

**UNIVERSITY OF BOLTON**  
**SCHOOL OF ENGINEERING**  
**BEng (HONS) CIVIL ENGINEERING**  
**SEMESTER ONE EXAMINATION 2023/2024**  
**ENGINEERING MATHEMATICS & STRUCTURES**  
**MODULE NO: CIE5004**

Date: Thursday 11<sup>th</sup> January 2024

Time: 10:00 – 1:00

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**INSTRUCTIONS TO CANDIDATES:**

There are FOUR Questions.

Answer ALL questions.

All questions carry equal marks

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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### Question 1

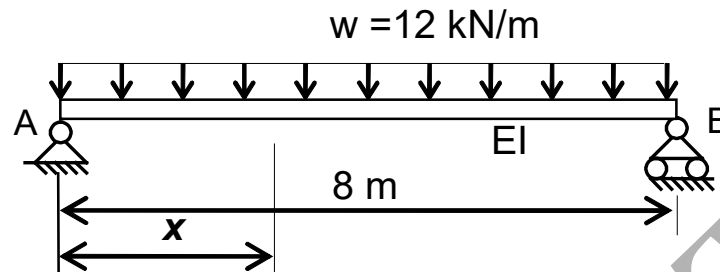


Figure Q1

Figure Q1 shows a simply supported beam AB with a span of 8m. The beam carries a uniform distributed load of  $w = 12 \text{ kN/m}$  as shown. The beam has uniform rigidity  $EI$ .

- a) Find the vertical reaction at support A. (2 marks)
- b) Write the bending moment  $M$  in terms of  $x$ . (3 marks)
- c) Use the method of Macaulay to write, in terms of  $x$  and  $EI$ :
  - i. The equation of the rotation (slope) (7 marks)
  - ii. The equation of the vertical deflection (7 marks)
- d) Calculate the values of the rotation and vertical deflection at  $x = 4 \text{ m}$  for the following data:

$$E = 25 \text{ kN/mm}^2$$

$$I = 120000 \text{ cm}^4$$

(6 marks)

Formula for the deflection of a beam:  $\frac{d^2v}{dx^2} = -\frac{M}{EI}$

(Total 25 marks)

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### Question 2

Figure Q2 (i) shows a three pin frame, pinned to supports at A and E, with a third pin at C. The frame is subjected to a horizontal point load of 50kN at position B.

- Calculate the value of the support reactions at A and E. (5 marks)
- Draw the axial force diagram (AFD) (4 marks)
- Draw the shear force diagram (SFD) (5 marks)
- Draw the bending moment diagram (BMD) (6 marks)

For b), c) and d), show all important values on the diagrams and produce accompanying calculations to show how these values have been derived.

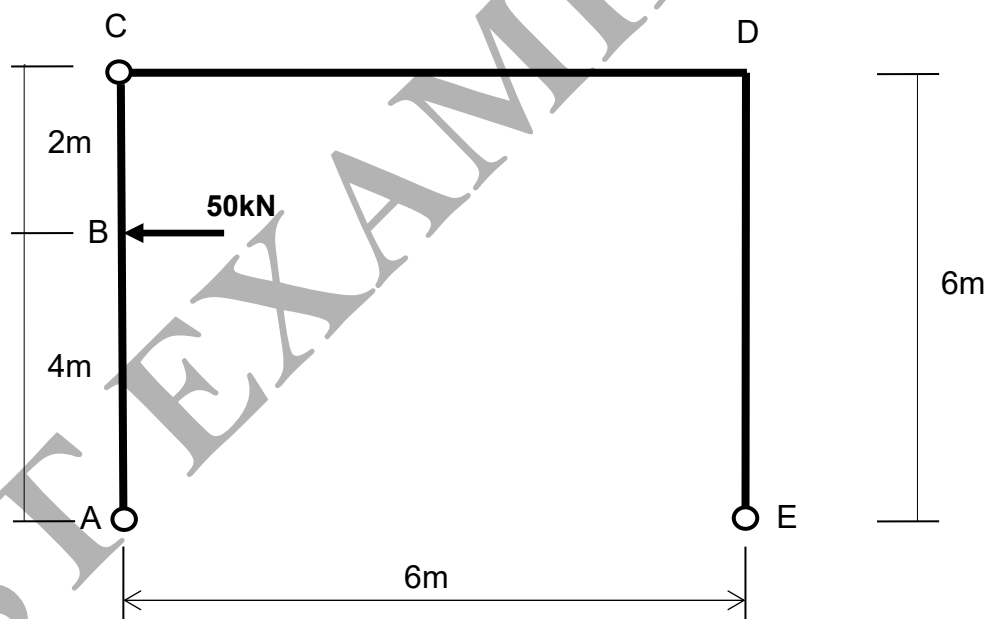


FIGURE Q2 (i)

Question 2 continued next page .....

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**Question 2 continued**

Figure Q2 (ii) shows a very similar three pin frame, pinned to supports at A and E, with the third pin at D (no longer at C). The horizontal point load remains the same as Figure Q2 (i).

- e) Without doing any further calculations, sketch the Bending Moment Diagram (BMD) for the three pin frame shown in Figure Q2 (ii).

Do not attempt to calculate the values of the bending moments in the frame.

(5 marks)

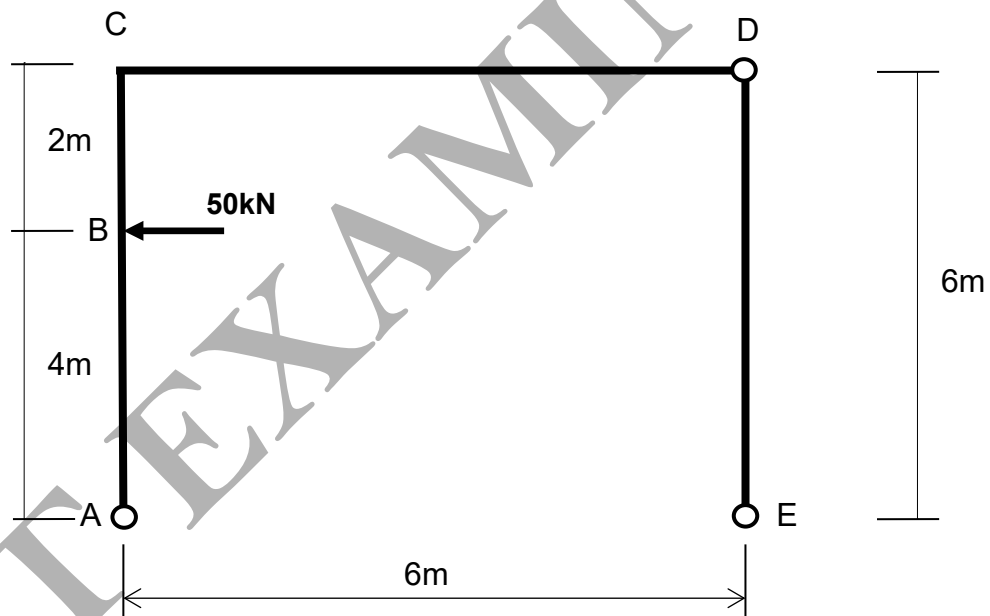


FIGURE Q2 (ii)

(Total 25 marks)

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**Question 3**

Figure Q3 shows a reinforced concrete frame structure that includes slabs, beams, columns and pad footings.

Calculate the mass of carbon emissions for the whole structure including the footings.

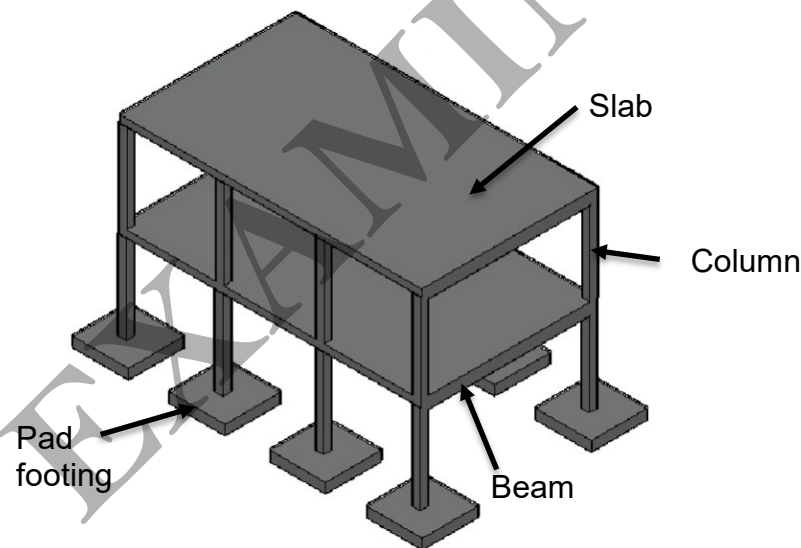
If you know:

Total volume of the concrete used in slabs is  $75\text{m}^3$

Total volume of the concrete used in beams is  $4.5\text{m}^3$

Total volume of the concrete used in columns is  $3.5\text{m}^3$

Total volume of the concrete used in footings is  $25\text{m}^3$



**Figure Q3**

Use the following data:

Density of concrete is  $2400\text{ kg/m}^3$

Estimated amount of reinforcement for slabs is:  $75\text{ kg/m}^3$  of concrete

Estimated amount of reinforcement for beams is:  $100\text{ kg/m}^3$  of concrete

Estimated amount of reinforcement for columns is:  $200\text{ kg/m}^3$  of concrete

Estimated amount of reinforcement for footings is:  $80\text{ kg/m}^3$  of concrete

Apply the wastage rate as 4%

Rate of embodied carbon for concrete is  $0.126\text{ kg CO}_2\text{e/kg}$

Rate of embodied carbon for steel is  $1.4\text{ kg CO}_2\text{e/kg}$

**(Total 25 marks)**  
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#### Question 4 – Moment Distribution Method

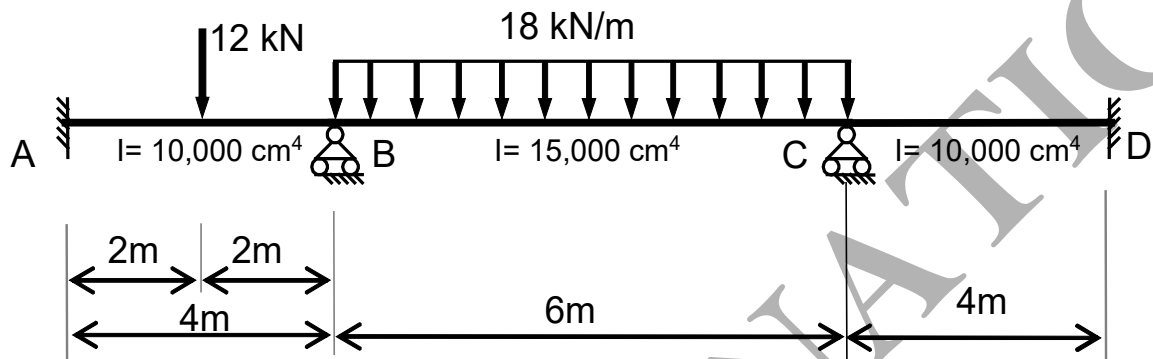


Figure Q4

Figure Q4 shows a 3-span continuous beam ABCD which is fixed to supports at A and D and simply supported at B and C. The  $I$  values of the members are shown, and all members have the same  $E$  value.

- Using moment distribution, calculate the bending moments at A, B, C and D. (17 marks)
- Sketch the bending moment diagram for the whole beam, showing values at important points. (8 marks)

Stiffness of a beam against end rotation  $K = 4EI/L$

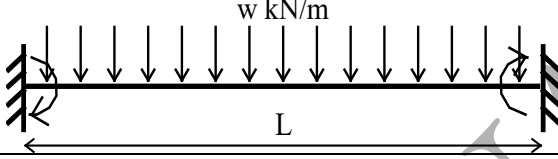
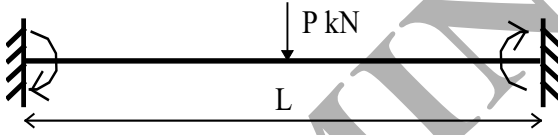
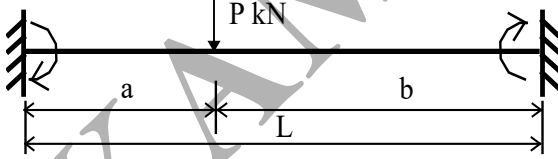
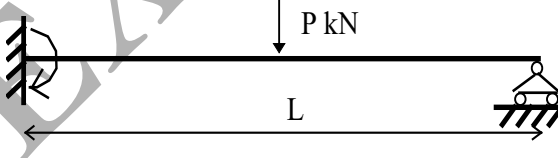
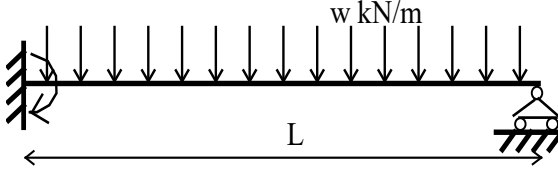
Table of Fixed-End Moments is shown in **Table 1** in page 7.

(Total 25 marks)

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**Table 1: Fixed End Moments to be used with Question 4**

FIXED-END MOMENTS			
$FEM_{AB}$	A	B	$FEM_{BA}$
$-\frac{wL^2}{12}$			$\frac{wL^2}{12}$
$-\frac{PL}{8}$			$\frac{PL}{8}$
$-\frac{Pab^2}{L^2}$			$\frac{Pa^2b}{L^2}$
$-\frac{3PL}{16}$ Reaction = $\frac{11P}{16}$			0 Reaction = $\frac{5P}{16}$
$-\frac{wL^2}{8}$ Reaction = $\frac{5wL}{8}$			0 Reaction = $\frac{3wL}{8}$

**END OF QUESTIONS**

**END OF PAPER**