[ENG25]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) BIOMEDICAL ENGINEERING

SEMESTER 1 EXAMINATIONS 2023/24

BIOMEDICAL ENGINEERING MODELLING & ANALYSIS

MODULE NO: BME5001

Date: Monday 8th January 2024 **Time: 10.00 - 12.00**

INSTRUCTIONS TO CANDIDATES:

This is an **open book** examination.

Please attempt **FOUR** of the **FIVE** questions.

For your guidance, the maximum mark that may be achieved for each question and part question is shown in brackets.

Tables for the normal distribution, the *t* distribution, and the chi-square distribution are included on pages 6 through 8.

School of Engineering
BEng (Hons) Biomedical Engineering
Semester 1 Examinations 2023/24
Biomedical Engineering Modelling & Analysis
Module No.: BME5001

Question 1

(a) Using the definition of the Laplace transform, show from first principles that the Laplace transform of $f(t) = 3e^{-4t}$ is

$$F(s) = \frac{3}{s+4}$$

(5 marks)

(b) Using Laplace transforms, solve the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 20y = 60$$

Initial conditions: at $t = 0$ we have $y = 0$ and $\frac{dy}{dt} = 2$.

(20 marks)

Question 2

A filter in a control system has transfer function $H(s) = \frac{s-4}{s+12}$

(a) List the zeros and poles of the system, and comment on stability.

(5 marks)

(b) Find the response to the step function $x(t) = 2$ for $t > 0$.

(7 marks)

(c) Find the response to the unit ramp function $x(t) = t$ for $t > 0$.

(7 marks)

(d) If the input is a sine wave of frequency $\omega = 8$ radians per second, find the gain and the phase shift of the filter.

(6 marks)

Please give your answers correct to *three significant figures*.

Question 3

A genetic condition has a prevalence of 1 in 100 of the population.

(i) Write down the probability that a person chosen at random from the population has the condition. (2 marks)

A test for the genetic condition has been developed, which is 84% sensitive and 92% specific.

(ii) Write down the probability that a person tests positive, given that they have the genetic condition. (2 marks)

(iii) Write down the probability that a person tests positive, given that they do *not* have the condition. (3 marks)

(iv) Calculate the overall probability that a person chosen at random from the population tests positive. (6 marks)

(v) Using Bayes' theorem, calculate the probability that a person has the condition, given that they have tested positive. (6 marks)

(vi) Calculate the probability that a person does *not* have the condition, given that they have tested negative. (6 marks)

Please give your answers correct to *four decimal places*.

Question 4

(a) In a population, it has been found that the blood serum levels of calcium are normally distributed with a mean value of 2.3 mmol/L and a standard deviation of $\sigma = 0.12$ mmol/L. Find the probability that a person chosen at random from the population has a mean calcium level of

(i) more than 2.5 mmol/L (2 marks)

(ii) less than 2.0 mmol/L (2 marks)

(iii) between 2.4 mmol/L and 2.6 mmol/L (4 marks)

(b) In a population who are not taking medication, the mean blood serum concentration of potassium is $m = 4.4$ mmol/L (millimoles per litre).

A new diuretic medication has been developed. There is concern that this may cause abnormally low levels of blood serum concentration of potassium as a side effect.

A sample of ten patients taking the new medication have been examined, and their sodium concentrations in millimoles per litre were as follows:

4.42 4.27 4.33 4.29 4.42 4.26 4.41 4.31 4.39 4.20

(i) Find estimates of the mean and standard deviation of sodium concentration for the *population of those who are taking the medication*.

(6 marks)

(ii) Calculate the standard error for the sample of ten patients.

(2 marks)

(iii) Using the *t* distribution, determine whether the sample gives evidence to reject the null hypothesis that the diuretic does not affect sodium levels, using the 5% significance level.

(6 marks)

(iv) Explain how it would affect the conclusion, if stronger evidence was required by adopting a 1% significance level.

(3 marks)

Question 5

Three antidepressant medications A, B and C were trialled with a group of 1000 patients. For each of the medications, the number of patients reporting significant improvement in symptoms were as follows:

	Improvement	***No improvement***
Medication A	310	90
Medication B	308	42
Medication C	182	68

(i) Based on the null hypothesis that there is *no difference* in effectiveness between the three medications, calculate the probabilities that a person chosen at random from the sample experiences significant improvement in symptoms.

(3 marks)

(ii) Still assuming the same null hypothesis, draw up a table of expected values of people experiencing significant improvement in symptoms or not.

(6 marks)

(iii) Calculate the chi-square statistic.

(6 marks)

(iv) State, with reasons, the number of degrees of freedom.

(3 marks)

(v) By comparing the chi-square statistic with the appropriate critical value for the 1% significance level, decide whether the null hypothesis should be rejected or not.

By looking at the data and the result based on the chi square value, suggest a plausible conclusion of the trial.

(7 marks)

END OF QUESTIONS

Distribution Tables follow over the page

PLEASE TURN THE PAGE

School of Engineering
BEng (Hons) Biomedical Engineering
Semester 1 Examinations 2023/24
Biomedical Engineering Modelling & Analysis
Module No.: BME5001

Normal Distribution

Table gives $1 - \Phi(z)$, the probability of a score greater than *z*.

z	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	0.5	0.496	0.492	0.488	0.484	0.48	0.476	0.472	0.468	0.464
0.1	0.46	0.456	0.452	0.448	0.444	0.44	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.39	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.33	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.23	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.2	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.14	0.138
1.1	0.136	0.134	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.1	0.099
1.3	0.097	0.095	0.093	0.092	0.09	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.04	0.039	0.038	0.038	0.037
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.03	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2	0.023	0.022	0.022	0.021	0.021	0.02	0.02	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.01	0.01	0.01	0.01	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

The *t* distribution

The rows are labelled with the number of degrees of freedom.

Each column shows the minimum values of the *t* statistic for significance at the level stated in the column heading.

	25%	**20%**	**10%**	**5%**	**1%**	**0.5%**	**0.1%**
1	1	1.376	3.078	6.314	31.82	63.66	318.3
2	0.816	1.08	1.886	2.92	6.965	9.925	22.33
3	0.765	0.978	1.638	2.353	4.541	5.841	10.21
4	0.741	0.941	1.533	2.132	3.747	4.604	7.173
5	0.727	0.92	1.476	2.015	3.365	4.032	5.893
6	0.718	0.906	1.44	1.943	3.143	3.707	5.208
7	0.711	0.896	1.415	1.895	2.998	3.499	4.785
8	0.706	0.889	1.397	1.86	2.896	3.355	4.501
9	0.703	0.883	1.383	1.833	2.821	3.25	4.297
10	0.7	0.879	1.372	1.812	2.764	3.169	4.144
11	0.697	0.876	1.363	1.796	2.718	3.106	4.025
12	0.695	0.873	1.356	1.782	2.681	3.055	3.93
13	0.694	0.87	1.35	1.771	2.65	3.012	3.852
14	0.692	0.868	1.345	1.761	2.624	2.977	3.787
15	0.691	0.866	1.341	1.753	2.602	2.947	3.733
16	0.69	0.865	1.337	1.746	2.583	2.921	3.686
17	0.689	0.863	1.333	1.74	2.567	2.898	3.646
18	0.688	0.862	1.33	1.734	2.552	2.878	3.61
19	0.688	0.861	1.328	1.729	2.539	2.861	3.579
20	0.687	0.86	1.325	1.725	2.528	2.845	3.552
21	0.686	0.859	1.323	1.721	2.518	2.831	3.527
22	0.686	0.858	1.321	1.717	2.508	2.819	3.505
23	0.685	0.858	1.319	1.714	2.5	2.807	3.485
24	0.685	0.857	1.318	1.711	2.492	2.797	3.467
25	0.684	0.856	1.316	1.708	2.485	2.787	3.45
26	0.684	0.856	1.315	1.706	2.479	2.779	3.435
27	0.684	0.855	1.314	1.703	2.473	2.771	3.421
28	0.683	0.855	1.313	1.701	2.467	2.763	3.408
29	0.683	0.854	1.311	1.699	2.462	2.756	3.396
30	0.68	0.85	1.31	1.7	2.46	2.75	3.38

The χ^2 distribution

The rows are labelled with the number of degrees of freedom.

Each column shows the minimum values of the χ^2 statistic for significance at the level stated in the column heading.

	10%	**5%**	**2.5%**	**1%**	**0.5%**
1	2.706	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.21	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.86
5	9.236	11.07	12.833	15.086	16.75
6	10.645	12.592	14.449	16.812	18.548
7	12.017	14.067	16.013	18.475	20.278
8	13.362	15.507	17.535	20.09	21.955
9	14.684	16.919	19.023	21.666	23.589
10	15.987	18.307	20.483	23.209	25.188
11	17.275	19.675	21.92	24.725	26.757
12	18.549	21.026	23.337	26.217	28.3
13	19.812	22.362	24.736	27.688	29.819
14	21.064	23.685	26.119	29.141	31.319
15	22.307	24.996	27.488	30.578	32.801

END OF PAPER