **[1 mark]**
- (ii) Find the magnitude of the force, in newtons. **[2 marks]**

The force due to the electric field propels the charged particle along the displacement vector

$$d = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}$$

where the values are given in millimetres.

- (iii) Find the distance that the particle moves, in millimetres. **[2 marks]**
- (iv) Find the work done by the field in displacing the particle, stating the units. **[2 marks]**
- (v) Find the angle between the electric field vector and the displacement vector. **[3 marks]**
- (b) Let A and B be the following matrices:

$$A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 4 \\ 5 & 3 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -4 & 0 \\ 1 & 6 & -2 \\ 0 & 3 & 5 \end{pmatrix}$$

Calculate the products AB and BA. **[8 marks]**

- (c) Write the following system of simultaneous linear equations as an equation of matrices:

$$3x + 7y = 10$$

$$2x + 8y = 15 \quad \mathbf{[2 marks]}$$

By finding the inverse of the square matrix, solve the system of equations.

[5 marks]

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Question 2

- (a) Find the complex solutions of the following quadratic equation:

$$x^2 + 6x + 90 = 0$$

Indicate the position of these on a sketch of the Argand diagram.

[6 marks]

- (b) A resistor of value $R = 4k\Omega$ is in series with a capacitor of value $C = 20nF$.

A sine wave voltage source with a frequency of 10000 radians per second is connected across the series pair of components.

Represent the combined impedance as a complex number.

Find the absolute value of the impedance, and the phase shift between the voltage source and the current.

[9 marks]

- (c) Let $z_1 = 36\angle 60^\circ$ and $z_2 = 3\angle 20^\circ$ be complex numbers in polar form.

Calculate each of the following in polar form:

- | | | | |
|-------|--------------|------|-------------------|
| (i) | $z_1 z_2$ | (ii) | $\frac{z_1}{z_2}$ |
| (iii) | $\sqrt{z_1}$ | (iv) | z_2^4 |

[10 marks]

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Question 3

(a) Differentiate each of the following functions to find $\frac{dy}{dt}$:

(i) $y = 2t^4 - 5t^3 + 7t - 4$

(ii) $y = e^{-4t} \sin 3t$

(iii) $y = (2t^4 - 5)^3$

[11 marks]

(b) Find the turning points of the function

$$y = -t^3 + 6t^2 + 135t - 30$$

Determine whether each turning point is a local maximum or a local minimum.

[10 marks]

(c) A voltage given by $v(t) = 10\sin(200t)$ is applied to a capacitor with value $250\mu F$.

Find an expression for the value of the current.

[4 marks]**Question 4**

(a) Evaluate each of the following definite integrals:

(i) $\int_2^3 3t^2 - 8t + 2 \cdot dt$

[6 marks]

(ii) $\int_0^{\frac{\pi}{3}} 12 \cos 6t + 8 \sin 2t \cdot dt$

[6 marks]

(b) Find each of the following integrals:

(i) $\int t^2 (t^3 + 4)^5 dt$

[5 marks]

(ii) $\int t^2 \cos 2t \cdot dt$

[8 marks]**PLEASE TURN THE PAGE**

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Question 5

- (a) Find the solution of the following differential equation by separating variables:

$$\frac{dy}{dt} = 6yt^2$$

The initial condition is $y = 8$ at $t = 0$.

[8 marks]

- (b) Consider the following linear differential equation:

$$\frac{dy}{dt} + 5y = 15t$$

- (i) Find the complementary function. **[2 marks]**
(ii) Find the particular integral. **[5 marks]**
(iii) Hence find the solution given that $y = 1$ when $t = 0$. **[3 marks]**

- (c) Find the general solution of the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 16\frac{dy}{dt} + 60y = 0$$

[7 marks]

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Question 6

- (a) The capacitances of ten capacitors in nanofarads are as follows:

49.2 51.4 52.0 48.8 47.2 50.6 50.8 48.5 49.9 52.6

Find the median value of the capacitances.

Find the interquartile range.

Calculate the mean and standard deviation of the capacitances.

[8 marks]

Please give answers to parts (b) and (c) correct to three decimal places.

- (b) A plant requires maintenance on average 0.8 times per week.

Calculate the probability that the plant requires maintenance exactly twice in a week.

Calculate the probability that the plant does not require maintenance over a period of two weeks.

Calculate the probability that the plant requires maintenance exactly five times over a period of four weeks.

[9 marks]

- (c) In a digital communication system operating under noisy conditions, the probability that a single bit is received correctly is 0.92.

In a block of eight bits, find the probability that a single error occurs.

In a block of eight bits, find the probability that exactly three errors occur.

In a block of seven bits, six or more bits are received correctly.

[8 marks]

END OF QUESTIONS

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FORMULA SHEET

Derivatives and integrals

Integral	Function	Derivative
$\int y dt$	y	$\frac{dy}{dt}$
t	1	0
$\frac{1}{n+1} t^{n+1}$	t^n	nt^{n-1}
$-\frac{1}{a} \cos at$	$\sin at$	$a \cos at$
$\frac{1}{a} \sin at$	$\cos at$	$-a \sin at$
$\frac{1}{a} e^{at}$	e^{at}	ae^{at}

Integration by parts

$$\int u \frac{dv}{dt} dt = uv - \int \frac{du}{dt} v dt$$

Binomial Distribution:

The probability of r successes in n trials is $\binom{n}{r} p^r q^{n-r}$

where p is the probability of success in a single trial and $p + q = 1$.

Poisson Distribution:

The probability of r successes is $\frac{m^r}{r!} e^{-m}$

where m is the expected number of successes.

END OF PAPER