## UNIVERSITY OF BOLTON

## SCHOOL OF ENGINEERING

## BENG (HONS) CIVIL ENGINEERING

## SEMESTER 1 EXAM 2021/2022

## MATHEMATICAL METHODS FOR CIVIL ENGINEERING

## MODULE NO: CIE4022

Date: Monday $17^{\text {th }}$ January 2022
Time: 10:00-13:00

INSTRUCTIONS TO CANDIDATES:
This is an OPEN book examination
There are FOUR questions
Answer ALL Questions
All questions carry equal marks.
Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

CANDIDATES REQUIRE:
Formula Sheets (attached following questions).

School of Engineering
BEng (Hons) Civil Engineering
Semester One Exam - 2021/2022
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Module No: CIE4022

## Question 1

(a) Differentiate the following
(i) $y=3 x^{3}+7 x-\frac{2}{x^{2}}$
(ii) $\quad y=\sqrt[3]{x^{7}}-\frac{1}{\sqrt[3]{x^{7}}}$
(iii) $\quad y=\left(3 x^{2}+7\right)^{11}$
(iv) $y=\ln \left(6 x^{3}+2 x-3\right)$ (4 marks)
(v) $y=5 \sin (3 x) \cos (4 x)$
(vi) $y=\frac{e^{5 x}}{7 x-2}$

## Question 2

(a) (i) Sketch the graph $y=(x+7)(x+1)(x-3)$ indicating where it crosses the $x$ axis.
(ii) Determine the gradient function of the equation from part (a)
(iii) Find the gradient where $x=1$
(iv) Find the $x$ co-ordinates where $\frac{d y}{d x}=8$.
(v) Find the $\times$ co-ordinates of the stationary points to two decimal places.
(vi) indicate, with justification, whether each stationary point is a local maxima or local minimum.
(b) If $h=7 e^{3 r^{2}}$

Show that: $\quad \frac{d^{2} h}{d r^{2}}=42 e^{3 r^{2}}\left(1+6 r^{2}\right)$

School of Engineering
BEng (Hons) Civil Engineering
Semester One Exam - 2021/2022
Mathematical Methods for Civil Engineering Module No: CIE4022

## Question 3

(a) Integrate each of the following
(i) $\int 3 x^{3}+7 x-\frac{2}{x^{2}} d x$
(ii) $\quad \int \frac{8}{\sqrt[3]{x^{7}}} d x$
(iii) $\int 2 x \cos \left(x^{2}\right) d x$
(iv) $\int \frac{60 x^{3}+18 x-21}{5 x^{4}+3 x^{2}-7 x+9} d x$
(v) $\int 2 x \cos (7 x) d x$
(vi) $\int 7 x^{2} \ln (x) d x$

School of Engineering
BEng (Hons) Civil Engineering
Semester One Exam - 2021/2022
Mathematical Methods for Civil Engineering
Module No: CIE4022

## Question 4

(a) The curve below is represented by the equation $y=3+4 x-x^{2}$. Find the area of the shaded region.

(b) (i) Sketch the graph of $y=\cos (\theta) \quad 0 \leq \theta \leq 2 \pi$
(ii) Evaluate $\int_{0}^{\pi / 2} \cos (\theta) d \theta$
iii) Evaluate $\int_{0}^{3 \pi / 2} \cos (\theta) d \theta$
(c) A function that passes through the point $(0,10)$ is differentiated to produce $\frac{d y}{d x}=3 e^{3 x}+7$.

Find the original function.
(d) Integrate the following,

$$
\int 6 x^{2}\left(x^{3}-3\right)^{-2 / 3} d x
$$

School of Engineering
BEng (Hons) Civil Engineering
Semester One Exam - 2021/2022
Mathematical Methods for Civil Engineering
Module No: CIE4022

FORMULA SHEET

| Function <br> $f(x)$ or $y$ | Differentiation <br> $f^{\prime}(x)$ or $\frac{d y}{d x}$ |
| :---: | :---: |
| $x^{n}$ | $n x^{n-1}$ |
| $e^{x}$ | $e^{x}$ |
| $e^{a x}$ | $a e^{a x}$ |
| $\ln (x)$ | $\frac{1}{x}$ |
| $\sin (x)$ | $\cos (x)$ |
| $\sin (a x)$ | $a \cos (a x)$ |
| $\cos (a x)$ | $-a \sin (a x)$ |


|  | Chain rule | Product rule | Quotient rule |
| :---: | :---: | :---: | :---: |
|  | $\begin{array}{r} y=f(g(x)) \quad u=g(x) \\ \frac{d y}{d x}=\frac{d y}{d u} \cdot \frac{d u}{d x} \end{array}$ | $\begin{gathered} y=u v \\ \frac{d y}{d x}=u \frac{d v}{d x}+v \frac{d u}{d x} \end{gathered}$ | $\begin{gathered} y=\frac{u}{v} \\ \frac{d y}{d x}=\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}} \end{gathered}$ |
|  | By parts |  |  |
|  | $\mathrm{y}=u \frac{d v}{d x}$ $\int u \frac{d v}{d x}=u v-\int v \frac{d u}{d x}$ |  |  |

