[ENG06]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BSc (HONS) MECHATRONICS (TOP-UP)

EXAMINATION FOR SEMESTER 1 - 2022/2023

ADVANCED MECHATRONIC SYSTEMS

MODULE NO: MEC6002

Date: Monday 9th January 2023 **Time: 10:00 - 12:00**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

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Question 1

If the position control system has experienced a disturbance D(s), and a gain K has been inserted into the system as shown in Figure Q1, determine the:

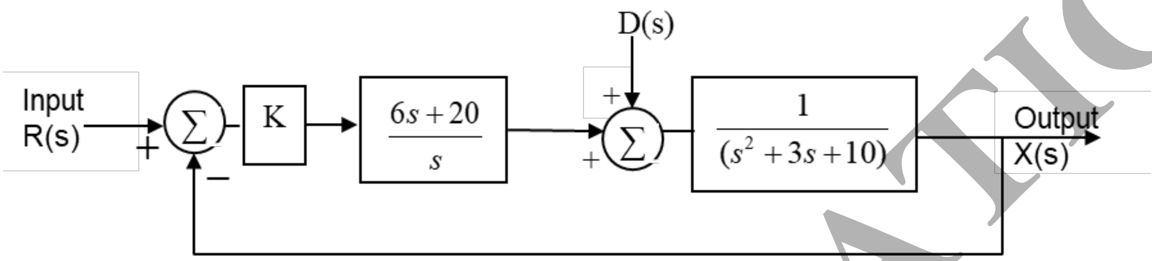

Figure Q1: Mechatronic system with disturbance

a) whole system's output X(s) function **[8 marks]**

b) Range of values of K for the system which will result in stability, using Routh-Hurwitz stability criterion. **[10 marks]**

c) Steady-state error if the disturbance D(s) = 0, R(s) is a unit ramp input, and K = 2. **[7 marks]**

Total 25 marks

Question 2

Figure Q2 shows a mechatronic control system, which requires high accuracy for position and velocity response. If the mechatronic system has a transfer function of:

$$G_p(s) = \frac{1}{(2s+6)(s+5)}$$

and a PID controller is used in the system.

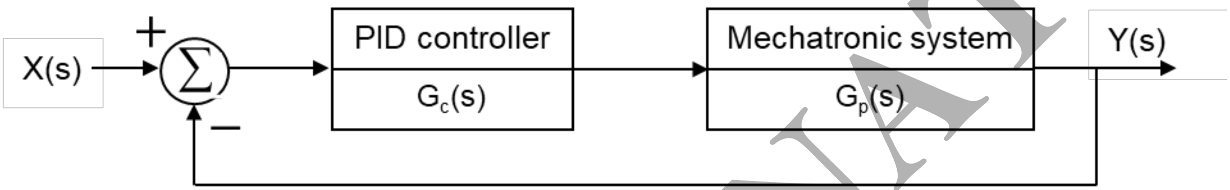

Figure Q2: A Mechatronic Control System

The design criteria for this system include Settling time < 0.8 sec, Overshoot < 5% and Steady state error = 0.5 (for a unit ramp input = $1/s^2$). Using these information,

a) Design a PID controller to determine the parameters K_p, K_i, and K_d and clearly identify the design procedure. **[19 Marks]**

b) Explain the effects on a control system of including Proportional controller (P), Proportional + Integral (PI) controller, Proportional + Derivative (PD) controller, and PID controller. **[6 marks]**

Total 25 marks

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Question 3

a) If a controller has a 16 bit Analogue to Digital Converter with the signal range between -32 Volts to +32 Volts:

(i) What is the resolution of the AD converter? **[4 marks]**

(ii) What integer number represented a value of 12 Volts? **[4 marks]**

(b) A controller consists of a Digital to Analogue Converter (DAC) with zero order element that is connected in series with the processing centre. The transfer function of the processing centre is represented as:

$$G_{p(s)} = \frac{10}{s+10}$$

and the full DAC system is as shown in Figure Q3(b).

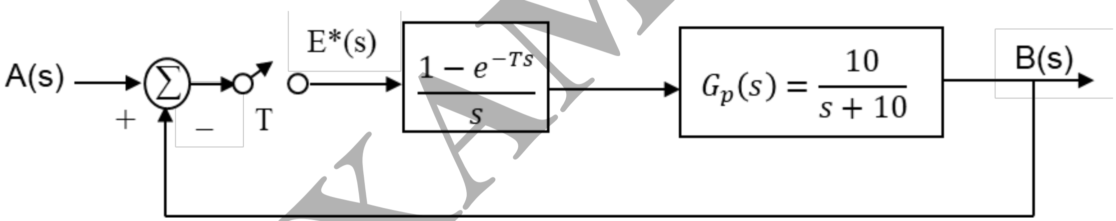

Figure Q3(b) Digital Feedback Control System

(i) Find the sampled-data transfer function G(z) for the DAC control system. Assume that the sampling time, T, is 0.05 seconds. **[10 marks]**

(ii) Determine the z-transfer of the closed-loop system T(z) **[2 marks]**

(iii) Determine whether the system is stable or not. **[5 marks]**

Total 25 marks

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Question 4

a) Z-transform and Laplace transform are useful tools in signal processing. Considering an digital and analogue system,

i) Describe the difference between Z-transform and Laplace transform. **[2 marks]**

ii) Using suitable illustrative examples, explain the relationships between z-plane and s-plane. **[6 marks]**

b) If the transfer function of the mechatronic system is given by

$$H(s) = \frac{s+2}{s^2 - 5s + 6}$$

i) Determine the poles and zeros of the system. **[3 marks]**

ii) A robot control system can be represented by the block diagram shown in Figure Q4(a). Using block diagram reduction techniques, find the θ_o for this control system if a unit step input is applied. **[14 marks]**

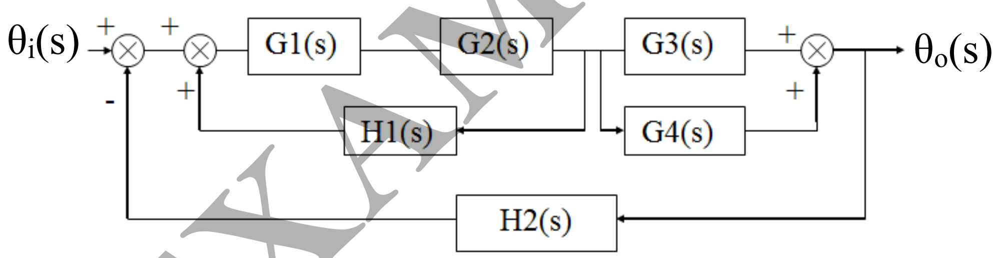

Figure Q4 (a) A robot control system, where $\theta_i(s)$ is the input and $\theta_o(s)$ is the output.

Total 25 marks

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Question 5

NASA has designed a mechatronic communication satellite to communicate with a Mars Exploration Rover which will be used to collect data from the Rover on its mission in space. The transfer function of the Rover system is expressed as

$$H(s) = \frac{s^2 + 0.2s + 1}{s^2 + 0.7s + 1} .$$

After the Mar Exploration Rover was launched into the Mars planetary space, a step input signal was sent to assess that the Rover landed successfully and it is able to communicate reliable data. The unit step input response received was transmitted to the Earth's ground station as shown in Figure Q5.

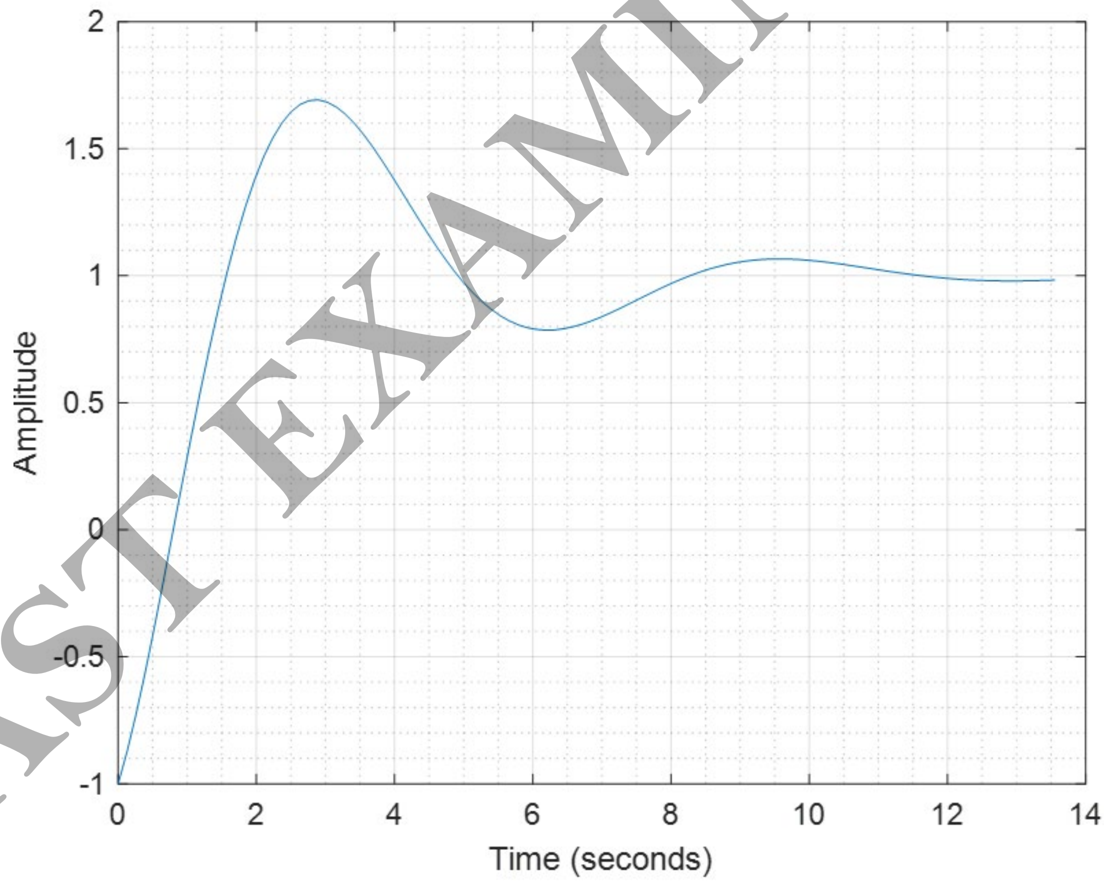

Figure Q5: Unit step input response of the system

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Q5 cont…

Based on the information provided above and using the unit step response as shown in Figure Q5, determine the:

(i) 100% rise time t_r, **[3 marks]**

(ii) percentage maximum overshoot, **[3 marks]**

(iii) Peak time t_p of the output. **[3 marks]**

(iv) 2% settling time t_s, **[3 marks]**

(v) damping factor ζ, **[3 marks]**

(vi) system's natural angular frequency, ω_n, **[5 marks]**

(vii) Damped angular frequency ω_d. **[5 marks]**

Total 25 marks

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Question 6

(a) A RLC circuit is shown in Figure Q6(a), where C is the Capacitance, L is the Inductance, R is Resistance, $I_1(t)$ is the total current and V(t) is the source voltage.

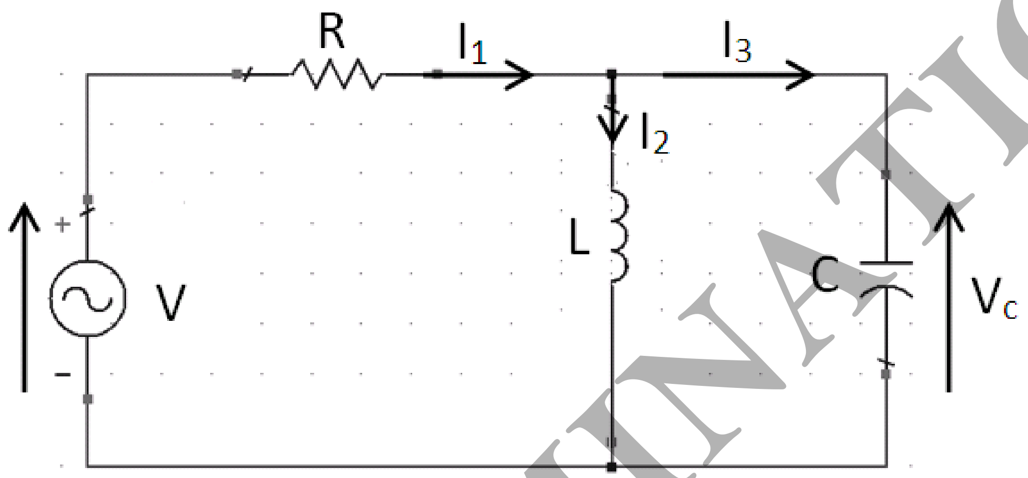

Figure Q6(a): RLC electrical circuit

(i) Develop a differential equation for the RLC electrical circuit shown in Figure Q6(a). **[8 marks]**

(ii) Determine the Laplace transforms of the differential equations obtained from (i) above. Assume that the system is subjected to a unit step input, the initial conditions of the system are zeros (i.e. at time = 0, *x*, *x'*, *x''* are all zeros) and the capacitor is initially discharged as the following expression. **[2 marks]**

(iii) Determine the transfer function $G(s) = V_c(s)/V(s)$, if R = 0.4 Ω, C = 0.5 F and L = 333.33×10^{-3} H. **[2 marks]**

Question 6 continues the next page…

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(b) A suspension system for a mobility-scooter is shown in Figure 6(b), where f(t) is the input force, y1(t) and y2(t) represents the output displacements, k1, k2 and k3 are the spring stiffness constants, C is the viscous damping coefficient.

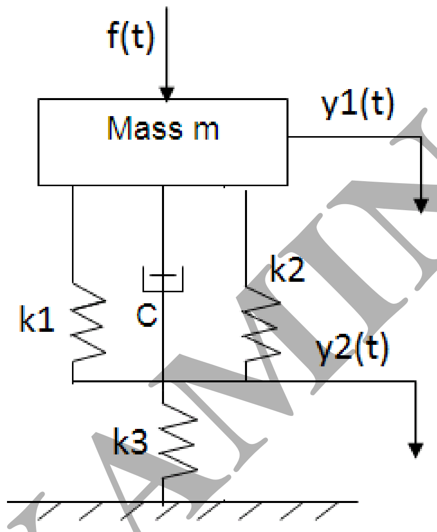

Figure Q6(b) A suspension system of a mobility-scooter

(i) Develop the differential equations for the suspension system. **[3 marks]**

(ii) Determine the Laplace transforms of the differential equations obtained from Q6(b)(i). Assume that the system is subjected to a unit step input, y(0)=0 and y'(0)=0. **[6 marks]**

(iii) Determine the transfer function G(s) = Y1(s)/F(s), if m =1 kg, k1 = 2k2 N/m, k2 =1 N/m, k3 = 0.5 N/m, C =1 Ns/m and F = 1 N. **[4 marks]**

Total 25 marks

END OF QUESTIONS

Formula Sheets follow over the page

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FORMULA SHEET

F1. The Laplace and Z-Transform Table

Laplace Transform	Time Function	z-Transform
1	Unit impulse $\delta(t)$	1
$\frac{1}{s}$	Unit step $u_s(t)$	$\frac{z}{z-1}$
$\frac{1}{1-e^{-Ts}}$	$\delta_T(t) = \sum_{n=0}^{\infty} \delta(t - nT)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
$\frac{1}{s^{n+1}}$	$\frac{t^n}{n!}$	$\lim_{\alpha \to 0} \frac{(-1)^n}{n!} \frac{\partial^n}{\partial \alpha^n}\left[\frac{z}{z-e^{-\alpha T}}\right]$
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$	$\frac{z}{z-e^{-\alpha T}}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$	$\frac{Tze^{-\alpha T}}{(z-e^{-\alpha T})^2}$
$\frac{\alpha}{s(s+\alpha)}$	$1 - e^{-\alpha t}$	$\frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}$
$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$
$\frac{\omega}{(s+\alpha)^2+\omega^2}$	$e^{-\alpha t} \sin \omega t$	$\frac{ze^{-\alpha T}\sin \omega T}{z^2 - 2ze^{-\alpha T}\cos \omega T + e^{-2\alpha T}}$
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$
$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$	$e^{-\alpha t} \cos \omega t$	$\frac{z^2 - ze^{-\alpha T}\cos \omega T}{z^2 - 2ze^{-\alpha T}\cos \omega T + e^{-2\alpha T}}$

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F2. Laplace Transforms od Specific Functions

A unit impulse function	1
A unit step function	$\frac{1}{s}$
A unit ramp function	$\frac{1}{s^2}$

F3. Steady-state error (Unity feedback) for different inputs

$$\boxed{e_{ss}(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\,\frac{R(s)}{1+G(s)}}$$

For Unit step input:
$$\boxed{e_{ss}(\infty) = e_{step_{ss}}(\infty) = \lim_{s\to 0}\frac{1}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)}}$$

For Ramp input:
$$\boxed{e_{ss}(\infty) = e_{ramp_{ss}}(\infty) = \lim_{s\to 0}\frac{1}{s(1+G(s))} = \frac{1}{\lim_{s\to 0} sG(s)}}$$

For Parabola input:
$$\boxed{e_{ss}(\infty) = e_{para_{ss}}(\infty) = \lim_{s\to 0}\frac{1}{s^2(1+G(s))} = \frac{1}{\lim_{s\to 0} s^2G(s)}}$$

F4. Blocks with feedback loop

G(s) = $\frac{Go(s)}{1+Go(s)H(s)}$ (for a negative feedback)

G(s) = $\frac{Go(s)}{1-Go(s)H(s)}$ (for a positive feedback)

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F5. Steady-State Errors

$e_{ss} = \lim_{s\to 0}[s(1 - G_O(s))\theta_i(s)]$ (for an open-loop system)

$e_{ss} = \lim_{s\to 0}[s\dfrac{1}{1+G_o(s)}\theta_i(s)]$ (for the closed-loop system with a unity feedback)

$e_{ss} = \lim_{s\to 0}[s\dfrac{1}{1+\dfrac{G_1(s)}{1+G_1(s)[H(s)-1]}}\theta_i(s)]$ (if the feedback H(s) ≠ 1)

$e_{ss} = \lim_{s\to 0}[-s\cdot\dfrac{G_2(s)}{1+G_2(G_1(s)+1)}\cdot\theta_d]$ (if the system subjects to a disturbance input)

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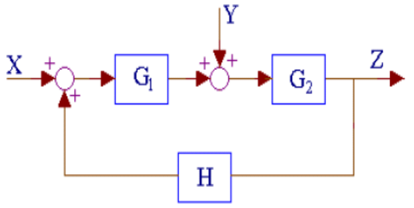
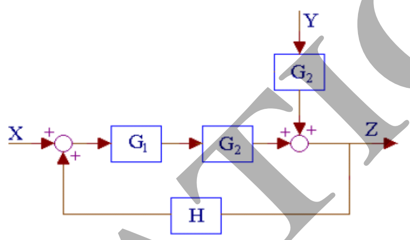
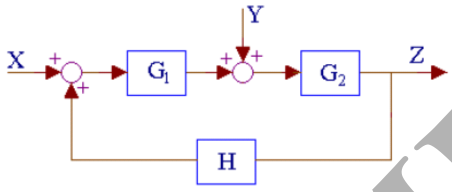
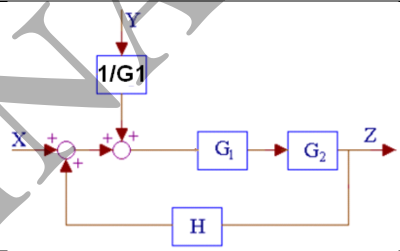
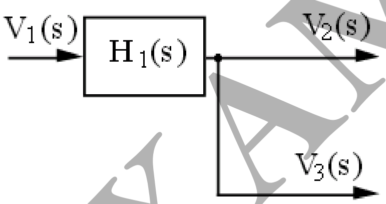
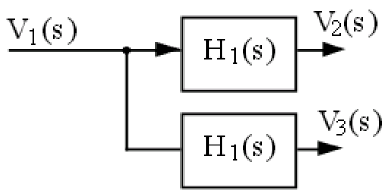
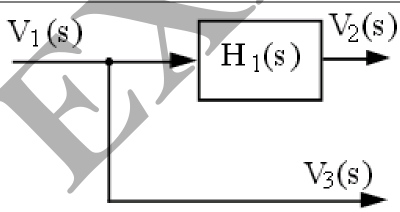
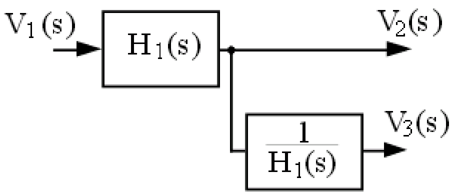
F6. Block Diagram Algebra

Rule	Original Diagram	Equivalent Diagram
1. Moving a summing point beyond a block		
2. Moving a summing point in front a block		
3. Moving a takeoff point to front of a block		
4. Moving a takeoff point to beyond a block		

F7. First order Systems

$$G(s) = \frac{\theta_o}{\theta_i} = \frac{G_{ss}(s)}{\tau s + 1}$$

$$\tau\left(\frac{d\theta_o}{dt}\right) + \theta_o = G_{ss}\theta_i$$

$\theta_O = G_{ss}(1 - e^{-t/\tau})$ (for a unit step input)

$\theta_o(t) = G_{ss}[t - \tau(1 - e^{-(t/\tau)})]$ (for a unit ramp input)

$\theta_o(t) = G_{ss}(\frac{1}{\tau})e^{-(t/\tau)}$ (for an impulse input)

F8. First order System (non-zero initial condition)

$$\theta_{o(total)}(t) = \theta_{o(final)} + \theta_{o(initial)}(t)$$

where $\theta_{o(initial)}(t) = \theta_o(0)\left[e^{-(t/\tau)}\right]$

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F9. Second order Systems

$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega_n \frac{d\theta_o}{dt} + \omega_n^2\theta_o = b_o\omega_n^2\theta_i$$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{b_o\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_d t_s = (1/2)\pi$ $\quad \omega_d t_p = \pi$

Percentage Overshoot (P.O) = exp $\left(\frac{-\zeta\pi}{\sqrt{(1-\zeta^2)}}\right) \times 100\%$

For 2% settling time: $t_s = \frac{4}{\zeta\omega_n}$

For 5% settling time: $t_s = \frac{3}{\zeta\omega_n}$

$\omega_d = \omega_n\sqrt{1-\varsigma^2}$

Subsidence ratio: $= e^{\left(\frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$

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