[ENG31]

UNIVERSITY OF BOLTON

NATIONAL CENTRE FOR MOTORSPORT ENGINEERING

BEng (HONS) AUTOMOTIVE PERFORMANCE ENGINEERING (MOTORSPORT)

SEMESTER 1 EXAMINATION 2021/2022

ENGINEERING MATHEMATICS

MODULE NUMBER MSP4017

Date: Tuesday 11th January 2022 **Time: 14:00 – 16:00**

INSTRUCTIONS TO CANDIDATES:

This is a TWO HOUR examination.

This paper has FIVE questions.

Answer all FIVE questions.

The maximum marks possible for each question are shown in brackets.

Electronic calculators may be used if data and program storage memory is cleared prior to the examination.

Mobile phones or tablets may-not be used as calculators.

Statistical table attached on page 5.

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Question 1

a) Calculate the resultant of the vectors $\vec{p}$ and $\vec{q}$ as depicted in figure 1. Give your answer in polar coordinates.

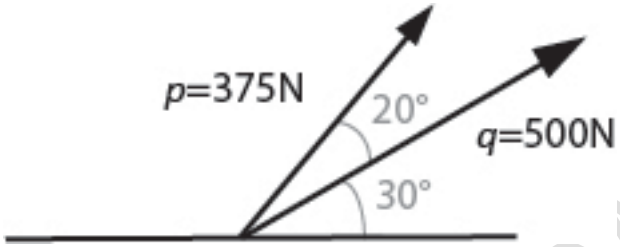

Figure 1

(8 marks)

b) Find the angle between the vectors $\vec{a}$ and $\vec{b}$

$\vec{a} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$
$\vec{b} = 9\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

(8 marks)

Question 2

Find the moment of force $\vec{F}$ about the point O, as depicted in figure 2.

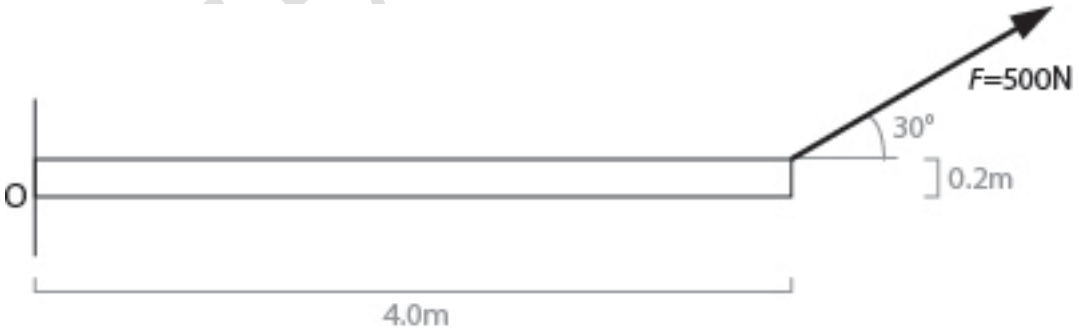

Figure 2

(16 marks)

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Question 3

a) Differentiate the following function from first principles:

$y = f(x) = 2x^2 + 4x + 6.$

(6 marks)

b) Calculate the derivative of the following functions:

I. $y = f(x) = x^2.e^{3x}sin4x$

(6 marks)

II. $y = f(x) = \cos(\sin 2x)$

(6 marks)

c) Determine the stationary point(s) of the function and state whether they are local maxima or local minima:

$y = f(x) = x^3 - 6x^2 - 63x + 14$

(8 marks)

Question 4

a) Calculate the following integrals:

I. $\int e^{2x} \sin 2x \, . dx$

(8 marks)

II. $\int (3x + 3) \cos 3x \, . dx$

(8 marks)

III. $\int_{t=0}^{3} e^{-2t} \, . dt$

(8 marks)

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Question 5

A sample of parts produced by a machining process is taken and a critical diameter is measured. The sample data is shown below:

10.03mm	10.05mm	10.05mm	10.05mm	10.07mm
10.01mm	10.03mm	10.07mm	10.08mm	10.04mm

a) Find the mean, median, and mode of the sample.

(6 marks)

b) Find the range, variance, and standard deviation of the sample

(6 marks)

c) Calculate the 95% confidence interval for the mean and explain how these relate to an estimate of the process mean.

(6 marks)

END OF QUESTIONS

STATISTICAL TABLE FOLLOWS OVER THE PAGE

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Statistical Table

Percentage points for a one-tailed test using Student's t distribution

	Level of Significance								
Degrees of Freedom	0.25	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850

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END OF PAPER

MSP4017 Engineering Mathematics
Formulae Sheet

2021/22

Derivatives

[in all cases a is a constant]

Function $f(x)$	Derivative $\frac{d}{dx} f(x)$	
x^n	nx^{n-1}	
e^{ax}	ae^{ax}	
$\sin ax$	$a\cos ax$	
$\cos ax$	$-a\sin ax$	
$\tan ax$	$a\sec^2 ax$	
$\ln x$	$\frac{1}{x}$	
$\ln(ax+b)$	$\frac{a}{(ax+b)}$	
$\ln[u(x)]$	$\frac{1}{u}\frac{du}{dx}$	If $u(x)>0$
$a\sinh x$	$a\cosh ax$	
$a\cosh ax$	$a\sinh ax$	
$\tanh ax$	$a(1-\tanh^2 ax)$	

Integrals

[in all cases a is a constant, and constants of integration are omitted]

Function $f(x)$	**Integral** $\int f(x)\,dx$	
x^n	$\frac{1}{n+1}x^{n+1}$	$n \neq -1$
$\frac{1}{x}$	$\ln x$	
e^{ax}	$\frac{1}{a}e^{ax}$	
$\sin ax$	$-\frac{1}{a}\cos ax$	
$\cos ax$	$\frac{1}{a}\sin ax$	
$\tan x$	$-\ln(\cos x)$	
$\ln(ax)$	$x\ln(ax)-x$	
$\frac{1}{(ax+b)}$	$\frac{1}{a}\ln(ax+b)$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(x/a\right)$	$a > x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(x/a\right)$	
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	

Calculus Rules – Differentiation

product rule : $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule : $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

chain rule : $$\frac{d}{dx}\left[y(u(x))\right] = \frac{dy}{du}\frac{du}{dx}$$

Parametric Differentiation

First derivative: $$\frac{dy}{dx} = \frac{dt}{dx}\cdot\frac{dy}{dt} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Second derivative: $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Calculus Rules – Integration

integration by parts : $\int u\ dv = uv - \int v\,du$

or : $\int u\frac{dv}{dx}\ dx = uv - \int v\frac{du}{dx}dx$

with limits : $\int_a^b u\frac{dv}{dx}\ dx = \left[uv\right]_a^b - \int_a^b v\frac{du}{dx}\ dx$

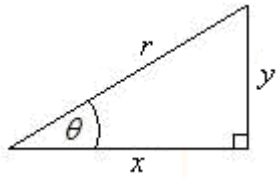
integration by substitution : $\int f(u)\ \frac{du}{dx}\ dx = \int f(u)\,du$

for expressions in the form

$$\int_a^b k\left[f(t)\right]f'(t)dt$$

Use the substitution $u = f(t)$

Trigonometric & Hyperbolic functions



$$\sin\theta = \frac{y}{r} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$$

Euler formula $\qquad e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right) \qquad \sinh\theta = \frac{1}{2}\left(e^{\theta} - e^{-\theta}\right)$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \qquad \cosh\theta = \frac{1}{2}\left(e^{\theta} + e^{-\theta}\right)$$

$$\tanh\theta = \frac{\sinh\theta}{\cosh\theta} = \frac{\left(e^{\theta} - e^{-\theta}\right)}{\left(e^{\theta} + e^{-\theta}\right)}$$

Sine Rule

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\sin 2\theta = 2\sin\theta\cos\theta$

$\sec^2\theta - \tan^2\theta = 1$ $\qquad$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$ $\qquad$ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\qquad$ $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$ $\qquad$ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$2 \sin A \sin B = -\cos(A+B) + \cos(A-B)$

$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

Matrices

Determination of a 2x2 matrix:

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - cb$$

Inverse of a 2x2 matrix

$$A^{-1} = \frac{1}{ad - cd}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{where } \det(A) \neq 0$$

Determination of a 3x3 matrix:

$$\text{if matrix A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det A = a\left[\det\begin{pmatrix} e & f \\ h & i \end{pmatrix}\right] - b\left[\det\begin{pmatrix} d & f \\ g & i \end{pmatrix}\right] + c\left[\det\begin{pmatrix} d & e \\ g & h \end{pmatrix}\right]$$

Inverse of any square matrix is given by:

$$A^{-1} = \frac{adjA}{\det A}$$

$$adjA = C^T$$