[ENG26]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (HONS) MECHANICAL, ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER ONE EXAMINATION 2022/23

ENGINEERING MODELLING AND ANALYSIS

MODULE NO: AME5014

Date: Monday 9th January 2023 **Time: 14:00 – 16:00**

INSTRUCTIONS TO CANDIDATES:

There are EIGHT questions.

Answer ANY FIVE questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used if data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached following questions).

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Q1: Differentiation-Integration

(a) The voltage $v = 1 - e^{-\frac{t}{2\times10^{-6}}}$, across a capacitor of a capacitance, $C = 0.2 \times 10^{-6}F$, has a current, i, given by

$$i = C\frac{dv}{dt}$$

Determine i.

(10 Marks)

(b) The force, F in N, required to compress a spring is given by

$$F = 1000x + 50x^3$$

where x is the displacement from its unstretched length. The work done, w, to compress a spring by $0.3m$ is given by

$$W = \int_0^{0.3} F dx$$

Determine W.

(10 Marks)

Total 20 Marks

Q2: Second Order Differential Equation

Solve ONE of the TWO parts below:

Part 1:

(a) The motion of a car can be modelled by the following equation

$$s(t)'' + 2s(t)' - 3s(t) = 0$$

For which y is the displacement in function of the time (t). The initial conditions are $s(0) = 0m$ and $s'(0) = 2m/s$.

Solve analytically the equation above to find the displacement, $s(t)$, and state the nature of displacement response.

(14 Marks)

(b) Create a table listing displacement, $s(t)$, found in (a) for $t = 3, 6$ and $9\ (s)$.

(6 Marks)

Total 20 Marks

Part 2:

(a) By applying Kirchhoff's current law to a circuit we obtain the second order differential equaion

$$i(t)'' + 7i(t)' + 10i(t) = 0$$

For which the natural response of the circuit is i, the current in function of the time (t). The initial conditions are $i(0) = 0\ A$ and $i'(0) = 2A/s$.

Solve analytically the equation above to find the current, $i(t)$, and state the nature of response of the current.

(14 Marks)

(b) Create a table listing the current, $i(t)$ found in (a) for $t = 2{,}4$ and $6\ (s)$.

(6 Marks)

Total 20 Marks

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Q3: First Order Differential Equation

Solve <u>ONE</u> of the <u>TWO</u> parts below:

<u>Part 1:</u>

(a) Consider a tank full of water is being drained out through an outlet. The height H in m of water in the tank at t in s is given by

$$\frac{dH}{dt} = -(2.8 \times 10^{-3})\sqrt{H}$$

Find the general solution of the above equation giving the H in function of time t Given that when $t = 0sec$, $H = 4m$, find an expression of H in terms of t.

(12 Marks)

(b) When $t = 0sec$, $H = 4m$, find the fully defined equation of H in terms of t.

(8 Marks)

Total 20 Marks

<u>Part 2:</u>

(a) A battery supplies contant voltage, $E(t)$ of $40V$, and if inductance, L is $2H$ and resistance, R is 10Ω. The current I in A of that circuit at t is s is given by

$$L\frac{dI}{dt} + RI = E(t)$$

Find the general solution of the above equation givning the I in terms of t.

(12 Marks)

(b) In the beginning of the time ($t = 0s$), the current, $i,$ was $0A$, find the fully defined equation of I in terms of t.

(8 Marks)

Total 20 Marks

Q4: Laplace Transforms

Solve <u>ONE</u> of the <u>TWO</u> parts below:

<u>Part 1:</u>

(a) The temperature gradient, $\frac{dT(t)}{dt}$, of an aluminium rod can be modelled by

$$\frac{dT(t)}{dt} = -1 \times 10^{-4}(T(t) - 20)$$

Given the rod is initially ($t = 0s$) placed into a oven at $220^{\circ}C$ till equilibrium.

Use the method of **Laplace transforms** to derive an expression for $T(t)$.

(12Marks)

(b) Estimate the time t in s taken for the rod to cool down to $100\ ^{\circ}C$ when its taken out of the oven and kept at room temperature of $20\ ^{\circ}C$.

(8 Marks)

Total 20 Marks

<u>Part 2:</u>

(a) An electric circuit can be modelled using the following equation:

$$8Q' + 25Q = 150$$

where Q in C is the instantantaneous charge respectively at time t is s.

Given: initial condition, $Q(0) = 0$ when $t = 0\ s$.

Use the method of **Laplace transforms** to derive an expression for $Q(t)$.

(12 Marks)

(b) Estimate the time t (in s) taken for the Q to reach $5C$.

(8 Marks)

Total 20 Marks

Q5: Fourier transform

An electronic/mechanical signal can be modelled by the following equations in the time domain t.

$$f(t) = 6; \qquad for\ |t| \leq 3$$
$$f(t) = 0; \qquad for\ |t| > 3$$

(a) Sketch the signal waveform from the equations and comment on the result.

(6 Marks)

(b) Calculate the Fourier transform $F(\omega)$ of the signal waveform and comment on the result.

(14 Marks)

Total 20 Marks

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Q6: Matrices

Solve <u>ONE</u> of the <u>TWO</u> parts below:

<u>Part 1:</u>

A electric circuit obays the following equations.

$$\frac{d\vec{x}}{dt} = A\vec{x}$$

$$\vec{x}(t) = e^{\lambda t}\vec{u}$$

where,

$$A = \begin{pmatrix} -0.5 & 0.5 \\ -1.5 & -2.5 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} I \\ V \end{pmatrix}$$

t is time, λ is eigen value and is $\vec{u}$ eigen vector and I is current and v is voltage.

a) Find the eigenvalues of matrix A.

(8 Marks)

b) Find the eigenvectors of matrix A.

(12 Marks)

Total 20 Marks

<u>Part 2:</u>

The natural period, T, of vibrations of a building is given by

$$T = \frac{2\pi}{\sqrt{-\lambda}}$$

where λ is the eigenvalue of a fiven matrix A.

$$A = \begin{pmatrix} -20 & 10 \\ 10 & -10 \end{pmatrix}$$

(a) Find the eigenvalues of matrix A and calculate two natural periods in s.

(10 Marks)

(b) Find the eigenvectors of matrix A.

(10 Marks)
Total 20 marks

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Q7: Simpson's rule

Solve ONE of the TWO parts below:

Part 1:

A force, F, acting on a pariticle varies with time, t, according to the table below.

Time - t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
Force - F (N)	3.2	5.6	7.0	7.7	8.4	9.9	11.6

(a) Sketch the graph of force, F versus time, t from the data given in the table and annotate the graph appropriately.

(6 Marks)

(b) Find an approximate value for the impulse of this force $\int_0^3 Fdt$ using Simpson's rule.

(14 Marks)
Total 20 Marks

Part 2:

The mean voltage, $\bar{v}$, is given by

$$\bar{v} = \frac{1}{0.6}\int_0^{0.6} vdt$$

where, v is values of voltages measured at intervals of 0.1s as shown in the table below.

t (s)	0	0.1	0.2	0.3	0.4	0.5	0.6
Voltage (v) (V)	4	3.92	3.86	3.77	3.61	3.52	3.41

(a) Sketch the graph of the voltage, v versus the time, t from the data given in the table and annotate the graph appropriately.

(6 Marks)

(b) Find an approximate value of mean voltage, $\bar{v}$ using Simpson's rule.

(**14 marks**)

Total 20 Marks

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Q8: Partial derivative and double integrals

(a) If f is given as

$$f = e^{y} * \sin(x)$$

Evaluate Z, so that:

$$Z = \frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial x \partial y}$$

If $x = \pi/2$ and $y = -3$.

(**10 Marks**)

(b) Evaluate the following double integrals

$$\int_{x=0}^{x=6} \int_{y=0}^{y=8} (3\,x^3 - 2\,y^3 + 4)\,dy\,dx$$

(**10 Marks**)

Total 20 Marks

<u>**END OF QUESTIONS**</u>

FORMULA SHEET FOLLOWS ON NEXT PAGES

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FORMULA SHEET

Partial Fractions

$$\frac{F(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\frac{F(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

$$\frac{F(x)}{(x^2+a)} = \frac{Ax+B}{(x^2+a)}$$

Small Changes

z = f(u, v, w)

$$\delta z \simeq \frac{\partial z}{\partial u}.\delta u + \frac{\partial z}{\partial v}.\delta v + \frac{\partial z}{\partial w}.\delta w$$

Total Differential

z = f(u, v, w)

$$dz = \frac{\partial z}{\partial u}du + \frac{\partial z}{\partial v}dv + \frac{\partial z}{\partial w}dw$$

Rate of Change

z = f(u, v, w)

$$\frac{dz}{dt} = \frac{\partial z}{\partial u}.\frac{du}{dt} + \frac{\partial z}{\partial v}.\frac{dv}{dt} + \frac{\partial z}{\partial w}.\frac{dw}{dt}$$

Eigenvalues

$$|A - \lambda \mathrm{I}| = 0$$

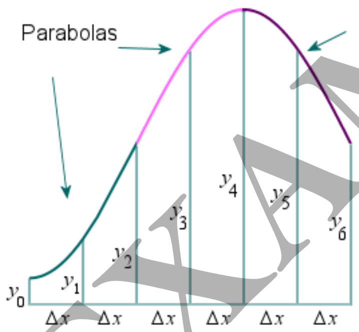
<u>Eigenvectors</u>

$$(A - \lambda_r \mathrm{I})x_r = 0$$

<u>Integration</u>

$$\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx$$

<u>Simpson's rule</u>
To calculate the area under the curve which is the integral of the function **Simpson's Rule** is used as shown in the figure below:



The area into *n* equal segments of width *Δx*. Note that in Simpson's Rule, *n* must be EVEN. The approximate area is given by the following rule:

$$Area = \int_a^b f(x)dx = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 \ldots + 4y_{n-1} + y_n)$$

Where $\Delta x = \frac{b-a}{n}$

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Differential equation

Homogeneous form:

$$a\ddot{y} + b\dot{y} + cy = 0$$

Characteristic equation:

$$a\lambda^2 + b\lambda + c = 0$$

Quadratic solutions :

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i. If $b^2 - 4ac > 0$, λ_1 and λ_2 are distinct real numbers then the general solution of the differential equation is:

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

A and B are constants.

ii. If $b^2 - 4ac = 0$, $\lambda_1 = \lambda_2 = \lambda$ then the general solution of the differential equation is:

$$y(t) = e^{\lambda t}(A + Bx)$$

A and B are constants.

iii. If $b^2 - 4ac < 0$, λ_1 and λ_2 are complex numbers then the general solution of the differential equation is:

$$y(t) = e^{\alpha t}[Acos(\beta t) + Bsin(\beta t)]$$

$$\alpha = \frac{-b}{2a} \quad and \quad \beta = \frac{\sqrt{4ac - b^2}}{2a}$$

A and B are constants.

Inverse of 2x2 matrices:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The inverse of A can be found using the formula:

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Modelling growth and decay of engineering problem

$C(t) = C_0 e^{kt}$

$k > 0$ gives exponential growth

$k < 0$ gives exponential decay

First order system

$$y(t) = k(1 - e^{-\frac{t}{\tau}})$$

Transfer function: $\frac{k}{\tau s+1}$

Derivatives table:

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$
$\text{cosec}\, x = \frac{1}{\sin x}$	$-\text{cosec}\, x \cot x$
$\sec x = \frac{1}{\cos x}$	$\sec x \tan x$
$\cot x = \frac{\cos x}{\sin x}$	$-\text{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$

<u>Integral table:</u>

$f(x)$	$\int f(x)\,\mathrm{d}x$
k, any constant	$kx + c$
x	$\frac{x^2}{2} + c$
x^2	$\frac{x^3}{3} + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$
e^x	$\mathrm{e}^x + c$
e^{kx}	$\frac{1}{k}\mathrm{e}^{kx} + c$
$\cos x$	$\sin x + c$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin x$	$-\cos x + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
$\tan x$	$\ln(\sec x) + c$
$\sec x$	$\ln(\sec x + \tan x) +$
$\operatorname{cosec} x$	$\ln(\operatorname{cosec} x - \cot x) +$
$\cot x$	$\ln(\sin x) + c$
$\cosh x$	$\sinh x + c$
$\sinh x$	$\cosh x + c$
$\tanh x$	$\ln\cosh x + c$
$\coth x$	$\ln\sinh x + c$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$

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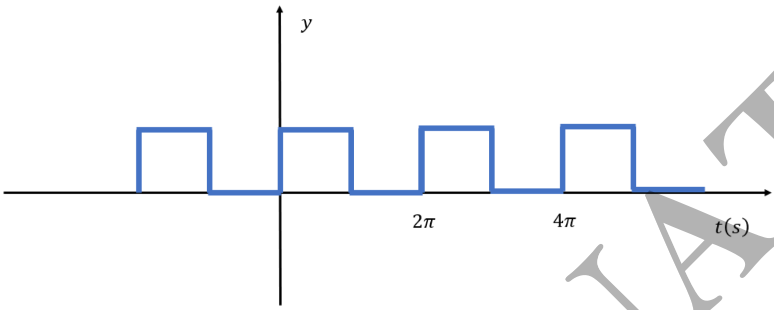
Laplace table:

$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\frac{1}{s}$		$u_c(t)$	$\frac{e^{-cs}}{s}$
t	$\frac{1}{s^2}$		$\delta(t)$	1
t^n	$\frac{n!}{s^{n+1}}$		$\delta(t-c)$	e^{-cs}
e^{at}	$\frac{1}{s-a}$		$f'(t)$	$sF(s)-f(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$		$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos bt$	$\frac{s}{s^2+b^2}$		$(-t)^n f(t)$	$F^{(n)}(s)$
$\sin bt$	$\frac{b}{s^2+b^2}$		$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$		$e^{ct}f(t)$	$F(s-c)$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$		$\delta(t-c)f(t)$	$e^{-cs}f(c)$

Fourier Series

The periodic square wave with Fourier Series and the coefficients of the Fourier Series



The function which represent the periodic square wave can be represented by

$$y = f(t)$$

Period of the function:

$$T = 2\pi \frac{sec}{cycle}$$

Fourier series of the function:

$$f(t) = a_0 + a_1\cos(t) + a_2\cos(2t) + a_3\cos(3t) + \cdots + a_n\cos(nt)$$
$$+b_1\sin(t) + b_2\sin(2t) + b_3\sin(3t) + \cdots b_n\sin(nt)$$

Where, $n = 1,2,3,4,5, \dots \dots.$

Alternatively,

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n\cos(nt) + b_n\sin(nt))$$

Fourier Coefficients:

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(nt)\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(nt)\,dt$$

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Useful Equations for Fourier transform

Fourier transform equation

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$$

Inverse Fourier transform equation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} d\omega$$

Euler's formula for trigonometric identities

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

Where, $j = \sqrt{-1}$

For any arbitrary function

$$\int_a^b f(t)\delta(t - t_0)\, dt = f(t_0)$$

End of the Formula Sheet

<u>END OF PAPER</u>