[ENG02]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) BIOMEDICAL ENGINEERING

SEMESTER 1 EXAMINATIONS 2022/23

BIOMEDICAL ENGINEERING MODELLING & ANALYSIS

MODULE NO: BME5001

Date: Monday 9th January 2023 **Time: 10.00 - 12.00**

INSTRUCTIONS TO CANDIDATES:

This is an **OPEN BOOK** examination, so candidates may make use of books, notes, and other materials.

Please attempt **FOUR** of the FIVE questions.

For your guidance, the maximum mark that may be achieved for each question and part question is shown in brackets.

Tables for the normal distribution, the *t* distribution, and the χ^2 distribution are included on pages 6, 7 and 8.

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Question 1

(a) Using integration by parts, show that the Laplace transform of $f(t) = te^{-3t}$ is

$$F(s) = \frac{1}{(s+3)^2}$$

(5 marks)

(b) Using Laplace transforms, solve the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 14y = 2e^{-4t}$$

Initial conditions: $y = 0$, $\frac{dy}{dt} = 1$ at $t = 0$.

(20 marks)

Question 2

A first order control system has transfer function $H(s) = \frac{s-400}{s+200}$

(a) List the zeroes and poles of the system.

(4 marks)

(b) Find the response to the unit step function $x(t) = 1$ for $t > 0$.

(7 marks)

(c) Find the response to the unit ramp function $x(t) = t$ for $t > 0$.

(7 marks)

(d) If the input is a sine wave of frequency $\omega = 300$ radians per second, find the gain and the phase shift of the system.

(7 marks)

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Question 3

A virus has a prevalence of 1 in 200 of the population.

(i) Write down the probability that a person chosen at random from the population is infected with the virus. (2 marks)

A test for the virus has been developed, which is 90% sensitive and 98% specific.

(ii) Write down the probability that a person tests positive, given that they are infected. (2 marks)

(iii) Write down the probability that a person tests positive, given that they are *not* infected. (3 marks)

(iv) Calculate the overall probability that a person chosen at random from the population tests positive. (6 marks)

(v) Using Bayes' theorem, calculate the probability that a person is infected, given that they have tested positive for the virus. (6 marks)

(vi) Calculate the probability that a person is *not* infected, given that they have tested negative for the virus. (6 marks)

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Question 4

Three vaccines are being tested to evaluate their effectiveness in preventing disease. From a sample of 200 people who received one of the three vaccines, the number of people who became sick is shown in the following table:

	Vaccine A	Vaccine B	Vaccine C
Became sick	17	15	8
Remained well	53	65	42
Total	70	80	50

(i) Based on the null hypothesis that there is *no difference* in effectiveness between the three different vaccines, calculate the probabilities that a person chosen from the sample gets sick or remains well.

(5 marks)

(ii) Still assuming the null hypothesis, draw up a table of expected values of people becoming sick or remaining well for each of the vaccines administered.

(8 marks)

(iii) Calculate the chi-square statistic.

(5 marks)

(iv) State, with reasons, the number of degrees of freedom.

(3 marks)

(v) By comparing the chi-square statistic with the appropriate critical value for the 10% significance level, decide whether the null hypothesis should be accepted or rejected.

(4 marks)

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Question 5

(a) In a population, the mean blood serum concentration of potassium is $m = 4.2$ mmol/L (millimoles per litre) with a standard deviation of $\sigma = 0.4$ mmol/L. Find the probability that a person chosen at random from the population has a blood serum potassium concentration of:

(i) More than 5.1 mmol/L (2 marks)

(ii) Less than 3.9 mmol/L (2 marks)

(iii) Between 4.3 mmol/L and 4.9 mmol/L (4 marks)

(b) In a population who are not taking medication, the mean blood serum concentration of calcium is $\mu = 2.41$ mmol/L (millimoles per litre).

A new diuretic medication has been developed. There is concern that this may cause abnormally high levels of blood serum concentration of calcium as a side effect.

A sample of ten patients taking the new medication have been examined, and their calcium concentrations in millimoles per litre were as follows:

2.61	2.83	2.37	2.42	2.76	2.49	2.62	2.71	2.53	2.56

(i) Find estimates of the mean and standard deviation of calcium concentration for the *population of those who are taking the medication* (6 marks)

(ii) Calculate the standard error for the sample of ten patients. (2 marks)

(iii) Using the *t* distribution, determine whether or not the calcium levels in the sample are abnormally high, compared with the general population, at the 5% significance level. (6 marks)

(iv) Explain how it would affect the conclusion, if stronger evidence of a reduction in blood serum calcium concentration was required by adopting a 1% significance level. (3 marks)

END OF QUESTIONS

Distribution tables follow over the page

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Normal Distribution

Table gives $1 - \Phi(z)$, the probability of a score greater than z

z	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	0.5	0.496	0.492	0.488	0.484	0.48	0.476	0.472	0.468	0.464
0.1	0.46	0.456	0.452	0.448	0.444	0.44	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.39	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.33	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.23	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.2	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.14	0.138
1.1	0.136	0.134	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.1	0.099
1.3	0.097	0.095	0.093	0.092	0.09	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.04	0.039	0.038	0.038	0.037
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.03	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2	0.023	0.022	0.022	0.021	0.021	0.02	0.02	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.01	0.01	0.01	0.01	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

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The *t* distribution

The rows are labelled with the number of degrees of freedom.

Each column shows the minimum values of the *t* statistic for significance at the level stated in the column heading.

	25%	**20%**	**10%**	**5%**	**1%**	**0.5%**	**0.1%**
1	1	1.376	3.078	6.314	31.82	63.66	318.3
2	0.816	1.08	1.886	2.92	6.965	9.925	22.33
3	0.765	0.978	1.638	2.353	4.541	5.841	10.21
4	0.741	0.941	1.533	2.132	3.747	4.604	7.173
5	0.727	0.92	1.476	2.015	3.365	4.032	5.893
6	0.718	0.906	1.44	1.943	3.143	3.707	5.208
7	0.711	0.896	1.415	1.895	2.998	3.499	4.785
8	0.706	0.889	1.397	1.86	2.896	3.355	4.501
9	0.703	0.883	1.383	1.833	2.821	3.25	4.297
10	0.7	0.879	1.372	1.812	2.764	3.169	4.144
11	0.697	0.876	1.363	1.796	2.718	3.106	4.025
12	0.695	0.873	1.356	1.782	2.681	3.055	3.93
13	0.694	0.87	1.35	1.771	2.65	3.012	3.852
14	0.692	0.868	1.345	1.761	2.624	2.977	3.787
15	0.691	0.866	1.341	1.753	2.602	2.947	3.733
16	0.69	0.865	1.337	1.746	2.583	2.921	3.686
17	0.689	0.863	1.333	1.74	2.567	2.898	3.646
18	0.688	0.862	1.33	1.734	2.552	2.878	3.61
19	0.688	0.861	1.328	1.729	2.539	2.861	3.579
20	0.687	0.86	1.325	1.725	2.528	2.845	3.552
21	0.686	0.859	1.323	1.721	2.518	2.831	3.527
22	0.686	0.858	1.321	1.717	2.508	2.819	3.505
23	0.685	0.858	1.319	1.714	2.5	2.807	3.485
24	0.685	0.857	1.318	1.711	2.492	2.797	3.467
25	0.684	0.856	1.316	1.708	2.485	2.787	3.45
26	0.684	0.856	1.315	1.706	2.479	2.779	3.435
27	0.684	0.855	1.314	1.703	2.473	2.771	3.421
28	0.683	0.855	1.313	1.701	2.467	2.763	3.408
29	0.683	0.854	1.311	1.699	2.462	2.756	3.396
30	0.68	0.85	1.31	1.7	2.46	2.75	3.38

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The χ^2 distribution

The rows are labelled with the number of degrees of freedom.

Each column shows the minimum values of the χ^2 statistic for significance at the level stated in the column heading.

	10%	**5%**	**2.5%**	**1%**	**0.5%**
1	2.706	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.21	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.86
5	9.236	11.07	12.833	15.086	16.75
6	10.645	12.592	14.449	16.812	18.548
7	12.017	14.067	16.013	18.475	20.278
8	13.362	15.507	17.535	20.09	21.955
9	14.684	16.919	19.023	21.666	23.589
10	15.987	18.307	20.483	23.209	25.188
11	17.275	19.675	21.92	24.725	26.757
12	18.549	21.026	23.337	26.217	28.3
13	19.812	22.362	24.736	27.688	29.819
14	21.064	23.685	26.119	29.141	31.319
15	22.307	24.996	27.488	30.578	32.801
16	23.542	26.296	28.845	32	34.267

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