UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) CIVIL ENGINEERING

SEMESTER 1 EXAMINATION 2023/24

MATHEMATICAL METHODS FOR CIVIL ENGINEERING

MODULE NO: CIE4022

Date: Wednesday 10th January 2024 **Time: 10:00am – 12:00 pm**

INSTRUCTIONS TO CANDIDATES:

This is a closed book examination

There are FIVE questions

Answer ANY FOUR Questions

A formula sheet is provided

The number of marks awarded for each question part is shown in square brackets

This examination carries a total of 80 marks.

Marks will be given for showing your working out.

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Q1) a. Given that $y=x^2+7$, show that $\frac{dy}{dx} = 2x$ using the first principles. [6 marks]

b. Show that [6 marks]

$$\frac{(x+\Delta)^4-x^4}{\Delta} \longrightarrow 4x^3$$

as $\Delta \rightarrow 0$

c. Find the local maxima and minima of the curve: [8 marks]

$$y = \frac{x^3}{3} - 4x + 1$$

Sketch the curve.

Total 20 marks

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Q2) a. Find $\frac{dy}{dx}$ when:

(i) y =$x^2 e^x$ [3 marks]

(ii) y = $\frac{e^x}{x^2}$ [3 marks]

(iii) y = e^{x^2} [3 marks]

b. Find $\frac{d^2 y}{dx^2}$ when: [7 marks]

y = x² sin(x)

c. The surface area of a closed hemi-sphere is given by S=3πr² [4 marks]

Show that: Δ_s = 6 π r Δ_r and interpret this result.

Total 20 marks

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Q3) a. Integrate:

(i) $\int x^2 dx$ [2 marks]
(ii) $\int \sin(x)\, dx$ [2 marks]
(iii) $\int 3x\text{-}1+e^x\, dx$ [3 marks]

b. Evaluate the definite integral: [5 marks]

$$\int_{-1}^{1} x^3\, dx$$

c. Use Simpsons Rule, with 4 strips, to find an approximation to [8 marks]

$$\int_{1}^{2} (4-x^2)dx$$

Total 20 marks

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Q4) a. Evaluate the definite integral: [8 marks]

$$\int_{-2}^{0} \frac{x^3}{3} - 4x + 1 \, dx$$

Mark the area this represents on your graph in question 1(c)

b. Evaluate the definite integrals:

(i) $\int_{-\pi}^{\pi} \sin(x) \, dx$ [6 marks]

(i) $\int_{-\pi}^{\pi} \cos(x) \, dx$ [6 marks]

Illustrate both integrals graphically.

Total 20 marks

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Q5) a. Integrate:

[8 marks]

$$\int \sqrt{1-x^2}\, dx$$

using the substitution x = cos Ø

b. Integrate:

[12 marks]

$$\int x^2 e^x\, dx$$

by parts.

Total 20 marks

END OF QUESTIONS

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CIE4002 Formula Sheet

$$y = x^n \qquad \frac{dy}{dx} = nx^{n-1}$$

$y = \sin\emptyset$	$\frac{dy}{dx} = \cos\emptyset$
$y = \cos\emptyset$	$\frac{dy}{dx} = -\sin\emptyset$
$y = -\sin\emptyset$	$\frac{dy}{dx} = -\cos\emptyset$
$y = -\cos\emptyset$	$\frac{dy}{dx} = \sin\emptyset$

$$y = e^x \qquad \frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = 0 \longrightarrow \text{turning point}$$

$$\frac{d^2y}{dx^2} > 0 \longrightarrow \text{min}$$

$$< 0 \longrightarrow \text{max}$$

$$= 0 \longrightarrow \text{point inflection}$$

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Product Rule

$$y = u\,v \qquad \frac{dy}{dx} = \frac{udv}{dx} + \frac{vdu}{dx}$$

Quotient Rule

$$y = \frac{u}{v} \qquad \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chan Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Approximation

$$\frac{dy}{dx} \cong \frac{\Delta y}{\Delta x}$$

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B Integration (analytic)

$$\int x^2\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sin\emptyset\,d\emptyset = -\cos\emptyset + c$$

$$\int -\cos\emptyset\,d\emptyset = -\sin\emptyset + c$$

$$\int -\sin\emptyset\,d\emptyset = \cos\emptyset + c$$

$$\int \cos\emptyset\,d\emptyset = \sin\emptyset + c$$

$$\int e^x\,dx = e^x + c$$

Trig Identity: $\sin^2\emptyset = \frac{1}{2} - \frac{1}{2}\cos 2\emptyset$

By Parts: $\int u\,dv = uv - \int v\,du$

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B Integration (numeric)

Simpsons Rule

X	x_0 x_1 x_2 ..... x_n
Y	y_0 y_1 y_2 ..... y_n

$$\text{Area} \cong \frac{h}{3} [y_0 + y_n + 4y_1 + 2y_2 + 4y_3 + \dots]$$

END OF FORMULA SHEET

END OF PAPER