[ENG32]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MECHANICAL ENGINEERING

SEMESTER 1 EXAMINATION 2023-2024

ADVANCED MATERIALS & STRUCTURES

MODULE NO: AME6012

Date: Monday 8th January 2024 **Time: 10:00 - 13:00**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Attempt FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

Q1.

a) A finite element analysis of a knee replacement part (shown in **Figs Q1a to Q1c** inclusive, shown on the following pages) estimated the following stress at one of the critical positions: Direct stresses: xx= 70 MPa tensile, yy= 15 MPa tensile. :zz= 35 MPa compressive accompanied by two shear stresses: xy= 38 MPa and yz= 23 MPa. Using this information:

(i) Sketch the elemental cube representing the state of stress. (3 marks)

(ii) Show that the characteristic equation representing the state of stress at this point is given as: $\sigma^3 - 50\sigma^2 - 3898\sigma + 23240 = 0$ and show the largest stress acting at this point is 90.3 MPa.
(7 marks)

(iii) Calculate direction of the largest stress and show this by a simple sketch.
(6 Marks)

b) If the yield stress of the material is 580 MPa determine the factor of safety at this point based upon the von Mises criterion.
(5 Marks)

c) The component was manufactured by stamping along the y direction. Explain how this would influence the choice of yield criteria and how this would change the von Mises criterion currently used. (4 Marks)

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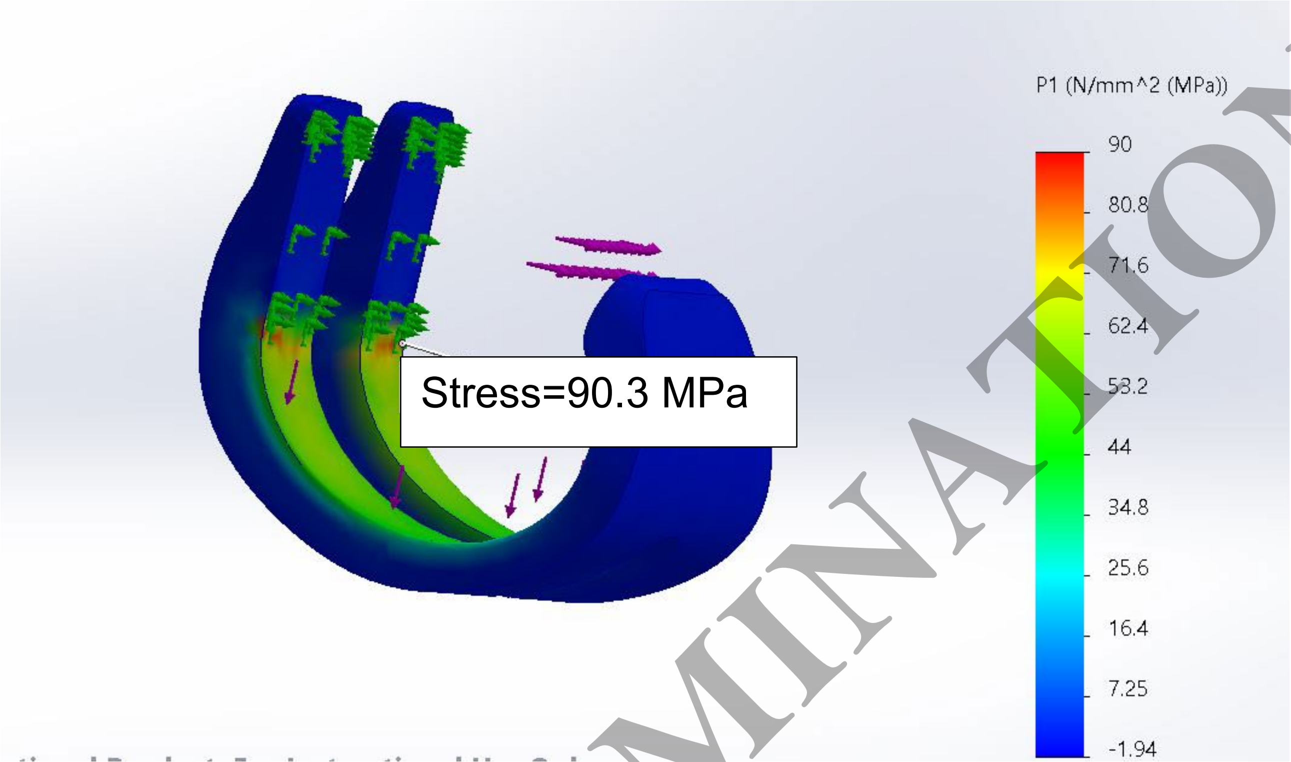

Fig Q1a P1 stress plot

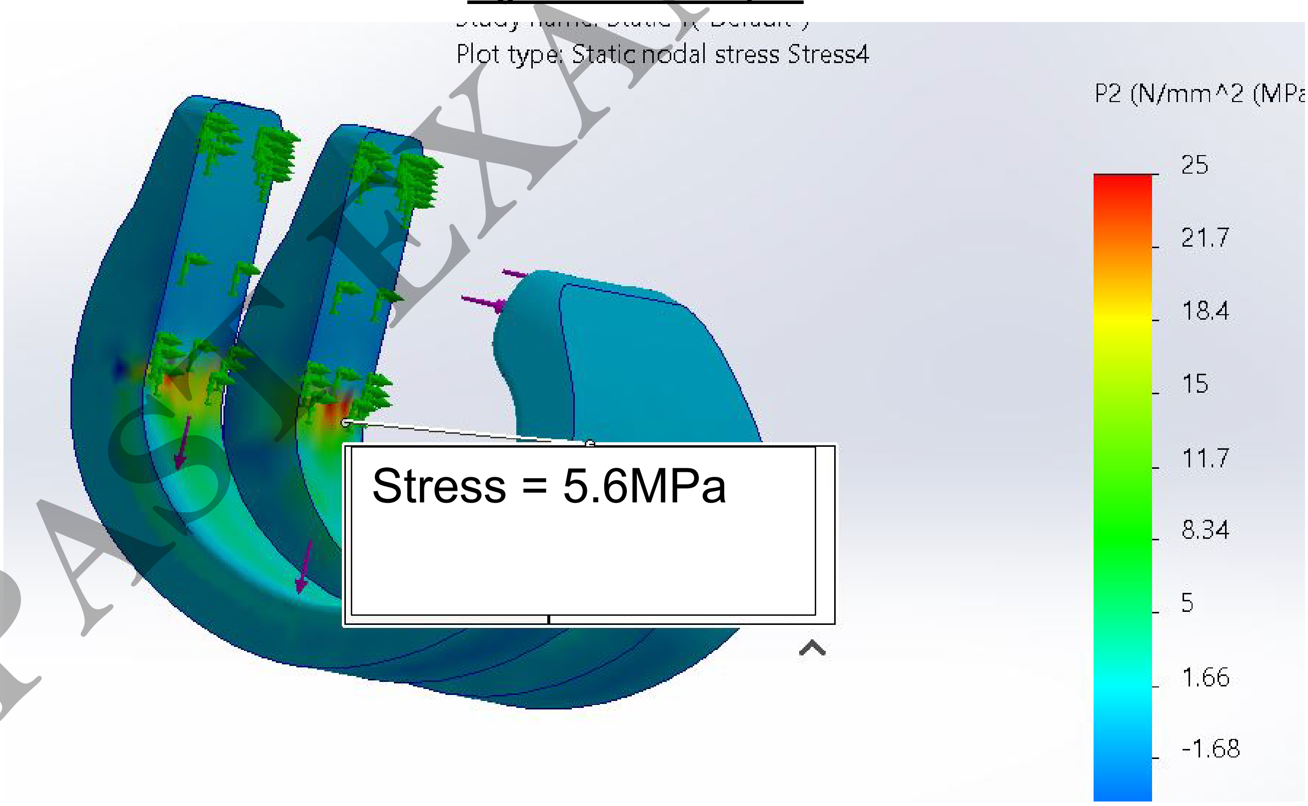

Fig Q1b P2 stress plot

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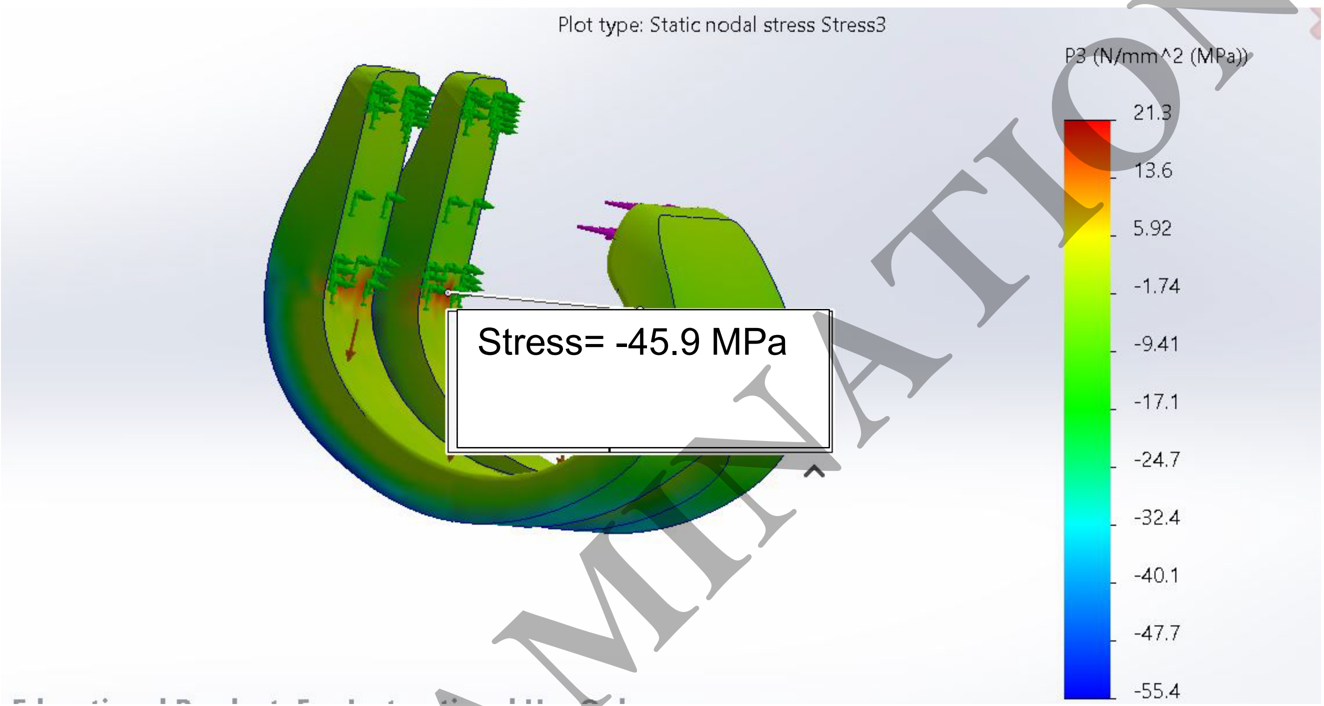

Fig Q1c P3 stress plot

Total 25 Marks

Q2.

a) A cryogenic chamber has a 25 mm wall thickness pipeline connected to it. The pipeline operates at -183°C. The pipeline can be susceptible to internal cracks at low temperatures. The material properties are given in **Table Q2**. The pipe is subjected to cyclic stresses ranging from 250MPa tensile to 150 MPa compressive every 30 minutes for fifteen hours per day on 6 days per week. The pipe is monitored regularly; however, the equipment used can only detect cracks larger than 3mm.

Using the above information and the material data in table Q2, determine the time taken for the crack to grow to 6mm. (11 Marks)

Table Q2	
Yield Strength	650 MPa
Young's Modulus	208 GPa
Poisson's Ratio	0.33
Fracture toughness at temperature T Kelvin	86-0.1T MPa.m$^{0.5}$
Paris coefficients M & C	3.1 & 1.2x10^{-12}
Shape factor Y	1.15

b) Also estimate how much longer life the pipe has under these conditions. (8 marks)

c) Explain briefly why this estimate is conservative and what other factors could be considered to improve the life predictions (6 Marks)

Total 25 Marks

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Semester 1 Examination 2023-24
Advanced Materials & Structures
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Q3.

a) **Figure Q3** shows schematically a portal frame representing a roll cage with worst case scenario load case with a horizontal load of 25 KN and a vertical load of 10 KN. Joints A, B and D can be assumed to be welded whilst joint C is a safety pin. Use this information to determine a suitable tubular section manufactured from steel with a yield stress of 640 MPa and a factor of safety of 4.

Assume for the analysis the material is rigid-perfectly plastic.

Take Z_p as D^2t where: D is the nominal bore and t the thickness of a tubular section.

(12 Marks)

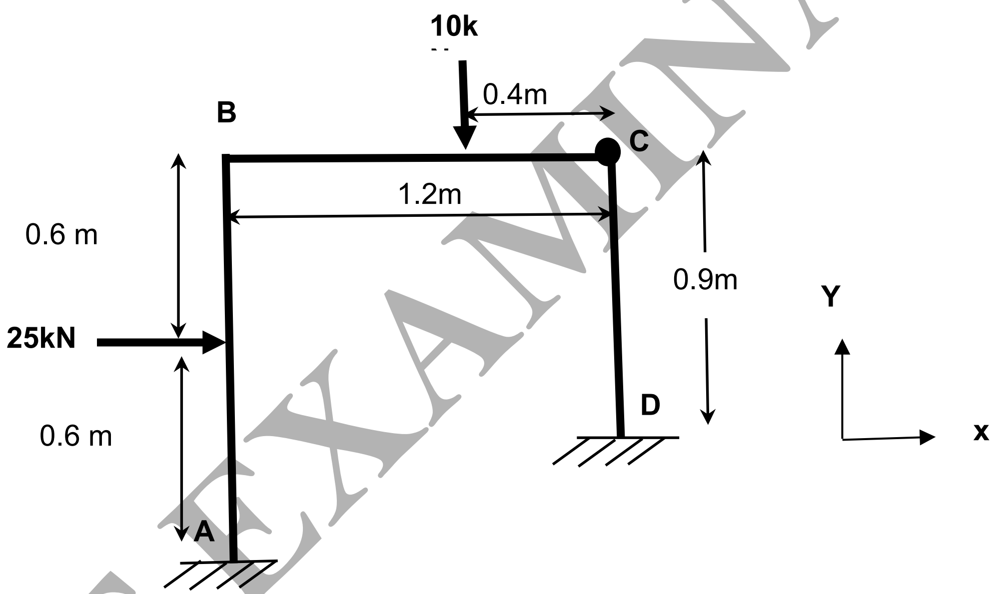

Fig Q3 frame set up

b) An alternative proposal is also considered with the same size tubing, but this time the 25KN load is acting 0.9m from A.and the safety pin is now at position D. Determine the new factor of safety.
(10 Marks)

c) Describe two other material models that could be used in place of the rigid perfectly plastic one stating in each case whether they would produce a higher or lower factor of safety. (4 Marks)

Total 25 Marks

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Q4.

a) A stage show musical has lifting mechanism for one of the actors (Figure Q4). In order to disguise the mechanism a beam fabricated from a rectangular cross section composite component using a high modulus carbon fibre reinforcement with an epoxy matrix in the form of a prepreg skin bonded to a 20mm thick balsa wood core to replace an existing metallic structure. The component is subject to both flexure and torsion; these loads are shown in **Table Q4**. Using this information determine a suitable lay up for the composite and illustrate this by a sketch.

(20 marks)

Fibre Modulus GPa	**Volume fraction %**	**Safe working strain %**	**Bond strength of skin MPa**	**Lamina Thickness mm**
380	65	0.5	12	0.125

Table Q4

b) If the component was to be used in varying temperatures due to the lighting system conditions describe what other factors, you would need to consider. (5 marks)

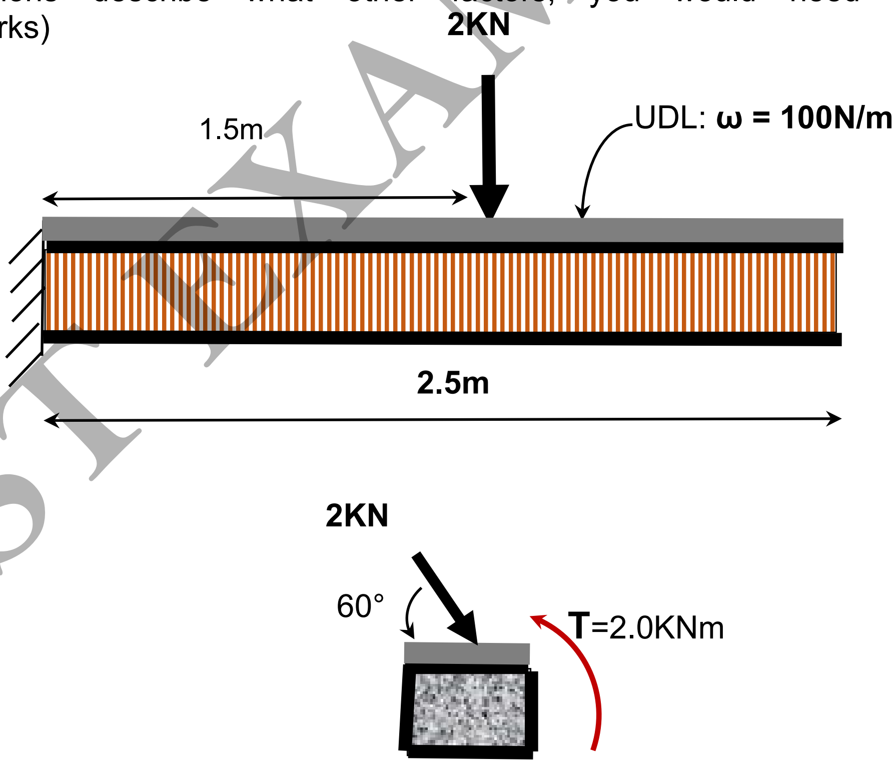

Fig Q4 schematic of the beam

Total 25 Marks

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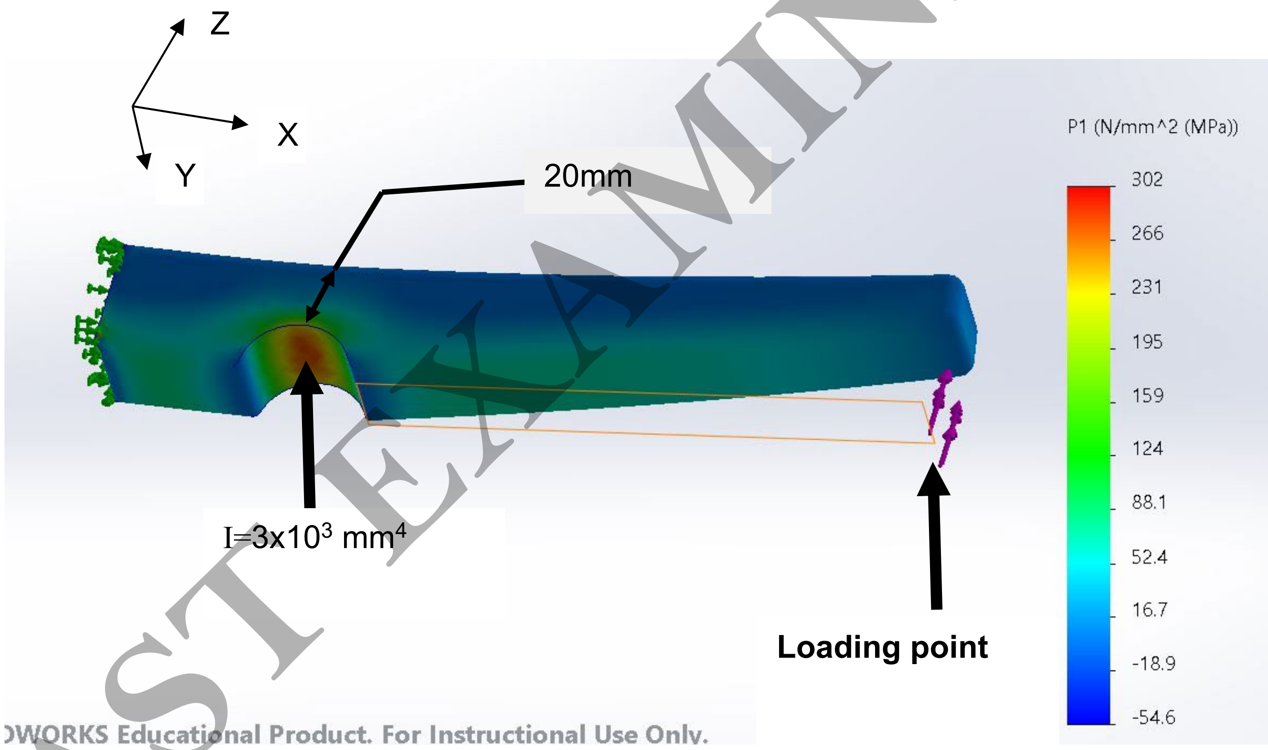
Q5.

a) A medical robot arm is manufactured from aluminium with a Young's modulus of 69 GPa and ν = 0.32 is to be evaluated for future use.

It is also expected that the component under its normal usage would be subjected to repeated cyclic loading with a maximum bending moment of 60Nm along with a lower load of 25Nm. Therefore a provisional FEA has been carried out and results for the P1 are shown in **fig Q5a** below.

Assuming at the position of largest stress, the 2nd moment of area is 3×10^3 mm^4 and maximum depth is 20 mm, hence, estimate the maximum stress and predict the life of the component under this condition. Given the S-N curve for material is shown in **Fig q5b** (shown on the following page). You can also assume for this geometry, K_t = 1.4 based on historic photoelastic test data and the notch sensitivity factor q= 0.8

(10 marks)



<ins>**FigQ5a FEA plot of predicted P1 stress values**</ins>

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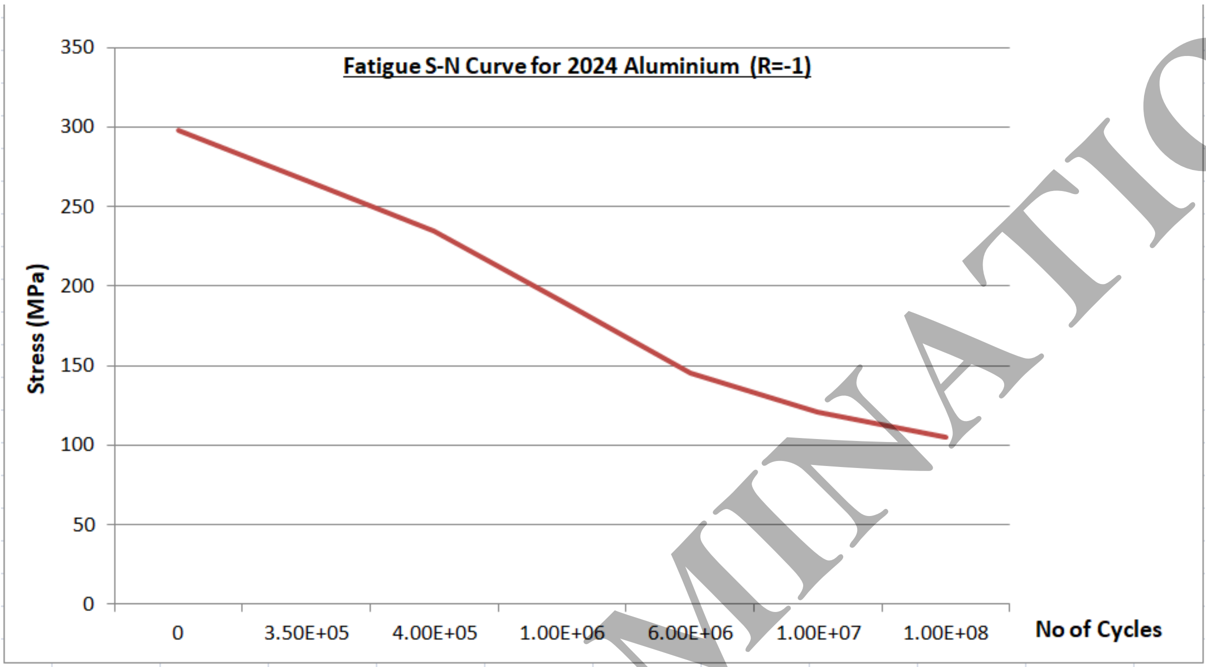

Fig Q5b Aluminium 2024 S-N curve for a fully reversed fatigue loading condition

b) In order to verify this design strain gauge techniques were used to evaluate the calculations. This was done using a strain gauge rosette consisting of three gauges in the pattern shown in figure **Q5c** (shown on the next page) bonded to the surface at an angle of 15° to the axis of symmetry. The gauges had a gauge length of 2mm and bonded using an epoxy adhesive. The output results under the maximum load condition for the three gauges are given below

ε_0 = 3100 x 10^{-6} mm/mm (0°)

ε_{45} = 2055 x 10^{-6} mm/mm (45°)

ε_{90} = -2779 x 10^{-6} mm/mm (90°)

Using this data calculate the maximum strain obtained and compare with the predicted experimental stress that was obtained using the finite element method. Explain also why there is a difference between the two results and where the main source of error is likely to occur.

(15 Marks)

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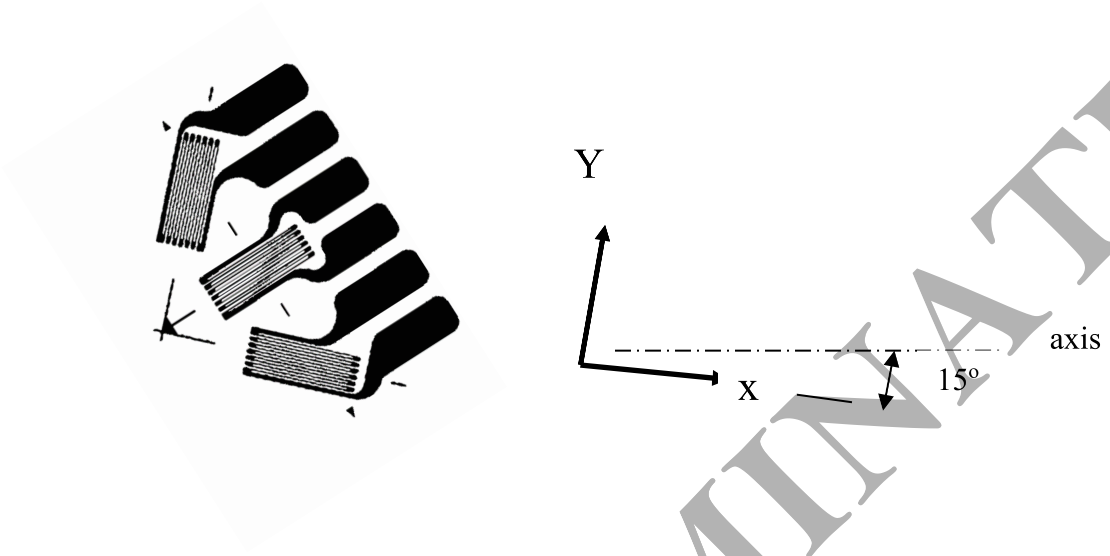

Fig Q5c Strain Gauge set up

End of the Questions

Formula sheet follows over the page

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FORMULA SHEET

Formulae used in Structures and Materials Module

Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = (Stress\ Tensor)\begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Where a, b and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \qquad l = \cos\alpha,$$
$$m = bk \qquad m = \cos\theta,$$
$$n = ck \qquad n = \cos\varphi.$$

Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

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Yield Criterion
Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

Tresca

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

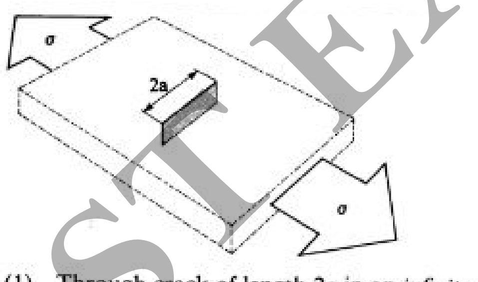
$$\sigma_{tresca} = 2\cdot\tau_{\max}$$

$$\tau_{\max} = \max\left(\frac{|\sigma_1-\sigma_2|}{2};\frac{|\sigma_1-\sigma_3|}{2};\frac{|\sigma_3-\sigma_2|}{2}\right)$$

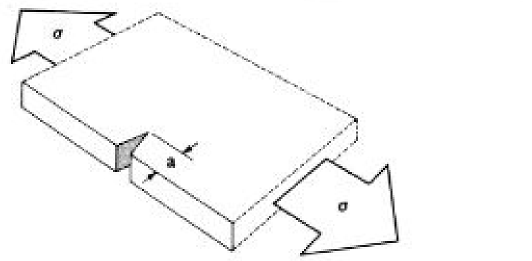
$$\frac{\sigma_{von\ Mises}}{\sigma_{Tresca}} = \frac{\sqrt{3}}{2}$$

Fracture mechanics

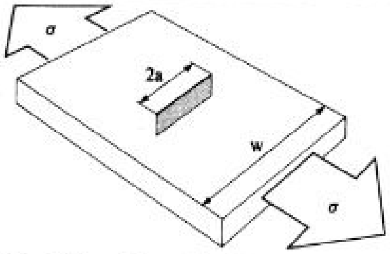
Table: Y values for plates loaded in tension



(1) Through crack of length $2a$ in an *infinite* plate
$Y = 1$

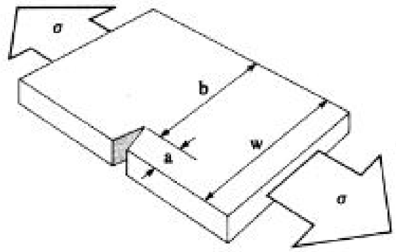


(2) Edge crack of length a in an *infinite* plate
$Y = 1.12$
Because plane strain and plane stress have identical stress fields, this calibration is also for an edge scratch of depth a on a large body carrying tensile stress σ.



(3) Through crack of length $2a$ in a plate of width w.

$$Y = \left(\sec\frac{\pi a}{w}\right)^{1/2}, \quad \frac{2a}{w} \leq 0.7$$



(4) Edge crack of length a in a plate of width w.

$$Y = 0.265\left(\frac{b}{w}\right)^4 + \frac{0.875 + 0.265a/w}{(b/w)^{3/2}}$$

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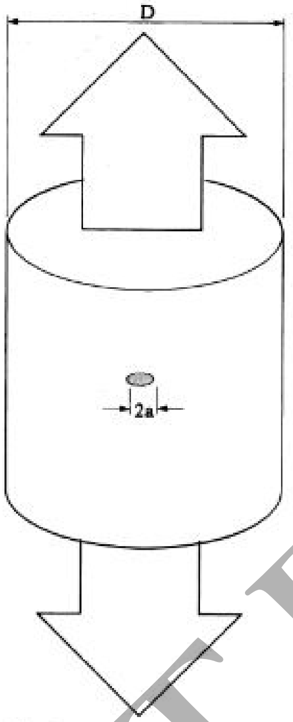
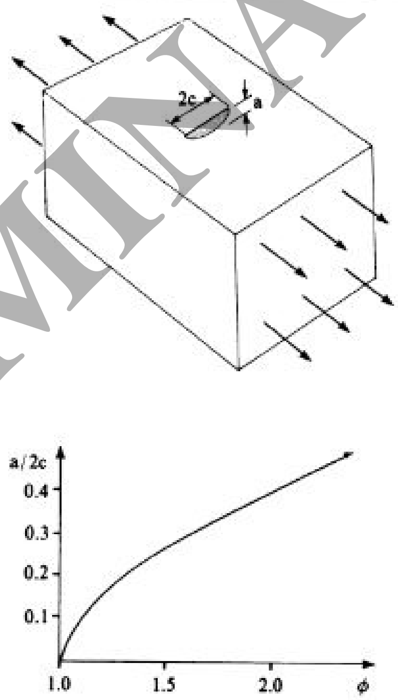


(5) Penny-shaped internal crack of radius a.

$$Y = \frac{2}{\pi}, \quad a \ll D$$



(6) Semi-elliptical surface flaw

$$Y = \frac{1.12}{\phi^{1/2}}$$

Life Calculations

$$\frac{da}{dN} = C\left(\Delta K\right)^m$$

$$N = \frac{1}{CY^m \sigma_a{}^m \pi^{\frac{m}{2}}} \int_{a_0}^{a_1} \frac{da}{a^{\frac{m}{2}}}$$

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Composite materials

$$E_{composite} = E_{fibre}V_{fibre} + E_{matrix}(1 - V_{fibre})$$

Fracture Toughness

Table: Fracture toughness of some engineering materials

Material	K_{IC} (MNm$^{-3/2}$)	E (GN/m^2)	$\mathcal{G}_{IC}$ (kJ/m^2)
Plain carbon steels	140 - 200	200	100 - 200
High strength steels	30 - 150	200	5 - 110
Low to medium strength steels	10 - 100	200	0.5 - 50
Titanium alloys	30 – 120	120	7 – 120
Aluminium alloys	22 – 33	70	7 - 16
Glass	0.3 – 0.6	70	0.002 – 0.008
Polycrystalline alumina	5	300	0.08
Teak – crack moves across the grain	8	10	6
Concrete	0.4	16	1
PMMA (Perspex)	1.2	4	0.4
Polystyrene	1.7	3	0.01
Polycarbonate (ductile)	1.1	0.02	54
Polycarbonate (brittle)	0.4	0.02	6.7
Epoxy resin	0.8	3	0.2
Fibreglass laminate	10	20	5
Aligned glass fibre composite – crack across fibres	10	35	3
Aligned glass fibre composite – crack down fibres	0.03	10	0.0001

Aligned carbon fibre composite – crack across fibres	20	185	2

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Strain relationships

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

where ε_x = normal strain at a point in x-direction

ε_y = normal strain at a point in y- direction

γ_{xy} = shear strain at a point on x face in y direction

$$\text{2D Strain tensor} = \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_y \end{bmatrix}$$

Hooke's Law in 2D

$$\sigma_1 = \frac{E}{(1-\upsilon^2)}(\varepsilon_1 + \upsilon\varepsilon_2)$$

$$\sigma_2 = \frac{E}{(1-\upsilon^2)}(\varepsilon_2 + \upsilon\varepsilon_1)$$

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TABLE 13.2 Formulas for Values of the Maximum Principal Stresses and Maximum Deflections in Circular Plates as Obtained by Theory of Flexure of Plates[a]

Support and loading	Principal stress (σ_{max})	Point of maximum stress	Maximum deflection (w_{max})
Edge simply supported; load uniform ($r_0 = a$)	$\frac{3}{8}(3+\nu)\,p\,\frac{a^2}{h^2}$	Center	$\frac{3}{16}(1-\nu)(5+\nu)\frac{pa^4}{Eh^3}$
Edge fixed; load uniform ($r_0 = a$)	$\frac{3}{4}p\,\frac{a^2}{h^2}$	Edge[b]	$\frac{3}{16}(1-\nu^2)\frac{pa^4}{Eh^3}$
Edge simply supported; load at center. $P = \pi r_0^2 p$, $r_0 \to 0$, but $r_0 > 0$	$\frac{3(1+\nu)}{2\pi h^2}P\left(\frac{1}{\nu+1} + \ln\frac{a}{r_0} - \frac{1-\nu}{1+\nu}\frac{r_0^2}{4a^2}\right)$	Center	$\frac{3(1-\nu)(3+\nu)Pa^2}{4\pi Eh^3}$
Fixed edge; load at center. $P = \pi r_0^2 p$, $r_0 \to 0$, but $r_0 > 0$	$\frac{3(1+\nu)}{2\pi h^2}P\left(\ln\frac{a}{r_0} + \frac{r_0^2}{4a^2}\right)$ a must be $> 1.7r_0$	Center	$\frac{3(1-\nu^2)Pa^2}{4\pi Eh^3}$

[a] a = radius of plate; r_0 = radius of central loaded area; h = thickness of plate; p = uniform load per unit area; ν = Poisson's ratio.
[b] For thicker plates ($h/r > 0.1$), the deflection is $w_{max} = C\left(\frac{3}{16}\right)(1-\nu^2)(pa^4/Eh^3)$, where the constant C depends on the ratio h/a as follows: $C = 1 + 5.72(h/a)^2$.

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$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=-\frac{Q_r}{D}$$

Hooke's law is expressed in terms of w, as follows

$$\sigma_r=\frac{E}{1-\upsilon^2}(\varepsilon_r+\upsilon\varepsilon_\theta)=-\frac{Ez}{1-\upsilon^2}\left(\frac{d^2w}{dr^2}+\frac{\upsilon}{r}\frac{dw}{dr}\right)$$

$$\sigma_\theta=\frac{E}{1-\upsilon^2}(\varepsilon_\theta+\upsilon\varepsilon_r)=-\frac{Ez}{1-\upsilon^2}\left(\frac{1}{r}\frac{dw}{dr}+\upsilon\frac{d^2w}{dr^2}\right)$$

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Bending moment and shear force

$$M_r=-D\left(\frac{d^2w}{dr^2}+\frac{\upsilon}{r}\frac{dw}{dr}\right),\ D=\frac{Et^3}{12\left(1-\upsilon^2\right)}$$

$$M_\theta=-D\left(\frac{1}{r}\frac{dw}{dr}+\upsilon\frac{d^2w}{dr^2}\right)$$

$$Q_r=-\frac{1}{2\pi r}\int_0^{2\pi}\int_b^r qr\,dr\,d\theta=-\frac{1}{r}\int_b^r qr\,dr$$

Governing equation

$$\nabla^4w=\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)w=\frac{q}{D}$$

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Related Mathematics

Cubic Equations-General form

,

$\sigma^3 + F_1 \sigma^2 + F_2\sigma + F_3 = 0$ where: F_1, F_2, & F_3 are constants then the solution has three roots, say a, b & c, giving: $(\sigma\text{-}a).(\sigma\text{-}b).(\sigma\text{-}c) = 0$,

hence,

$\sigma^3 – \sigma^2 (a+b+c) + \sigma (a+c)b – abc = 0$

as a general form.

If either a, b or c is known a simple quadratic equation based upon the other two unknowns can derived and solved.

Position of the Maximum moment of a propped cantilever length L is given by:

(√2-1) L from the prop end

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Finding determinants using cofactors

Sign of cofactor

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

Find determinants

$$2\begin{vmatrix} 0 & 4 \\ -1 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$$

$$2[(0\times 2)-(-1\times 4)]-4[(1\times 2)-(2\times 4)]-3[(1\times -1)-(0\times 2)]$$

$$8+24+3=35$$

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