[ENG19]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) BIOMEDICAL ENGINEERING

SEMESTER 1 EXAMINATIONS 2021/22

BIOMEDICAL ENGINEERING MODELLING & ANALYSIS

MODULE NO: BME5001

Date: Wednesday 12th January 2022 **Time: 10.00 - 12.00**

INSTRUCTIONS TO CANDIDATES:

Please attempt **ALL FIVE** questions.

For your guidance, the maximum mark that may be achieved for each question and part question is shown in brackets.

Tables for the normal distribution and the *t* distribution are included on pages 5 and 6.

School of Engineering
BEng (Hons) Biomedical Engineering
Semester 1 Examinations 2021/22
Biomedical Engineering Modelling & Analysis
BME5001

Question 1

Using Laplace transforms, solve the following second order linear differential equation:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 2\sin t$$

Initial conditions: $y = 0$, $\frac{dy}{dt} = 2$ at $t = 0$.

(20 marks)

Question 2

A filter in a control system has transfer function $H(s) = \frac{s+30}{s+50}$

(a) Find the response to the unit step function $x(t) = 1$ for $t > 0$.

(7 marks)

(b) Find the response to the unit ramp function $x(t) = t$ for $t > 0$.

(7 marks)

(c) If the input is a sine wave of frequency $\omega = 20$ radians per second, find the gain and the phase shift of the filter.

(6 marks)

PLEASE TURN THE PAGE

Question 3

A virus has a prevalence of 1 in 250 of the population.

(i) Write down the probability that a person chosen at random from the population is infected with the virus. (2 marks)

A test for the virus has been developed, which is 96% sensitive and 99% specific.

(ii) Write down the probability that a person tests positive, given that they are infected. (2 marks)

(iii) Write down the probability that a person tests positive, given that they are *not* infected. (3 marks)

(iv) Calculate the overall probability that a person chosen at random from the population tests postitive. (6 marks)

(v) Using Baye's theorem, calculate the probability that a person is infected, given that they have tested positive for the virus. (6 marks)

(vi) Calculate the probability that a person is *not* infected, given that they have tested negative for the virus. (6 marks)

Question 4

In a population, it has been found that the red blood cell count (RBC) is normally distributed with a mean of $\mu = 5.05 \times 10^6$ cells per litre, and a standard deviation of $\sigma = 0.43 \times 10^6$ cells per litre.

(a) State, with reasons, whether or not an RBC of 5.83×10^6 cells per litre should be considered abnormally high for a person chosen from the population, at the 5% significance level. (6 marks)

(b) A sample of six people chosen at random from the population had the following RBC counts in millions of cells per litre:

5.21	5.48	5.33	5.7	5.32	5.36

(i) Calculate the sample mean. Find the standard error of the sample mean based on the standard deviation of the population. (6 marks)

(ii) State, with reasons, whether or not this sample should be considered abnormal, at the 5% significance level. (6 marks)

PLEASE TURN THE PAGE

Question 5

In a population who are not taking medication, the mean blood serum concentration of sodium is $\mu = 139.8$mmol/L (millimoles per litre).

A new diuretic medication has been developed. There is concern that this may cause abnormally low levels of blood serum concentration of sodium as a side effect.

A sample of nine patients taking the new medication have been examined, and their sodium concentations in millimoles per litre were as follows:

139.2	137.1	135.2	140.4	144.4	132.2	135.3	137.3	133.7

(i) Find estimates of the mean and standard deviation of sodium concentration for the *population of those who are taking the medication*.

(6 marks)

(ii) Calculate the standard error for the sample of nine patients.

(2 marks)

(iii) Using the *t* distribution, determine whether or not the sodium levels in the sample are abnormally low, compared with the general population, at the 5% significance level.

(6 marks)

(iv) Explain how it would affect the conclusion, if stronger evidence of a reduction in blood serum sodium concentration was required by adopting a 1% significance level.

(3 marks)

END OF QUESTIONS

School of Engineering
BEng (Hons) Biomedical Engineering
Semester 1 Examinations 2021/22
Biomedical Engineering Modelling & Analysis
BME5001

Normal Distribution

Table gives $1-\Phi(z)$, the probability of a score greater than z

z	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	0.5	0.496	0.492	0.488	0.484	0.48	0.476	0.472	0.468	0.464
0.1	0.46	0.456	0.452	0.448	0.444	0.44	0.436	0.433	0.429	0.425
0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.39	0.386
0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
0.4	0.345	0.341	0.337	0.334	0.33	0.326	0.323	0.319	0.316	0.312
0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
0.7	0.242	0.239	0.236	0.233	0.23	0.227	0.224	0.221	0.218	0.215
0.8	0.212	0.209	0.206	0.203	0.2	0.198	0.195	0.192	0.189	0.187
0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
1	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.14	0.138
1.1	0.136	0.134	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.1	0.099
1.3	0.097	0.095	0.093	0.092	0.09	0.089	0.087	0.085	0.084	0.082
1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
1.7	0.045	0.044	0.043	0.042	0.041	0.04	0.039	0.038	0.038	0.037
1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.03	0.029
1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
2	0.023	0.022	0.022	0.021	0.021	0.02	0.02	0.019	0.019	0.018
2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
2.3	0.011	0.01	0.01	0.01	0.01	0.009	0.009	0.009	0.009	0.008
2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

School of Engineering
BEng (Hons) Biomedical Engineering
Semester 1 Examinations 2021/22
Biomedical Engineering Modelling & Analysis
BME5001

The *t* distribution

The rows are labelled with the number of degrees of freedom.

Each column shows the minimum values of the *t* statistic for significance at the level stated in the column heading.

	25%	**20%**	**10%**	**5%**	**1%**	**0.5%**	**0.1%**
1	1	1.376	3.078	6.314	31.82	63.66	318.3
2	0.816	1.08	1.886	2.92	6.965	9.925	22.33
3	0.765	0.978	1.638	2.353	4.541	5.841	10.21
4	0.741	0.941	1.533	2.132	3.747	4.604	7.173
5	0.727	0.92	1.476	2.015	3.365	4.032	5.893
6	0.718	0.906	1.44	1.943	3.143	3.707	5.208
7	0.711	0.896	1.415	1.895	2.998	3.499	4.785
8	0.706	0.889	1.397	1.86	2.896	3.355	4.501
9	0.703	0.883	1.383	1.833	2.821	3.25	4.297
10	0.7	0.879	1.372	1.812	2.764	3.169	4.144
11	0.697	0.876	1.363	1.796	2.718	3.106	4.025
12	0.695	0.873	1.356	1.782	2.681	3.055	3.93
13	0.694	0.87	1.35	1.771	2.65	3.012	3.852
14	0.692	0.868	1.345	1.761	2.624	2.977	3.787
15	0.691	0.866	1.341	1.753	2.602	2.947	3.733
16	0.69	0.865	1.337	1.746	2.583	2.921	3.686
17	0.689	0.863	1.333	1.74	2.567	2.898	3.646
18	0.688	0.862	1.33	1.734	2.552	2.878	3.61
19	0.688	0.861	1.328	1.729	2.539	2.861	3.579
20	0.687	0.86	1.325	1.725	2.528	2.845	3.552
21	0.686	0.859	1.323	1.721	2.518	2.831	3.527
22	0.686	0.858	1.321	1.717	2.508	2.819	3.505
23	0.685	0.858	1.319	1.714	2.5	2.807	3.485
24	0.685	0.857	1.318	1.711	2.492	2.797	3.467
25	0.684	0.856	1.316	1.708	2.485	2.787	3.45
26	0.684	0.856	1.315	1.706	2.479	2.779	3.435
27	0.684	0.855	1.314	1.703	2.473	2.771	3.421
28	0.683	0.855	1.313	1.701	2.467	2.763	3.408
29	0.683	0.854	1.311	1.699	2.462	2.756	3.396
30	0.68	0.85	1.31	1.7	2.46	2.75	3.38

END OF PAPER