OCD028

UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

WESTERN INTERNATIONAL COLLEGE

BENG(HONS) ELECTRICAL AND ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATIONS 2022/2023

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Tuesday, 10 January 2023 Time: 10:00 – 12:30

INSTRUCTIONS TO CANDIDATES:

There are SIX questions on this paper.

Answer ANY FOUR questions.

Silent calculators may be used.

This is a closed book assessment.

All questions carry equal marks.

Q1.

A. A. Given point P (- 2, 6, 3) and vector A = $ya_x + (x + z) a_y$, express P and A in cylindrical and spherical coordinates. Evaluate A at Pin the Cartesian coordinate system.

[10 marks]

B. Two uniform vector fields are given by E = $-5a_\rho + 10a_\varnothing + 3a_z$ and F = $a_\rho + 2a_\varnothing - 6a_z$. Calculate |E x F|

[3 marks]

C. Consider the object shown in Figure 1.

Calculate

(i) The length BC

[4 marks]

(ii) The surface area ABO

[2 marks]

(iii) The length CD

[2 marks]

(iv) The surface area ABCD

[3 marks]

(v) The volume ABDCFO.

[3 marks]

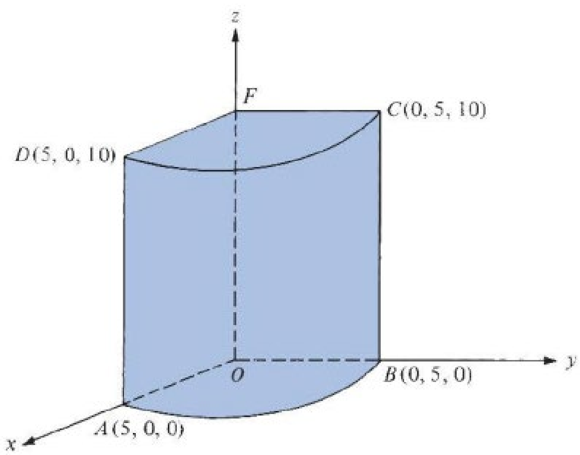

Figure 1

Total 25 marks

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Q2.

A. Find the gradient of the scalar field: $V = e^{-z} \sin 2x \cosh y$

[3 marks]

B. Determine the divergence and curl of the vector field: $P = x^2 yza_x + xza_z$

[7 marks]

C. The finite sheet $0 < x < 1$, $0 < y < 1$ on the $z = 0$ plane has a charge density Ps $= xy(x^2 + y^2 + 25)^{3/2}$ nC/m^2. Find

(i) The total charge on the sheet

[7 marks]

(ii) The electric field at (0, 0, 5)

[8 marks]

Total 25 marks

Q3.

A. Given that $D = z\rho\cos^2 a_z$ C/m^2, calculate the charge density at (1, Π/4,3) and the total charge enclosed by the cylinder of radius 1 m with $- 2 < z < 2$ m.

[7 marks]

B. Given the potential $V = \frac{10}{r^2}\sin\theta\ \cos\text{Ø}$, find the electric flux density D at (2, Π/2, 0).

[2 marks]

C. If $J = \frac{1}{r^3}(2\cos\theta\, a_r + \sin\theta\, a_{\text{Ø}})$ A/m^2 calculate the current passing through a hemispherical shell of radius 20 cm, 0 < θ < Π/2, 0 < Ø < 2Π.

[8 marks]

Q3 continued over the page…

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Q3 continued…

D. Current-carrying components in high-voltage power equipment can be cooled to carry away the heat caused by ohmic losses. A means of pumping is based on the force transmitted to the cooling fluid by charges in an electric field. Electro hydrodynamic (EHD) pumping is modelled in Figure 2. The region between the electrodes contains a uniform charge ρ_0, which is generated at the left electrode and collected at the right electrode. Calculate the pressure of the pump if $\rho_0 = 25$ mC/m^3 and $V_0 = 22$ kV.

[8 marks]

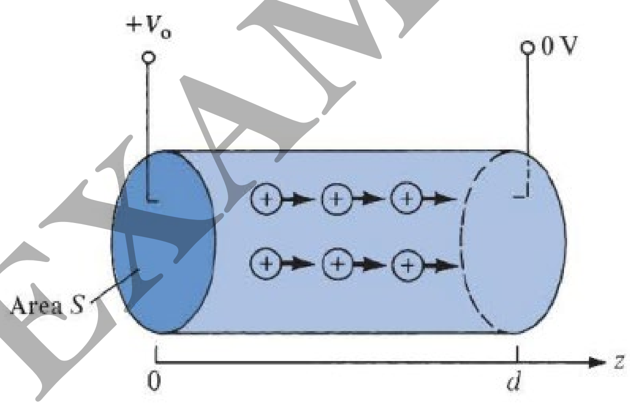

Figure 2

Total 25 marks

Q4.

A. Given the magnetic vector potential

$$A = -\frac{\rho^2}{4} Wb/m$$

Calculate the total magnetic flux crossing the surface

$$\emptyset = \frac{\Pi}{2}, 1 \le \rho \le 2m, 0 \le z \le 5m$$

[5 marks]

B. An electric field in free space is given by $E = 50\ \cos(10^8 T + {}^{\beta}x)\ a_y\ V/m$

(i) Find the direction of wave propagation.

[5 marks]

(ii) Calculate β and the time it takes to travel a distance of ***λ***/2.

[5 marks]

(iii) Sketch the wave at t = 0

[2 marks]

C. An EM wave travels in free space with the electric field component

$$E = 100\ e^{j(0.866y+0.5z)} a_y V/m$$

Determine

(i) ω and ***λ***

[3 marks]

(ii) The magnetic field component

[3 marks]

(iii) The time average power in the wave

[3 marks]

Total 25 marks

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Engineering Electromagnetism
Module No. EEE6012

Q5.

A. A transmission line 2 m long operating at ω= 10^6 rad/s has α = 8 dB/m, β = 1 rad/m, and Z_0= 60 + j40 Ω. If the line is connected to a source of 10 < 0° V, Z_g = 40 Ω and terminated by a load of 20 + j50 Ω, determine

(i) The input impedance

[7 marks]

(ii) The sending-end current

[3 marks]

(iii) The current at the middle of the line.

[12 marks]

B. A lossless transmission line with Z_0 = 50 Ω is 30 m long and operates at 2 MHz. The line is terminated with a load Z_i = 60 + j40Ω. If u = 0.6c on the line, find

(i) The reflection coefficient r

[2 marks]

(ii) The standing wave ratio s

[1 mark]

Total 25 marks

Q6.

An antenna with a circular aperture of diameter 3meters is desired to operate at 5GHz.The radiation resistance of it is given as 72Ω and a loss resistance as 8Ω.

A. Determine the power being radiated by the antenna which is drawing a current of 8 Ampere

[2 marks]

B. Calculate

(i) the capture area

[5 marks]

(ii) directivity gain in dB

[5 marks]

(iii) Q factor of the antenna

[4 marks]

C. An S-band radar transmitting at 3 GHz radiates 200 kW. Determine the signal power density at ranges 100 and 400 nautical miles if the effective area of the radar antenna is 9 m^2 . With a 20 m^2target at 300 nautical miles, calculate the power of the reflected signal at the radar.

[9 marks]

Total 25 marks

END OF QUESTIONS

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EQUATION SHEET

CIRCULAR CYLINDRICAL COORDINATES (ρ, ϕ, z)

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

SPHERICAL COORDINATES (r, θ, ϕ)

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \quad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

DIFFERENTIAL LENGTH, AREA, AND VOLUME

A. Cartesian Coordinate Systems

1. Differential displacement is given by

$$d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$$

2. Differential normal surface area is given by

$$d\mathbf{S} = \begin{array}{l} dy\, dz\, \mathbf{a}_x \\ dx\, dz\, \mathbf{a}_y \\ dx\, dy\, \mathbf{a}_z \end{array}$$

3. Differential volume is given by

$$dv = dx\, dy\, dz$$

B. Cylindrical Coordinate Systems

1. Differential displacement is given by

$$d\mathbf{l} = d\rho\, \mathbf{a}_\rho + \rho\, d\phi\, \mathbf{a}_\phi + dz\, \mathbf{a}_z$$

2. Differential normal surface area is given by

$$d\mathbf{S} = \begin{array}{l} \rho\, d\phi\, dz\, \mathbf{a}_\rho \\ d\rho\, dz\, \mathbf{a}_\phi \\ \rho\, d\rho\, d\phi\, \mathbf{a}_z \end{array}$$

and illustrated in Figure 3.4.

3. Differential volume is given by

$$dv = \rho\, d\rho\, d\phi\, dz$$

C. Spherical Coordinate Systems

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2. The differential normal surface area is

$$d\mathbf{S} = r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a}_r \\ r \sin\theta\, dr\, d\phi\, \mathbf{a}_\theta \\ r\, dr\, d\theta\, \mathbf{a}_\phi$$

3. The differential volume is

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

DEL OPERATOR

$$\nabla = \frac{\partial}{\partial x}\mathbf{a}_x + \frac{\partial}{\partial y}\mathbf{a}_y + \frac{\partial}{\partial z}\mathbf{a}_z$$

$$\nabla = \mathbf{a}_\rho \frac{\partial}{\partial \rho} + \mathbf{a}_\phi \frac{1}{\rho}\frac{\partial}{\partial \phi} + \mathbf{a}_z \frac{\partial}{\partial z}$$

$$\nabla = \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \mathbf{a}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

GRADIENT OF A SCALAR

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

DIVERGENCE OF A VECTOR

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \mathbf{A}_\phi}{\partial \phi}$$

CURL OF A VECTOR

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\,\mathbf{a}_\phi & \mathbf{a}_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \mathbf{a}_r & r\,\mathbf{a}_\theta & r\sin\theta\,\mathbf{a}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{A}\, dv = 0$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{Q}$$

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\mathbf{a}_r$$

$$\mathbf{E} = \int_S \frac{\rho_S\, dS\, \mathbf{a}_R}{4\pi\varepsilon_o r^2} = \int_S \frac{\rho_S\, dS\,(\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o |\mathbf{r} - \mathbf{r}'|^3}$$

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$$Q = \int_L \rho_L \, dl \qquad \text{for line charge}$$

$$Q = \int_S \rho_S \, dS \qquad \text{for surface charge}$$

$$Q = \int_v \rho_v \, dv \qquad \text{for volume charge}$$

$$\mathbf{D} = \varepsilon_o \mathbf{E}$$

ELECTRIC FLUX DENSITY

$$\boxed{\mathbf{D} = \varepsilon_o \mathbf{E}}$$

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

$$\boxed{\rho_v = \nabla \cdot \mathbf{D}}$$

$$\boxed{\mathbf{E} = -\nabla V}$$

electric flux through a surface S is

$$\Psi = \int_S \mathbf{D} \cdot d\mathbf{S}$$

$$I = \oint \mathbf{J} \cdot d\mathbf{S} = \int \nabla \cdot \mathbf{J} \, dv$$

$$\boxed{\mathbf{J} = \sigma \mathbf{E}}$$

$$\rho_v = ne$$

$$J = \sigma E$$

$$\mathbf{D} = \varepsilon_o (1 + \chi_e) \mathbf{E} = \varepsilon_o \varepsilon_r \mathbf{E}$$

$$\boxed{\mathbf{D} = \varepsilon \mathbf{E}}$$

$$\boxed{\varepsilon = \varepsilon_o \varepsilon_r}$$

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$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

$$\mathbf{F} = \int_v \rho_v \mathbf{E}\, dv$$

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\mathbf{B} = \mu_o \mathbf{H}$$

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B}\, dv = 0$$

$$\nabla \cdot \mathbf{D} = \rho_v \qquad \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v\, dv$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\nabla \times \mathbf{E} = \mathbf{0} \qquad \oint_L \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\nabla \cdot \mathbf{A} = 0$$

$$\mathbf{F} = \oint_L I\, d\mathbf{l} \times \mathbf{B}$$

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$F_m = I\ell B$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\mathcal{P} = \mathbf{E} \times \mathbf{H}$$

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$k = \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\mathcal{P}_{\text{ave}} = \frac{1}{2}\text{Re}(\mathbf{E}_s \times \mathbf{H}_s^*) = \frac{E_o^2}{2\eta}\mathbf{a}_k$$

TRANSMISSION LINES

1 Np= 8.686db

Propagation constant

$\gamma = \alpha + j\beta$

Wave velocity, $u = \frac{\omega}{\beta} = f\lambda$

Wavelength, $\lambda = \frac{2\pi}{\beta}$

Input impedance

$$Z_{in} = Z_o \left[\frac{Z_L + Z_0 \tanh\gamma\ell}{Z_0 + Z_L \tanh\gamma\ell} \right]$$

$$\tanh(x \pm jy) = \frac{\sinh 2x}{\cosh 2x + \cos 2y} \pm j \frac{\sinh 2y}{\cosh 2x + \cos 2y}$$

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

Voltage and current at any point z

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_o)$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_o)$$

Sending end current and voltage

$$I_0 = \frac{V_g}{Z_{in} + Z_g}$$

$$V_0 = Z_{in} I_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g$$

Reflection coefficient

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing wave ratio

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Antenna

Wavelength

$$\lambda = \frac{c}{f}$$

Power radiated,

$P_{rad} or\ W = I_{rms}^2$ x R_{rad}

Effective area,

$A_e = \frac{\lambda^2}{4\pi}$ D

Capture area of a circular aperture,

$$A_e = \frac{\pi D^2}{4}$$

Radiation Efficiency

$\eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_\ell}$

Directivity

D = $\frac{4\pi\ U_{max}}{P_{rad}}$

U_{max} – Radiation intensity

$$D = \frac{4\pi}{\lambda^2} A_e$$

Gain of an Antenna

G= $\boldsymbol{\eta} D$

η – Radiation Efficiency

G = KD

$G = K \frac{4\pi}{\lambda^2} A_e$

K- antenna factor , 1 if no losses present

Gain in db, G_{db}= *10* $log_{10}G$

Q factor

Q= $\frac{f_r}{\Delta f}$

Δf *– Bandwidth*

1 nautical mile(nm) =1852m

Radar power density

P = $\frac{G_{dt}\, P_{rad}}{4\pi r^2}$

Power of the reflected signal at the radar

$$P_r = \frac{A_e\, \sigma G_d P_{rad}}{[4\pi r^2]^2}$$

END OF EQUATION SHEET

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