UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER 1 EXAMINATIONS 2022/23

INTRODUCTORY ENGINEERING MATHEMATICS

MODULE NO: EEE4011

Date: Wenesday 11th January 2023 **Time: 14:00 - 16:00**

INSTRUCTIONS TO CANDIDATES:

This assessment contributes 40% towards your final module mark.

Please attempt **FOUR** of the SIX questions.

For your guidance, the maximum mark that may be achieved for each question and part question is shown in brackets.

A formula sheet is provided on page 7.

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Question 1

(a) A uniform electric field is given by the vector

$$E = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$$

(i) If a particle with charge of 0.1 coulombs is placed in the field, find the vector representing the force on the particle. (1 mark)

(ii) Find the magnitude of the force, in newtons. (2 marks)

The force of the elctric field propels the charged particle along the displacement vector

$$d = \begin{pmatrix} 6 \\ 6 \\ -3 \end{pmatrix}$$

(iii) Find the distance that the particle moves, in metres. (2 marks)

(iv) Find the work done by the field in displacing the particle. (2 marks)

(v) Find the angle between the electric field vector and the displacement vector. (3 marks)

(b) Let A and B be the following matrices:

$$A = \begin{pmatrix} 1 & 3 & -5 \\ 0 & 2 & 4 \\ 6 & 1 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -6 & 4 \\ 1 & -3 & 0 \\ 4 & 5 & 3 \end{pmatrix}$$

Calculate the following matrices:

AB BA (4 marks for each)

(c) Write the following system of simultaneous linear equations as an equation of matrices:

$$8x + 7y = 3$$
$$6x + 5y = 9$$

(2 marks)

By finding the inverse of the square matrix, solve the system of equations.

(5 marks)

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Question 2

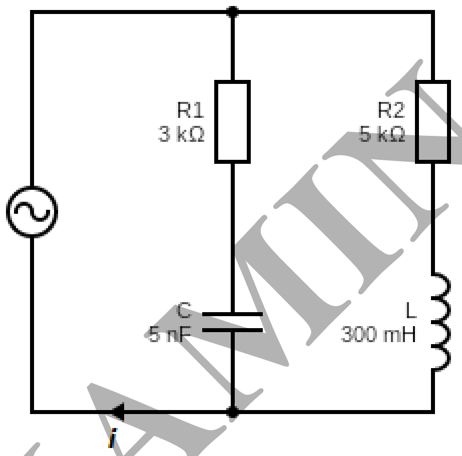
(a) Find the complex solutions of the following quadratic equation:

$$2x^2 + 10x + 17 = 0.$$

(5 marks)

Plot the solutions on a sketch of the Argand diagram. (1 marks)

(b) Consider the arrangement of two resistors, one capacitor and one inductor shown in the diagram.



The AC source has an rms voltage of 20volts, and frequency $\omega = 20000$ rad/s.

Represent the combined impedances of the components by a complex number.

(9 marks)

Hence find the magnitude of the impedance, the rms value of the current i and the phase shift between the current and the applied voltage.

(5 marks)

(c) A complex number in polar form is given by $z = 25\angle 70^\circ$

Find the two square roots of z in cartesian coordinates.

(5 marks)

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Question 3

(a) A capacitor has a value of $C = 50\mu F$. A voltage source is applied across the capacitor, given by the following time function:

$$v(t) = e^{-400t} \sin 600t$$

By differentiating the voltage, find an expression for the current $i(t)$ as a function of time t (8 marks)

(a) Differentiate each of the following functions to find $\frac{dy}{dt}$:

(i) $y = 2t^4 - 7t^3 + 5t^2 - 1$ (3 marks)

(ii) $y = \cos(t^3 + 2)$ (4 marks)

(b) Find the turning points of the following function:

$$y = t^3 - 15t^2 + 63t - 10.$$

Determine whether each turning point is a local maximum or a local minimum.

(10 marks)

Question 4

(a) Evaluate each of the following definite integrals:

(i) $\int_1^5 (12t^2 - 6t + 4)dt$

(6 marks)

(ii) $\int_0^{\frac{\pi}{3}} (12 \sin 6t + 15 \cos 3t)dt$

(6 marks)

(b) Find each of the following indefinite integrals

(i) $\int t^2 e^{-3t} dt$

(7 marks)

(ii) $\int t(3t^2 + 1)^4 dt$

(6 marks)

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Question 5

(a) Solve the following differential equation by separating variables:

$$\frac{dy}{dt} = \frac{8e^{-4t}}{2y+5}$$

The boundary condition is $y = 2$ when $t = 0$.

(8 marks)

(b) Consider the following linear differential equation:

$$\frac{dy}{dt} + 4y = 6e^{-2t}.$$

(i) Find the particular integral. (5 marks)

(ii) Find the complementary function. (2 marks)

(iii) Hence find the solution given that when $t = 0$ we have $y = 5$. (3 marks)

(c) Find the general solution of the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 24y = 0.$$

(7 marks)

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Question 6

(a) The resistaces in ohms of eight resistors are as follows:

1026	1011	992	1004
1001	998	987	1008

Calculate the standard deviation of the resistances. (4 marks)

Find the interquartile range of resistances. (4 marks)

(b) Zener diodes are being manufactured.

It is known that 6% of these fail quality control testing, so that the probability that a single dioded fails is 0.06.

Calculate to three decimal places the probability that in a batch of five of these diodes:

(i) none fail (3 marks)

(ii) exactly one fails (3 marks)

(iii) exactly three fail (3 marks)

(c) Calls to a helpline arrive at a rate of one call every twelve minutes, and calls takes a mean of ten minutes to complete

Find the expected number of calls received in one hour. (2 marks)

Calculate to four decimal places the probability

(i) exactly four calls are received in one hour (2 marks)

(ii) two or more calls are received in the same ten minute interval. (4 marks)

END OF QUESTIONS

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Formulae

Derivatives and Integrals:

Integral	Function	Derivative
$\int y\,dt$	y	$\frac{dy}{dt}$
t	1	0
$\frac{1}{n+1}t^{n+1}$	t^n	nt^{n-1}
$-\frac{1}{a}\cos at$	$\sin at$	$a\cos at$
$\frac{1}{a}\sin at$	$\cos at$	$-a\sin at$
$\frac{1}{a}e^{at}$	e^{at}	ae^{at}

Integration by Parts:

$$\int u\frac{dv}{dt}dt = uv - \int v\frac{du}{dt}dt$$

Binomial Distribution:

The probability of *r* successes in *n* trials is

$$\binom{n}{r}p^r q^{n-r}$$

where *p* is the probability of success in a single trial and $p + q = 1$.

Poisson Distribution:

The probability of *r* successes is

$$\frac{m^r}{r!}e^{-m}$$

where *m* is the expected number of successes.

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