[ENG33]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) ELECTRICAL & ELECTRONICS ENGINEERING

SEMESTER 1 EXAMINATION - 2022/2023

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Wednesday 11th January 2023 **Time: 14:00 – 16:30**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

QUESTION 1

a) A series RLC circuit is connected to a voltage source given by $v_S(t) = 150\cos \omega t\ V$. Find (i) the phasor current I and (ii) the instantaneous current $i(t)$ for R=400 Ω, L= 3 mH, C=16.67 n$F, and\ \omega = 10^5 rad/s$. **[7 marks]**

b) Two points in a cartesian coordinates are P1(2,3,3) and P2(-1,-5,-1). Find (i) the distance vector between P1 and P2. (ii) the angle between vectors $\overrightarrow{OP1}$ and $\overrightarrow{OP2}$ using the cross product between them. (iii) the angle between vector $\overrightarrow{OP2}$ and the y-axis. **[12 marks]**

c) Transform vector **A**=$\hat{\boldsymbol{x}}\,(x + y) + \hat{\boldsymbol{y}}\,(y - x) + \hat{\boldsymbol{z}}\,z$ from Cartesian to Cylindrical coordinates. **[6 marks]**

QUESTION 2

a) A scalar quantity of $V = rz^2cos2\phi$. Find its directional derivative along the direction **A**=$\hat{\boldsymbol{r}}\,\mathbf{2} - \hat{\boldsymbol{z}}\mathbf{3}$ and evaluate it at (1,0.5$\pi, 2$). **[9 marks]**

b) Find the divergence and the curl of the given vector A=$e^{-7y}(\hat{\boldsymbol{x}}\,sin3x$+$\hat{\boldsymbol{y}}$cos3x) at x=10 and y=1.0 **[4 marks]**

c) Four charges of 100 μC each are located in free space at points with Cartesian coordinates (-3,0,0), (3,0,0), (0, -3,0) and (0,3,0). Find the force on a 200 μC charge located at (0,0,4). All distances are in metres. **[12 marks]**

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QUESTION 3

(a) The potential difference between two points in volts is numerically equal to the work in joules per coulomb necessary to move a coulomb of charge between the two points. A two-wire airline (single-phase system) has conductors of straight cylindrical bare wires with identical radius of 25 mm and spacing of 0.544 m.

(i) What is the charge on each conductor? **[1 mark]**

(ii) What is the voltage drop between the two conductor and **[4 marks]**

(iii) Find the capacitance of a two-wire airline (single-phase system). Then calculate the capacitance of each wire to ground. **[5 marks]**

(b) Briefly explain the operation of the Linear Variable Differential Transformer LVDT sensor shown in **Figure Q3b**. **[8 marks]**

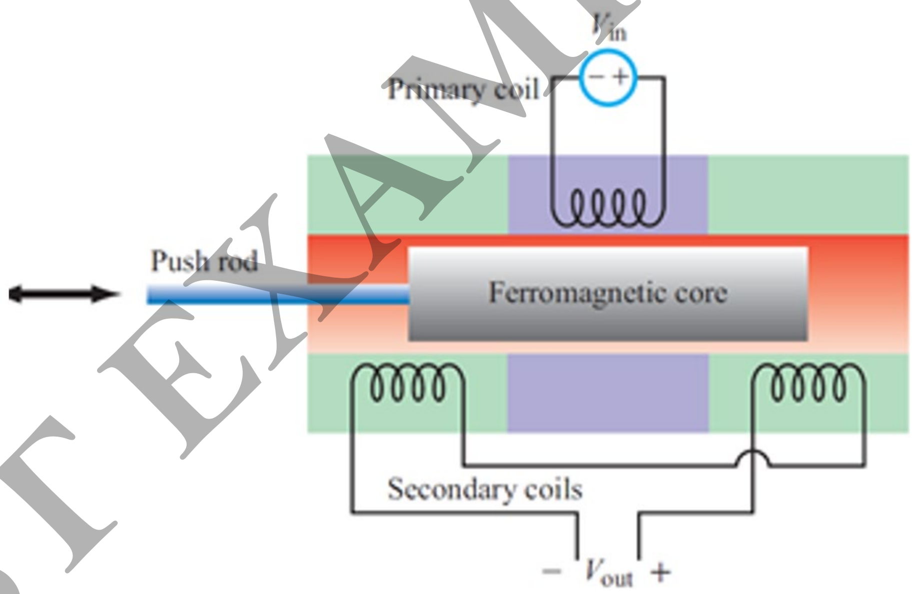

Figure Q3b

(c) A square coil of 200 turns and 0.5 m long sides is in a region with a uniform magnetic flux density of 0.2 T. If the maximum magnetic torque exerted on the coil is $4X10^{-2}$ N.m what is the current flowing in the coil? And what is the MMF produced by the coil? **[7 marks]**

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Question 4

(a) State the Faraday's law of electromotive force (voltage) induced by time-varying magnetic flux. **[5 Marks]**

(b) The transformer shown in the **Figure Q4** consists of a long wire coincident with the *z*-axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the $x - y$ plane and centred at the origin. The toroidal core uses iron material with relative permeability μ_r, around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the Figure Q4.

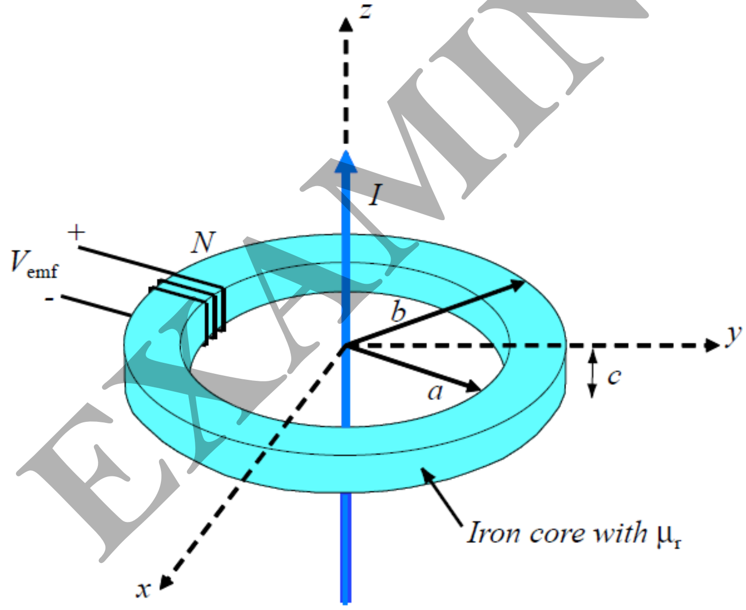

Figure Q4

i. Derive an expression for V_{emf}. **[9 Marks]**

ii. Calculate V_{emf} for f = 60 Hz, μ_r = 4000, a = 5 cm, b = 6 cm, c = 2 cm, and $I_0 = 50$ A. **[5 Marks]**

(c) Identify any three types of EMF sensors. **[6 Marks]**

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Question 5

(a) For a distortionless line, $R'C' = L'G'$, where R' is the resistance, C' is the capacitance, L' is the inductance and G' is the conductance. If α represents the attenuation constant, β represents the phase constant and Z_0 denotes the characteristic impedance, Show that:

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{\frac{L'}{C'}}.$$

[13 Marks]

(b) For a distortionless line with Z_0 = 50 W, α = 20 (mNp/m), u_p = 2.5 × 10^8 (m/s), find the line parameters and l at 100 MHz. **[12 Marks]**

Question 6

(a) Usually attached to antennas, a duplexer is useful in satellite communications because it separates the path of received signal from that of a transmitted signal. Explain why duplexers are sometimes referred to as transceiver switch.

[5 Marks]

(b) An electric field strength of 10 μV/m is to be measured at an observation point $\theta = \pi/2$, 500 km from a half-wave (resonant) dipole antenna operating in air at 50 MHz.

i. What is the length of the dipole? **[3 Marks]**
ii. Calculate the current that must be fed to the antenna. **[5 Marks]**
iii. Find the average power radiated by the antenna. **[3 Marks]**
iv. If a transmission line with $Z_0 = 75\ \Omega$ is connected to the antenna, determine the standing wave ratio. **[9 Marks]**

END OF QUESTIONS

Formula Sheet follows on the next page…

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EEE6012 Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\widetilde{Z}$, where $z(t) = \mathfrak{Re}\,[\widetilde{Z}e^{j\omega t}]$.

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\widetilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

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Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\quad \lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\quad \overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\quad \mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\quad \mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $\quad d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi$ $+ \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi$ $+ \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi$ $+ A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi$ $+ A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi$ $+ \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi$ $+ \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi$ $+ A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi$ $+ A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{12}}, \ \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3}, \ \mathbf{E} = \frac{\mathbf{F}}{Q}, \ \mathbf{E} = \int\frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\mathbf{a}_R, \ \mathbf{E} = \int\frac{\rho_S dS}{4\pi\varepsilon_0 R^2}\mathbf{a}_R, \ \mathbf{E} = \int\frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n, \ \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho, \ Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_v \rho_v dv, \ \nabla\cdot\mathbf{D} = \rho_v, \ W = -Q\int_A^B \mathbf{E}\cdot d\ell, \ V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\ell, \ V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint\mathbf{E}\cdot d\ell = 0, \ \nabla\times\mathbf{E} = 0, \ \mathbf{E} = -\nabla V, \ W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k, \ W_E = \frac{1}{2}\int\mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int\varepsilon_0 E^2 dv, \ \mathbf{J} = \rho_v\mathbf{u}, \ I = \int_S \mathbf{J}\cdot d\mathbf{S}, \ \mathbf{J} = \sigma\mathbf{E},$$

$$R = \frac{\mathrm{V}}{\mathrm{I}} = \frac{\int\mathbf{E}\cdot d\mathbf{l}}{\int\sigma\mathbf{E}\cdot d\mathbf{S}}, \ \mathbf{D} = \varepsilon\,\mathbf{E}, \ \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}, \ E_{1t} = E_{2t}, \ D_{1n} - D_{2n} = \rho_S, \ D_{1n} = D_{2n}, \ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}, \ \nabla^2 V = 0, \ C = \frac{Q}{V} = \frac{\varepsilon\oint\mathbf{E}\cdot d\mathbf{S}}{\int\mathbf{E}\cdot d\mathbf{l}}, \ W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}, \ C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}, \ C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}, \ RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12}F/m \quad , \mu_o = 4\pi X10^{-7}\ H/m$$

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MAGNETOSTATICS:

$$\mathbf{H} = \int_L \frac{Id\mathbf{l}\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H} = \int_S \frac{\mathbf{K}dS\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H} = \int_v \frac{\mathbf{J}dv\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi,\ \mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{a}_\phi = \mathbf{a}_\ell\times\mathbf{a}_\rho,$$

$$\oint \mathbf{H}\cdot d\mathbf{l} = I_{enc},\ \nabla\times\mathbf{H} = \mathbf{J},\ \mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{H} = \frac{1}{2}\mathbf{K}\times\mathbf{a}_n,\ \mathbf{B} = \mu\mathbf{H},\ \Psi = \int_S \mathbf{B}\cdot d\mathbf{S},\ \oint \mathbf{B}\cdot d\mathbf{S} = 0,\ \nabla\cdot\mathbf{B} = 0,\ \mathbf{H} = -\nabla V_m,$$

$$\mathbf{B} = \nabla\times\mathbf{A},\ \mathbf{A} = \int_L \frac{\mu_0 Id\mathbf{l}}{4\pi R},\ \mathbf{A} = \int_S \frac{\mu_0\mathbf{K}dS}{4\pi R},\ \mathbf{A} = \int_v \frac{\mu_0\mathbf{J}dv}{4\pi R},\ \Psi = \oint_L \mathbf{A}\cdot d\mathbf{l},\ \mathbf{F} = Q(\mathbf{E}+\mathbf{u}\times\mathbf{B}),\ d\mathbf{F} = Id\mathbf{l}\times\mathbf{B},\ \mathbf{B}_{1n} = \mathbf{B}_{2n},$$

$$(\mathbf{H}_1 - \mathbf{H}_2)\times\mathbf{a}_{n12} = \mathbf{K},\ \mathbf{H}_{1t} = \mathbf{H}_{2t},\ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2},\ L = \frac{\lambda}{I} = \frac{N\psi}{I},\ M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1\psi_{12}}{I_2},\ W_m = \frac{1}{2}\int \mathbf{B}\cdot\mathbf{H}dv = \frac{1}{2}\int \mu H^2 dv$$

WAVES AND APPLICATIONS:

$$V_{emf} = -\frac{d\psi}{dt}\quad ,V_{emf} = \oint_L \mathbf{E}\cdot d\mathbf{l} = -\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\quad ,\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}\quad ,V_{emf} = \oint_L \mathbf{E}_m\cdot d\mathbf{l} = \oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$V_{emf} = \oint_L \mathbf{E}\cdot d\mathbf{l} = -\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S} + \oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}\quad ,\mathbf{J}_d = \frac{\partial\mathbf{D}}{dt}\quad ,\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{dt}\quad ,\beta = \frac{2\pi}{\lambda},\ \underline{\gamma} = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]},\qquad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]},\ \mathbf{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\mathbf{a}_x$$

$$|\underline{\eta}| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}},\qquad \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon},\ \mathbf{H} = \frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\mathbf{a}_y,\ \tan\theta = \frac{\sigma}{\omega\varepsilon},\ \mathbf{a}_E\times\mathbf{a}_H = \mathbf{a}_k$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377\Omega,\ p(t) = \mathbf{E}\times\mathbf{H},\ p_{ave}(z) = \frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}^*_s),\ p_{ave}(z) = \frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta\mathbf{a}_z,\ P_{ave} = \int_S p_{ave}\cdot d\mathbf{S},$$

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1},\ \tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1},\ s = \frac{|\mathbf{E}_1|_{\max}}{|\mathbf{E}_1|_{\min}} = \frac{|\mathbf{H}_1|_{\max}}{|\mathbf{H}_1|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|},\qquad k_i\sin\theta_i = k_t\sin\theta_t,$$

$$\Gamma_\parallel = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i},\ \tau_\parallel = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i},\ \sin^2\theta_{B\parallel} = \frac{1-\mu_2\varepsilon_1/\mu_1\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2},$$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t},\ \tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t},\ \sin^2\theta_{B\perp} = \frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

$\omega = \beta c$

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}\qquad\qquad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

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Antenna and Radar formula

Dipole

Solid angle:

$$\Omega_p = \iint_{4\pi} F(\theta, \phi)\, d\Omega$$

Directivity:	**Shorted dipole**	**Hertzian monopole**
$D = \dfrac{4\pi}{\Omega_p}$ or $D = \dfrac{4\pi A_e}{\lambda^2}$	$S_0 = \dfrac{15\pi I_0^2}{R^2}\left(\dfrac{l}{\lambda}\right)^2$ $R_{rad} = 80\pi^2\left(\dfrac{l}{\lambda}\right)^2.$	$R_{rad} = 80\pi^2\left[\dfrac{dl}{\lambda}\right]^2$ $P_{rad} = \dfrac{1}{2} I_o^2 R_{rad}$
Half –wave dipole		
$\widetilde{E}_\theta = j\,60 I_0 \left\{\dfrac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}\left(\dfrac{e^{-jkR}}{R}\right),$ $\widetilde{H}_\phi = \dfrac{\widetilde{E}_\theta}{\eta_0}.$ $\lvert E_{\phi s}\rvert = \dfrac{\eta_o I_o \cos\left(\dfrac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$ $\lvert H_{\phi s}\rvert = \dfrac{I_o \cos\left(\dfrac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$		

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For Transmission line

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Distortionless line

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

Open-circuited line

$$\tilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\tilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0}\sin\beta d,$$

$$Z_{in}^{oc} = \frac{\tilde{V}_{oc}(l)}{\tilde{I}_{oc}(l)} = -jZ_0\cot\beta l.$$

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Short-circuited line

$$\widetilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin \beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0} \cos \beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan \beta d.$$

$$j\omega L_{eq} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \leq 0$$

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \right]$$

$$Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} \right]$$

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt} G_{dr}}{4\pi} \left[\frac{\lambda}{4\pi r_1 r_2} \right]^2 \sigma P_{rad}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

$$P_{rec} = P_t G_t G_r \left(\frac{\lambda}{4\pi R} \right)^2$$

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