ENG24

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) ELECTRICAL & ELECTRONIC ENGINEERING

SEMESTER 1 EXAMINATIONS 2021/22

INTRODUCTORY ENGINEERING MATHEMATICS

MODULE NO: EEE4011

Date: Friday 14th January 2022 **Time: 10:00 - 12:00**

INSTRUCTIONS TO CANDIDATES:

This assessment contributes 40% towards your final module mark.

Please attempt **FOUR** of the six questions.

For your guidance, the maximum mark that may be achieved for each question and part question is shown in brackets.

A formula sheet is provided on page 7.

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Question 1

(a) Consider the 3-dimensional vectors $u = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$ and $v = \begin{pmatrix} 8 \\ -4 \\ 7 \end{pmatrix}$

Calculate the following:

(i) $3u + 2v$ (2 marks)

(ii) $u.v$ (1 mark)

(iii) $|u|$ (1 mark)

(iv) $|v|$ (1 mark)

(v) the angle between u and v. (2 marks)

(b) Let A and B be the following matrices:

$$A = \begin{pmatrix} 4 & -1 & 5 \\ 3 & 2 & 0 \\ 8 & 7 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 8 & 2 \\ 6 & 1 & -4 \\ 7 & 0 & 5 \end{pmatrix}$$

Calculate the following matrices:

AB (5 marks)

BA (5 marks)

(c) Write the following system of simultaneous linear equations as an equation of matrices:

$$9x + 7y = 55$$
$$8x + 5y = 66$$

(2 marks)

By finding the inverse of the square matrix, solve the system of equations.

(6 marks)

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Question 2

(a) Find the complex solutions of the following quadratic equation:

$$x^2 - 14x + 74 = 0.$$

(5 marks)

Plot the solutions on a sketch of the Argand diagram. (2 marks)

(b) Let $z_1 = 8 + 3j$ and $z_2 = 2 - 4j$ be complex numbers. Calculate the following:

(i) $3z_1 - 4z_2$ (2 marks)

(ii) $z_1\overline{z_1}$ (2 marks)

(iii) $z_1 z_2$ (2 marks)

(iv) $\frac{z_1}{z_2}$. (3 marks)

(c) Let $z_1 = 100\angle 50^\circ$ and $z_2 = 20\angle 10^\circ$ be complex numbers in polar form.

Calculate the following complex numbers in polar form:

(i) $z_1 z_2$ (2 marks)

(ii) $\frac{z_1}{z_2}$ (2 marks)

(iii) z_2^3 (2 marks)

(iv) $\sqrt{z_1}$ (3 marks)

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Question 3

(a) Differentiate each of the following functions to find $\frac{dy}{dt}$:

(i) $y = 3t^4 + 5t^3 - 2t^2 + 7$ (3 marks)

(ii) $y = t^3 \sin 4t$ (4 marks)

(iii) $y = \sin(t^2 + 3)$ (4 marks)

(iv) $y = \frac{t^2 + 3t + 2}{e^{4t}}$ (4 marks)

(b) Find the turning points of the following function:

$$y = t^3 - 9t^2 - 48t + 25.$$

Determine whether each turning point is a local maximum or a local minimum.

(10 marks)

Question 4

(a) Evaluate each of the following definite integrals:

(i) $\int_2^3 (9t^2 + 8t - 3)dt$

(6 marks)

(ii) $\int_0^{\frac{\pi}{2}} (8\cos 2t + 12\sin 4t)dt$

(6 marks)

(b) Find each of the following indefinite integrals

(i) $\int t^2 \cos 5t dx$

(7 marks)

(ii) $\int t^2 (t^3 + 7)^9 dx$

(6 marks)

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Question 5

(a) Solve the following differential equation by separating variables:

$$\frac{dy}{dt} = \frac{6t+5}{4y-3}$$

The boundary condition is $y = 2$ when $t = 0$.

(8 marks)

(b) Consider the following linear differential equation:

$$\frac{dy}{dt} + 5y = 10t.$$

(i) Find the particular integral. (5 marks)

(ii) Find the complementary function. (3 marks)

(iii) Hence find the solution given that when $t = 0$ we have $y = 1$. (2 marks)

(c) Find the general solution of the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 28y = 0.$$

(7 marks)

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Question 6

(a) The ages in years of ten employees are as follows:

64	25	44	52	36
30	31	39	21	38

Find the median age and calculate the mean age. (4 marks)

Calculate the standard deviation of the ages. (4 marks)

(b) Operational amplifiers ("op amps") are being manufactured.

It is known that 3% of these fail quality control testing, so that the probability that a single op amp fails is 0.03.

Calculate to three decimal places the probability that in a batch of six of these op amps:

(i) none fail (3 marks)

(ii) exactly one fails (3 marks)

(iii) exactly two fail (3 marks)

(c) Customers arrive at a helpdesk at a mean rate of one customer every 6 minutes.

Find the expected number of customers that will arrive in one hour. (2 marks)

Calculate to four decimal palces the probability that in one hour

(i) exactly nine customers arrive (2 marks)

(ii) exactly eleven customers arrive (2 marks)

(iii) two or more customers arrive (2 marks)

END OF QUESTIONS

FORMULA SHEET OVER THE PAGE

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Formulae

Derivatives and Integrals:

Integral	Function	Derivative
$\int y dt$	y	$\frac{dy}{dt}$
t	1	0
$\frac{1}{n+1}t^{n+1}$	t^n	nt^{n-1}
$-\frac{1}{a}\cos at$	$\sin at$	$a\cos at$
$\frac{1}{a}\sin at$	$\cos at$	$-a\sin at$
$\frac{1}{a}e^{at}$	e^{at}	ae^{at}

Integration by Parts:

$$\int u\frac{dv}{dt}dt = uv - \int v\frac{du}{dt}dt$$

Binomial Distribution:

The probability of r successes in n trials is

$$\binom{n}{r}p^r q^{n-r}$$

where p is the probability of success in a single trial and $p + q = 1$.

Poisson Distribution:

The probability of r successes is

$$\frac{m^r}{r!}e^{-m}$$

where m is the expected number of successes.

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