[ENG12]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (HONS) CIVIL ENGINEERING

SEMESTER ONE EXAMINATION 2021/2022

ENGINEERING MATHEMATICS & STRUCTURES

MODULE NO: CIE5004

Date: Friday 14th January 2022 **Time: 10:00 – 13:00**

INSTRUCTIONS TO CANDIDATES:

This is an open book exam.

There are FOUR Questions.

Answer ALL questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Engineering Mathematics & Structures
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Question 1

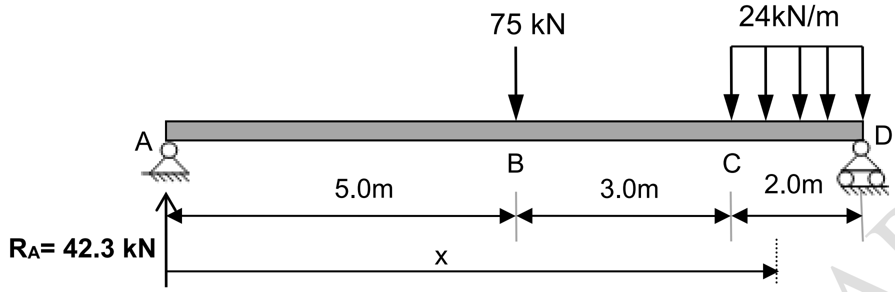

Figure Q1

Figure Q1 shows a beam ABCD which is simply supported with a span of 10 m. The beam carries one point load and one distributed load as shown in Figure Q1. The beam has modulus of elasticity **E = 200** kN/mm^2 and second moment of area **I = 10000** cm^4.

a Use the method of Macaulay to calculate

i. The rotation (slope) at A.

ii. The vertical deflection at B.

(20 marks)

b Estimate the value of x at which the rotation (slope) will be zero.

(5 marks)

Formula for the deflection of a beam: $M = -EI\dfrac{d^2v}{dx^2}$

Total 25 marks

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Question 2: Moment Distribution Method

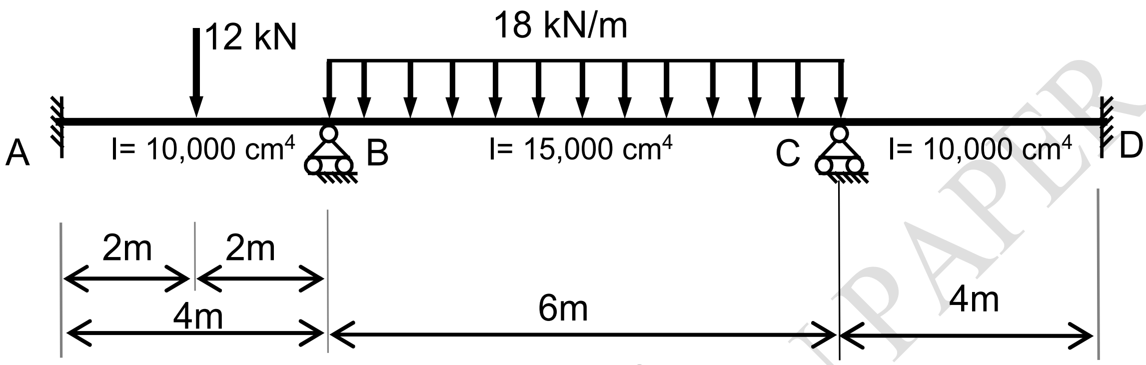

Figure Q2

Figure Q2 shows a 3-span continuous beam ABCD which is fixed to supports at A and D and simply supported at B and C. The I values of the members are shown, and all members have the same E value.

a. Using moment distribution, calculate the bending moments at A, B, C and D.

(17 marks)

b. Sketch the bending moment diagram for the whole beam, showing values at important points.

(8 marks)

Stiffness of a beam against end rotation K = 4EI/L

Tables of Fixed-End Moments are provided in **Table 1 shown on page 7**.

Total 25 marks

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Question 3

A span in a continuous beam is 6m long, placed at 5m spacing (Figure Q3).
Beam 1 in Figure Q3 has an ultimate bending moment of 800kNm at its supports.
The size of the beam is 600mm deep and 430mm wide, and it is monolithically cast with a floor slab. Assume the top reinforcement cover as 40mm.
The beam has a fire rating of 2 hours and the grade of concrete is C30/37.
Use **H32** for top bars and **H20** for bottom bars, fyk **= 500** N/mm^2

a. Determine the reinforcement required in the beam at its supports (15 marks)
b. Draw a cross section of the beam at the supports showing the reinforcement details. (3 marks)
c. Calculate the mass of carbon emissions for the slab and beams shown in Figure Q3.
   Data to be used:
   Total volume of the concrete slab is 38.5m^3
   Total volume of the concrete beams is 15.8m^3
   Density of concrete is 2400 kg/m^3
   Estimated amount of reinforcement for the slab is: 85 kg/m^3 of concrete
   Estimated amount of reinforcement for the beams is: 100 kg/m^3 of concrete
   Apply the wastage rate as 4%
   Rate of embodied carbon for concrete is 0.126 kg eCO2/kg
   Rate of embodied carbon for steel is 1.4 kg eCO2/kg

(7 marks)

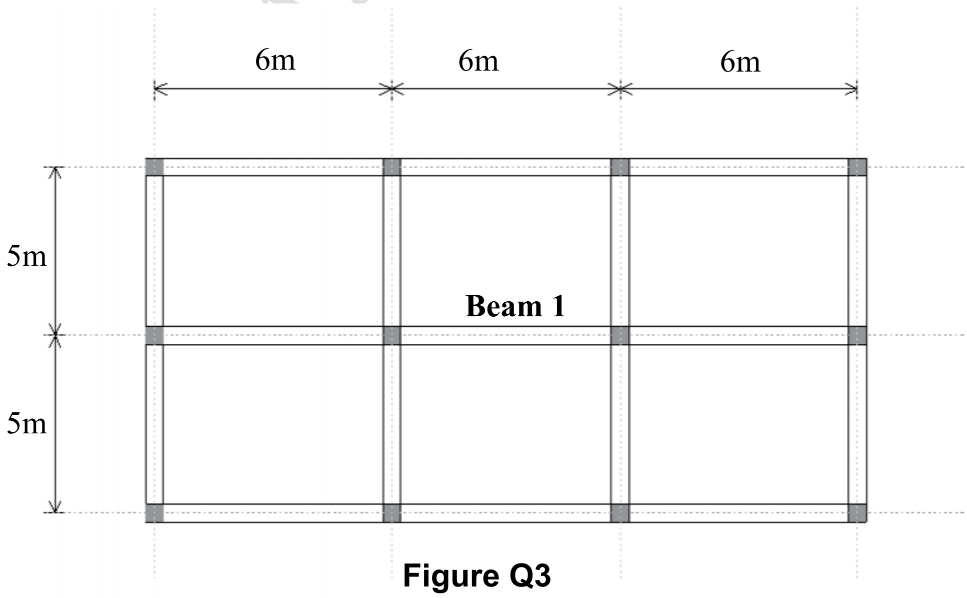

Figure Q3

Total 25 marks

Concrete design equations are provided in **Table 2 shown on pages 8 and 9**

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Question 4

Figure Q4.1 shows a reinforced concrete cantilever beam connected to a column which provides a rigid support at its support at A.
A point vertical load of 2500kN is applied at its free end at point B as shown in Figure Q4.1

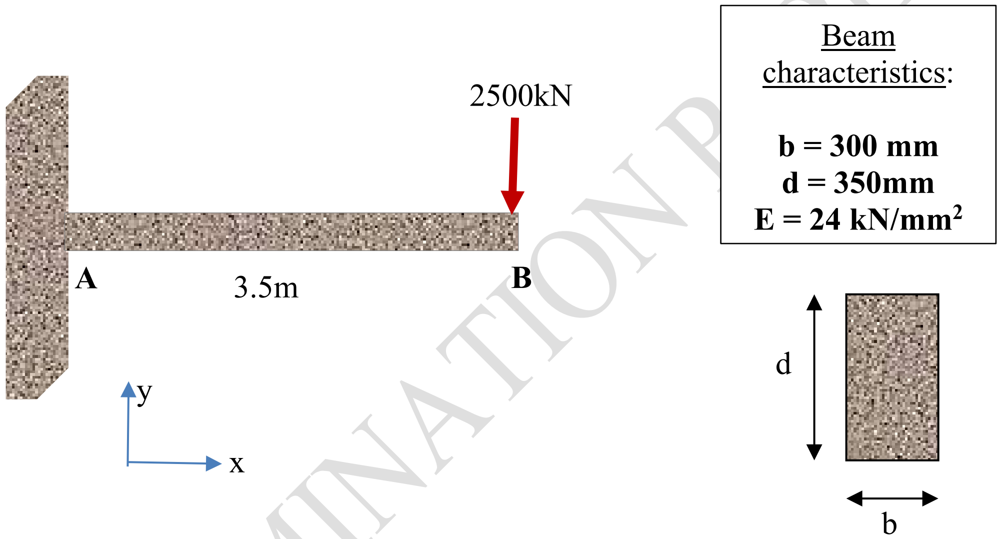

Fig. Q4.1

Using the stiffness matrix equation in Figure Q4.2 (next page) to represent the beam:

a) Write out the reduced stiffness matrix required to determine the deflection at the tip of the beam at B.

(5 marks)

b) Determine the inverse of the reduced stiffness matrix.

(15marks)

c) Determine the tip deflection and rotation of the beam at point B.

(5 marks)

Total 25 marks

Question 4 continues over the page….

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Question 4 continued..

STIFFNESS MATRIX FOR BEAM ELEMENT

$$\begin{bmatrix} P_{XA} \\ P_{YA} \\ M_A \\ P_{XB} \\ P_{YB} \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \times \begin{bmatrix} \delta_{XA} \\ \delta_{YA} \\ \theta_A \\ \delta_{XB} \\ \delta_{YB} \\ \theta_B \end{bmatrix}$$

Fig. Q4.2

Matrix formulae to be used with Question 4 are provided in **Table 3 shown on pages 10 and 11.**

END OF QUESTIONS

Tables for questions 2, 3 and 4 over the page….

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Table 1: Fixed End Moments to be used with <u>Question 2</u>

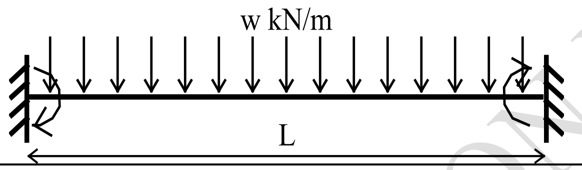
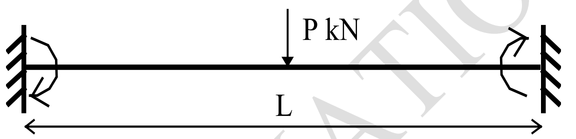
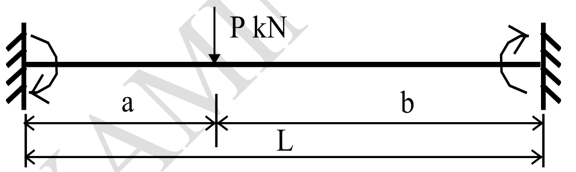
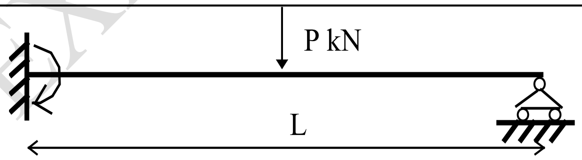
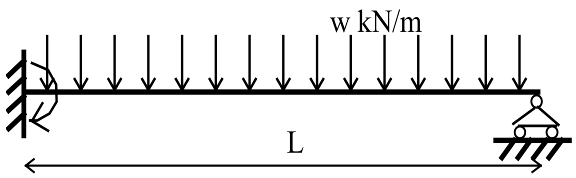
FIXED-END MOMENTS		
FEM_{AB}	A B	FEM_{BA}
$-\frac{wL^2}{12}$		$\frac{wL^2}{12}$
$-\frac{PL}{8}$		$\frac{PL}{8}$
$-\frac{Pab^2}{L^2}$		$\frac{Pa^2b}{L^2}$
$-\frac{3PL}{16}$ $\text{Reaction} = \frac{11P}{16}$		0 $\text{Reaction} = \frac{5P}{16}$
$-\frac{wL^2}{8}$ $\text{Reaction} = \frac{5wL}{8}$		0 $\text{Reaction} = \frac{3wL}{8}$

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Table 2: Concrete design equations to be used with <u>Question 3</u>

<u>Equations:</u>

Effective depth, d
d = h – a

Bending moment resistance of concrete:

$$M_u = 0.167\, f_{ck}\, b\, d^2$$

Equation for area of compression reinforcement:

$$As_2 = \frac{M - M_u}{0.87\, f_{yk}\, (d - d_2)}$$

Equation for area of tension reinforcement:

$$As = \frac{M_u}{0.87\, f_{yk}\, z} + As_2$$

Lever arm and effective depth relationship:

$$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{1.134}}$$

$$K = \frac{M_u}{f_{ck}\, b\, d^2}$$

Table 5.11 Fire resistance requirements for continuous beams

Standard fire resistance (R) in minutes	Possible combinations of a the average axis distance and b_{min} the width of beam Minimum dimensions (mm)			
R60	b_{min} = 120 a = 25[a]	200 12[a]		
R90	b_{min} = 150 a = 35[a]	250 25[a]		
R120	b_{min} = 200 a = 45	300 35[a]	450 35[a]	500 30[a]
R180	b_{min} = 240 a = 60	400 50	550 50	600 40
R240	b_{min} = 280 a = [illegible]	500 [illegible]	650 [illegible]	700 [illegible]

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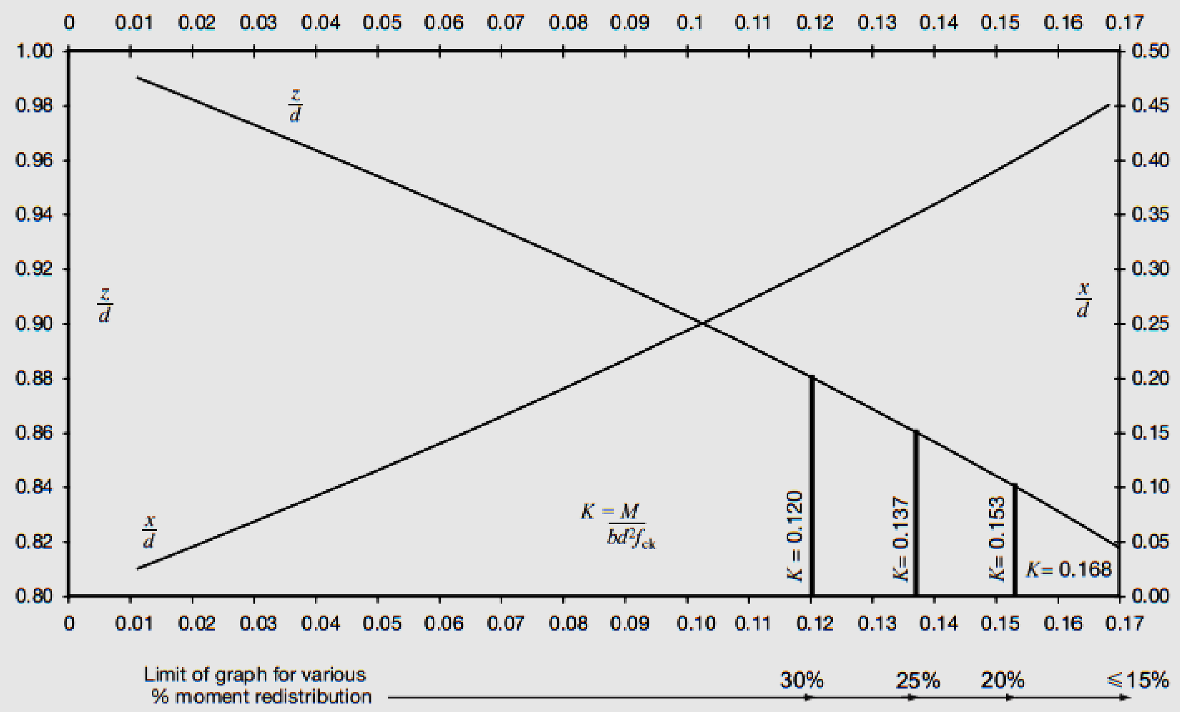
Table 2 continue …. Concrete design equations to be used with <u>Question 3</u>

Fig 5.5 Values of lever arm and neutral axis depth



Note
This figure has been prepared using the redistribution rules in the UK Naitonal Annex to Eurocode 2, with the additional limit of $\frac{x}{d} = 0.45$.

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Table 3: Matrix Formulae to be used with <u>Question 4</u>

(3×3) Matrices

The determinant of a (3×3) matrix A is given by:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \implies \det(A) = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$$
$$= -a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23}$$
$$= a_{31}M_{31} - a_{32}M_{32} + a_{33}M_{33}$$

where M_{ij} is the $(i,j)^{\text{th}}$ coefficient of the minor matrix M associated with A respectively, found by taking the determinant of the sub-matrix of A formed by removing the i^{th} row and j^{th} column of A. The cofactor matrix C associated with A is a (3×3) matrix with coefficients defined by $C_{ij} = (-1)^{i+j}M_{ij}$. The adjoint of A is given by the transpose of the cofactor matrix: $\text{adj}(A) = C^T$, where the transpose of a matrix consists of writing the rows of a matrix as its columns. The inverse of A exists if $\det(A) \neq 0$ and is given by:

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)}.$$

Table 3 continue on next page …..

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Table 3 Continue … Matrix Formulae to be used with <u>Question 4</u>
(Example how to find the inverse of 3x3 matrix)

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Example. Let A be the (3×3) matrix defined by

$$A = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 5 & -4 \end{pmatrix}.$$

To begin, we find the determinant of A using a Laplace expansion:

$$\begin{aligned} \det(A) &= \sum_{i=1}^{3} (-1)^{i+1} a_{i1} M_{i1} = (-1)^2 a_{11} M_{11} + (-1)^3 a_{21} M_{21} + (-1)^4 a_{31} M_{31} \\ &= 2 \cdot M_{11} - 0 \cdot M_{21} + 0 \cdot M_{31} = 2M_{11} \\ &= 2 \det \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 5 & -4 \end{pmatrix} = 2 \det \begin{pmatrix} 1 & -1 \\ 5 & -4 \end{pmatrix} = 2 \end{aligned}$$

Next we need to find the matrix of cofactors. To do this we must calculate all nine of its components, the first three of which are:

$$C_{11} = (-1)^{1+1} M_{11} = \det \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 5 & -4 \end{pmatrix} = \det \begin{pmatrix} 1 & -1 \\ 5 & -4 \end{pmatrix} = 1$$

$$C_{12} = (-1)^{1+2} M_{12} = -\det \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 5 & -4 \end{pmatrix} = -\det \begin{pmatrix} 0 & -1 \\ 0 & -4 \end{pmatrix} = 0$$

$$C_{13} = (-1)^{1+3} M_{13} = \det \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 5 & -4 \end{pmatrix} = \det \begin{pmatrix} 0 & 1 \\ 0 & 5 \end{pmatrix} = 0.$$

The remaining coefficients can be found to yield the cofactor matrix:

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 13 & -8 & -10 \\ -2 & 2 & 2 \end{pmatrix}.$$

Transposing C to obtain the adjoint then dividing by the determinant yields:

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)} = \frac{C^T}{\det(A)} = \frac{1}{2} \begin{pmatrix} 1 & 13 & -2 \\ 0 & -8 & 2 \\ 0 & -10 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{13}{2} & -1 \\ 0 & -4 & 1 \\ 0 & -5 & 1 \end{pmatrix}.$$

END OF PAPER