[ENG08]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATION 2021-2022

ADVANCED MATERIALS & STRUCTURES

MODULE NO: AME6012

Date: Monday 10th January 2022 **Time: 10:00 – 13:00**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions.

Attempt FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached from page 9).**

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QUESTION 1

a) An automotive suspension bracket is being developed. Part of the process is to test the bracket. Under the initial test conditions the following stresses at a point of interest were obtained measured from monitoring the strains for one of the critical components.

Direct stresses: xx= 205 MPa compressive, yy= 133 MPa tensile and zz= 165 MPa compressive. The direct stresses were accompanied by three shear stresses: xy= 95 MPa, xz= -32 MPa and yz= 75MPa.

Using this data,

(i) Sketch the elemental cube representing the state of stress. **(3 marks)**

(ii) Show that the characteristic equation representing the state of stress at this point is given as: $\sigma^3 - 93\sigma^2 - 54817\sigma + 5426 = 0$

And show the largest stress acting at this point is 229 MPa . **(7 marks)**

(iii) Calculate direction of the largest compressive stress of 236 MPa and show this by a simple sketch. **(6 Marks)**

b) If the yield stress of the material is 625 MPa determine the factor of safety at this point based upon the von Mises criterion assuming the other principal stress at this point is 100 MPa. **(5 Marks)**

c) The component was manufactured by initially rolling the stamping along the z direction. Explain how this would influence the choice of yield criteria and how this would change the von Mises criterion currently used. **(4 Marks)**

Total 25 Marks

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QUESTION 2

a) Part of a bulkhead on a submarine is manufactured from a plate with the properties given in **Table Q2**. The plate is subjected to cyclic stresses ranging from 260MPa tensile to 140 MPa compressive every 15 minutes for twelve hours per day when out at sea. The plate is susceptible to cracks in the central region and therefore is monitored regularly; however, the equipment used can only detect cracks larger than 3mm.

Using the above information and the material data in table Q2, determine the time taken for the crack to grow to 9mm. **(9 Marks)**

Table Q2	
Yield Strength	950 MPa
Young's Modulus	208 GPa
Poisson's Ratio	0.34
Fracture toughness	89 MPa.m$^{0.5}$
Paris coefficients M & C	3.0 & 1.2x10^{-12}
Shape factor Y	1.12

b) Also estimate how much longer life the plate has under these conditions.

(7 Marks)

c) Sketch also the graph of fatigue-crack growth rates da/dN, as a function of the applied stress-intensity range K in metallic materials, identifying the key elements of the graph?

(4 marks)

d) Explain briefly why this estimate is conservative and what other factors could be considered to improve the life predictions

(5 Marks)

Total 25 Marks

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QUESTION 3

a) **Figure Q3** shows schematically a portal frame representing a roll cage with worst case scenario load case with a horizontal load of 25 KN and a vertical load of 12 KN. Joints A, C and D can be assumed to be welded whilst joint B is a safety pin. Use this information to determine a suitable tubular section manufactured from steel with a yield stress of 663 MPa and a factor of safety of 3.

Assume for the analysis the material is rigid-perfectly plastic.

Take Z_p as D^2t where: D is the nominal bore and t the thickness of a tubular section.

(12 Marks)

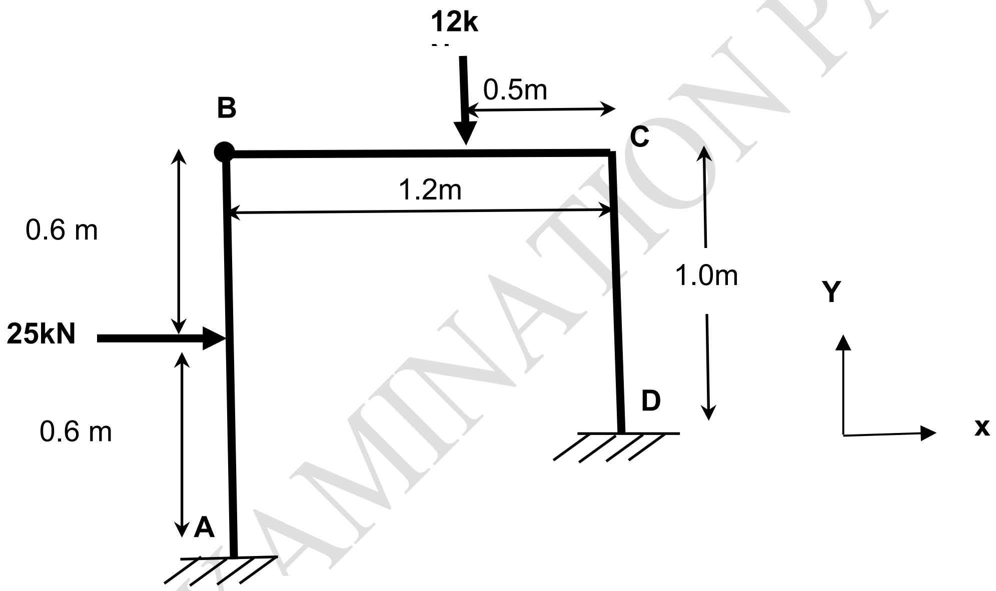

Fig Q3 frame set up

b) An alternative proposal is also considered with the same size tubing, but this time the 25KN load is acting 0.9m from A. Determine the new factor of safety.

(9 Marks)

c) Describe two other material models that could be used in place of the rigid perfectly plastic one stating in each case whether or not they would produce a higher or lower factor of safety

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(4 Marks)

Total 25 Marks

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QUESTION 4

a) A rectangular composite component is to be manufactured from carbon fibre reinforced epoxy skins (see **Table Q4**) with a 22mm thick foam core is proposed to replace an existing aluminum structure. The component is subject to both flexure and torsion; these loads are shown in **Fig Q4**. Using this information determine a suitable lay up for the composite and illustrate this by a sketch.

(20 marks)

Fibre Modulus GPa	Volume fraction %	Safe working strain %	Bond strength of skin MPa	Lamina Thickness mm
320	68	0.6	12	0.125

Table Q4

b) If the component was to be used in dessert conditions describe what other factors you would need to consider.

(5 marks)

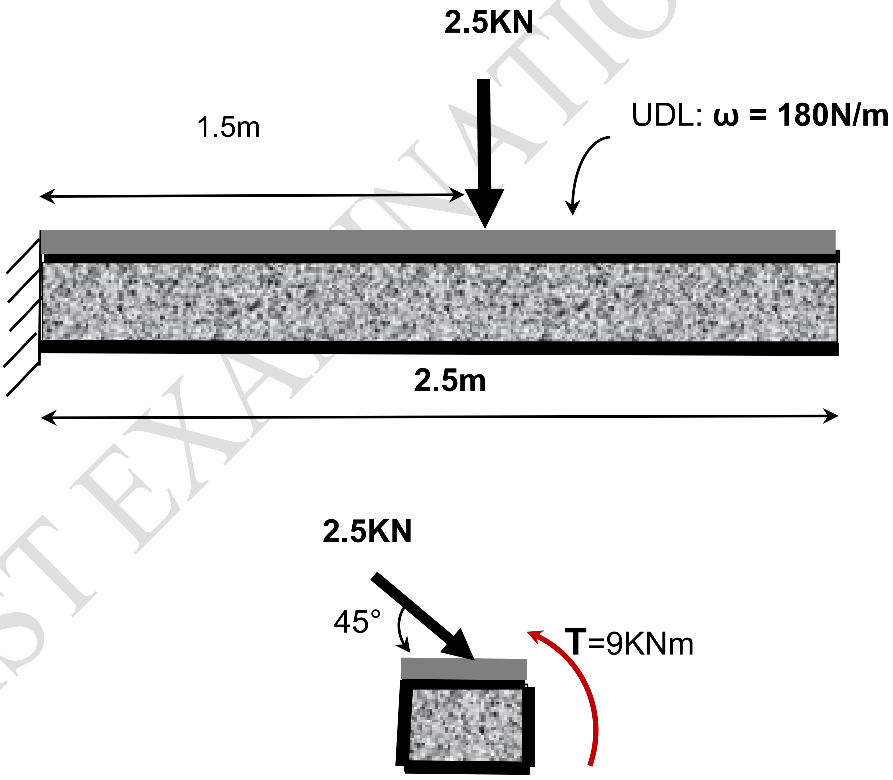

Fig Q4 schematic of the beam

Total 25 Marks

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QUESTION 5

a) A biomedical fastener is manufactured from CoCrMo shown in **Fig Q5a** has a Young's modulus of 104 GPa and $v = 0.3$ is to be evaluated for future use.
It is also expected that the component under its normal usage would be under repeated

cyclic loading with a maximum bending moment of 80Nm along with a lower load of 25Nm. Assuming at the position of largest stress the 2nd moment of area is $4.15\text{x}10^3$ mm^4 and maximum depth is 30 mm, hence, estimate the maximum stress and predict the life of the component under this condition. The S-N curve for CoCrMo is given in **Fig Q5c** and you can also assume for this geometry, $K_t = 1.45$ based on photoelastic test data and the notch sensitivity factor q= 0.85

(10 marks)

Fig Q5a Schematic of the implant

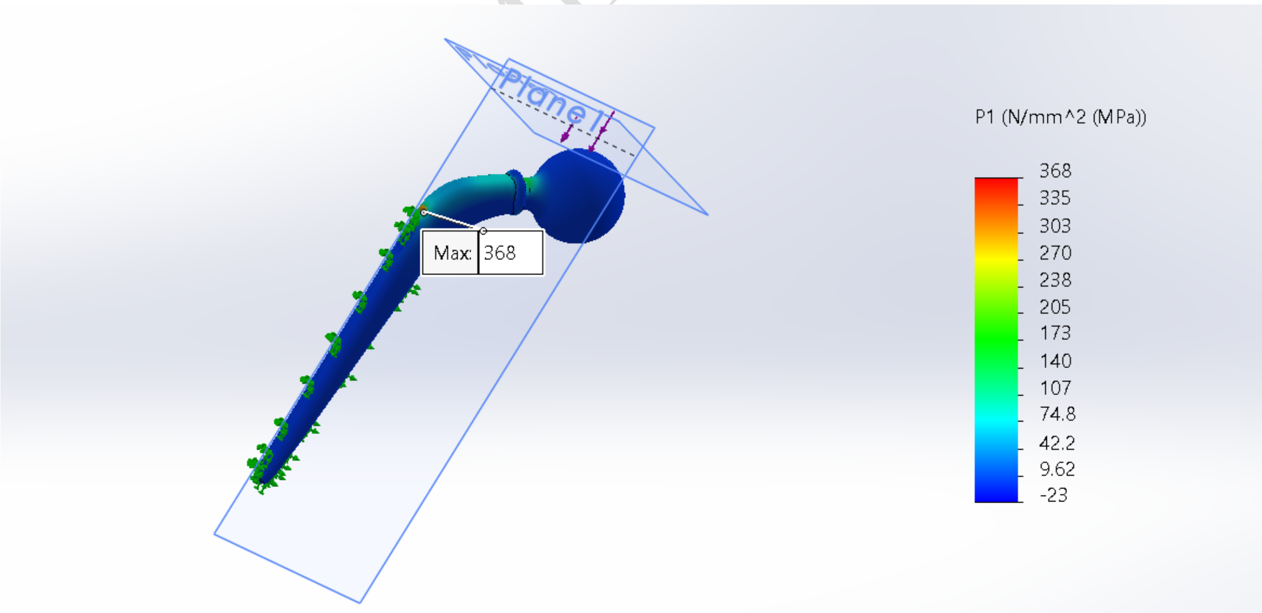

Fig Q5b FEA plot of the Principal stress under an inplane moment of 170Nm

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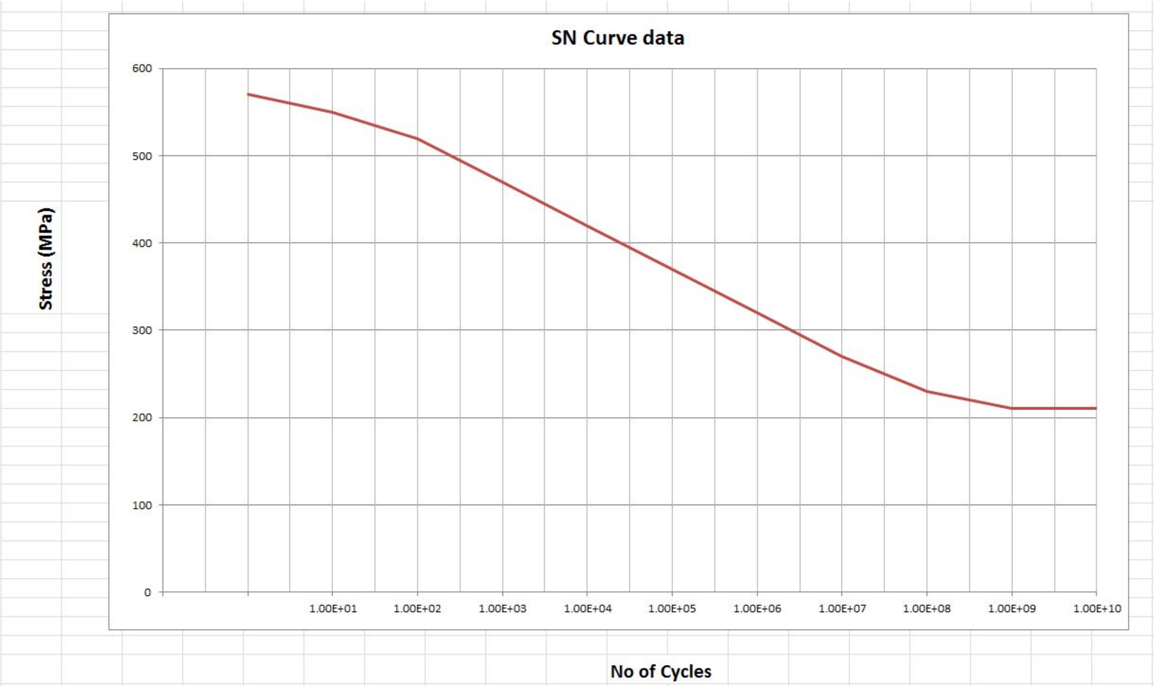

Fig Q5c- S-N Curve for CoCrMo material used.

b) In order to verify the behaviour both finite element analysis and strain gauge techniques were used to evaluate the design. The output from the finite element model is shown in figure **Q5b** indicating the principal stress values at the position of interest.

Further confirmation was achieved using a strain gauge rosette consisting of three gauges in the pattern shown in figure **Q5d** bonded to the surface at an angle of 10^o to the axis of symmetry. The gauges had a gauge length of 2mm and bonded using an epoxy adhesive. The output results under the maximum load condition for the three gauges are given below

ε_0 = 3653 x 10^{-6} mm/mm (0^o)
ε_{45} = 1785 x 10^{-6} mm/mm (45^o)
ε_{90} = -2604 x 10^{-6} mm/mm (90^o)

Using this data calculate the maximum strain obtained and compare with the predicted experimental stress that was obtained using the finite element method. Explain also why there is a difference between the two results and where the main source of error is likely to occur.

(15 Marks)

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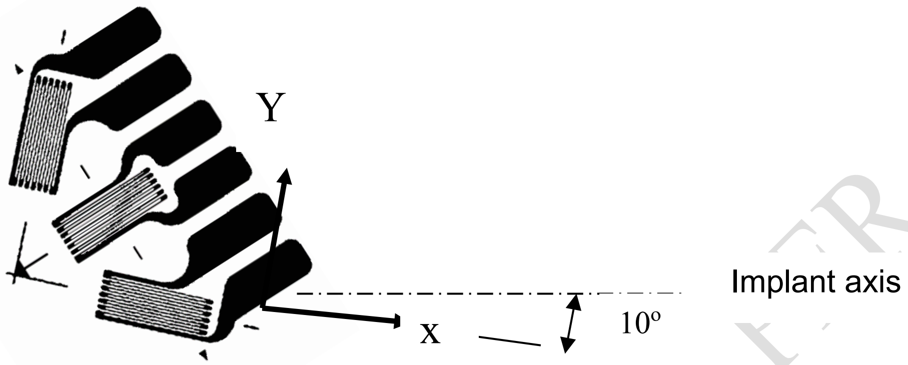

Fig Q5d Strain Gauge set up

END OF QUESTIONS

Formula sheet follows on the next page….

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FORMULA SHEET

Formulae used in Structures and Materials Module

Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = (Stress\ Tensor)\begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Where a, b and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \qquad l = \cos\alpha,$$
$$m = bk \qquad m = \cos\theta,$$
$$n = ck \qquad n = \cos\varphi.$$

Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

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Yield Criterion
Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

Tresca

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

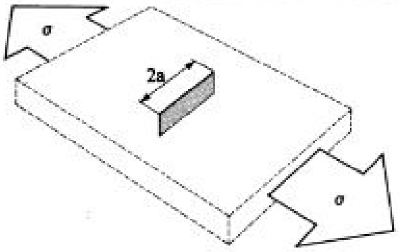
$$\sigma_{tresca} = 2\cdot\tau_{\max}$$

$$\tau_{\max} = \max\left(\frac{|\sigma_1-\sigma_2|}{2};\frac{|\sigma_1-\sigma_3|}{2};\frac{|\sigma_3-\sigma_2|}{2}\right)$$

$$\frac{\sigma_{von\ Mises}}{\sigma_{Tresca}} = \frac{\sqrt{3}}{2}$$

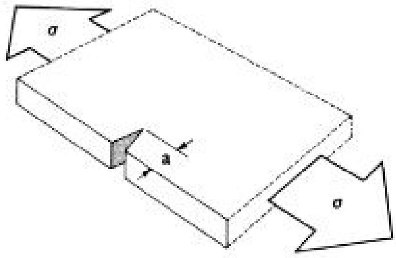
Fracture mechanics

Table: Y values for plates loaded in tension



(1) Through crack of length $2a$ in an *infinite* plate

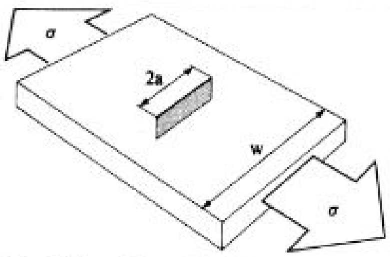
$Y = 1$



(2) Edge crack of length a in an *infinite* plate

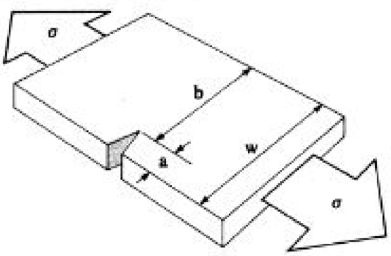
$Y = 1.12$

Because plane strain and plane stress have identical stress fields, this calibration is also for an edge scratch of depth a on a large body carrying tensile stress σ.



(3) Through crack of length $2a$ in a plate of width w.

$$Y = \left(\sec\frac{\pi a}{w}\right)^{1/2},\ \frac{2a}{w} \leq 0.7$$



(4) Edge crack of length a in a plate of width w.

$$Y = 0.265\left(\frac{b}{w}\right)^4 + \frac{0.875 + 0.265a/w}{(b/w)^{3/2}}$$

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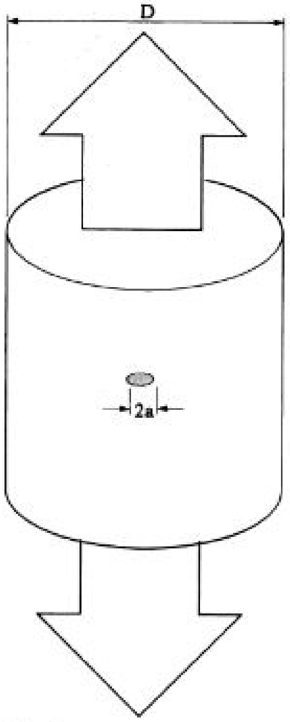
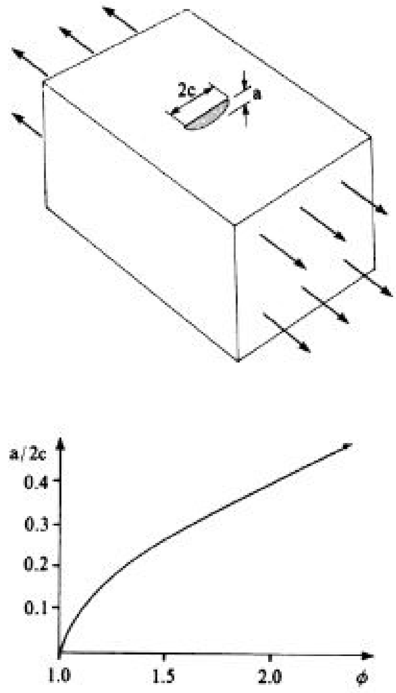


(5) Penny-shaped internal crack of radius a.

$Y = \frac{2}{\pi}, \quad a \ll D$



(6) Semi-elliptical surface flaw

$Y = \frac{1.12}{\phi^{1/2}}$

Life Calculations

$$\frac{da}{dN} = C\left(\Delta K\right)^m$$

$$N = \frac{1}{C Y^m \sigma_a{}^m \pi^{\frac{m}{2}}} \int_{a_0}^{a_1} \frac{da}{a^{\frac{m}{2}}}$$

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Composite materials

$$E_{composite} = E_{fibre}V_{fibre} + E_{matrix}(1 - V_{fibre})$$

Fracture Toughness

Table: Fracture toughness of some engineering materials

Material	K_{IC} (MNm$^{-3/2}$)	E (GN/m^2)	G_{IC} (kJ/m^2)
Plain carbon steels	140 - 200	200	100 - 200
High strength steels	30 - 150	200	5 - 110
Low to medium strength steels	10 - 100	200	0.5 - 50
Titanium alloys	30 – 120	120	7 – 120
Aluminium alloys	22 – 33	70	7 - 16
Glass	0.3 – 0.6	70	0.002 – 0.008
Polycrystalline alumina	5	300	0.08
Teak – crack moves across the grain	8	10	6
Concrete	0.4	16	1
PMMA (Perspex)	1.2	4	0.4
Polystyrene	1.7	3	0.01
Polycarbonate (ductile)	1.1	0.02	54
Polycarbonate (brittle)	0.4	0.02	6.7
Epoxy resin	0.8	3	0.2
Fibreglass laminate	10	20	5
Aligned glass fibre composite – crack across fibres	10	35	3
Aligned glass fibre composite – crack down fibres	0.03	10	0.0001
Aligned carbon fibre composite – crack across fibres	20	185	2

Strain relationships

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

where ε_x = normal strain at a point in x-direction

ε_y = normal strain at a point in y- direction

γ_{xy} = shear strain at a point on x face in y direction

Hooke's Law in 2D

$$\sigma_1 = \frac{E}{(1-\upsilon^2)}(\varepsilon_1 + \upsilon\varepsilon_2)$$

$$\sigma_2 = \frac{E}{(1-\upsilon^2)}(\varepsilon_2 + \upsilon\varepsilon_1)$$

TABLE 13.2 Formulas for Values of the Maximum Principal Stresses and Maximum Deflections in Circular Plates as Obtained by Theory of Flexure of Plates[a]

Support and loading	Principal stress (σ_{max})	Point of maximum stress	Maximum deflection (w_{max})
Edge simply supported; load uniform ($r_0 = a$)	$\frac{3}{8}(3+\nu)\,p\frac{a^2}{h^2}$	Center	$\frac{3}{16}(1-\nu)(5+\nu)\frac{pa^4}{Eh^3}$
Edge fixed; load uniform ($r_0 = a$)	$\frac{3}{4}\,p\frac{a^2}{h^2}$	Edge[b]	$\frac{3}{16}(1-\nu^2)\frac{pa^4}{Eh^3}$
Edge simply supported; load at center. $P = \pi r_0^2 p$, $r_0 \to 0$, but $r_0 > 0$	$\frac{3(1+\nu)}{2\pi h^2}P\left(\frac{1}{\nu+1} + \ln\frac{a}{r_0} - \frac{1-\nu}{1+\nu}\frac{r_0^2}{4a^2}\right)$	Center	$\frac{3(1-\nu)(3+\nu)Pa^2}{4\pi Eh^3}$
Fixed edge; load at center. $P = \pi r_0^2 p$, $r_0 \to 0$, but $r_0 > 0$	$\frac{3(1+\nu)}{2\pi h^2}P\left(\ln\frac{a}{r_0} + \frac{r_0^2}{4a^2}\right)$ a must be $> 1.7r_0$	Center	$\frac{3(1-\nu^2)Pa^2}{4\pi Eh^3}$

[a] a = radius of plate; r_0 = radius of central loaded area; h = thickness of plate; p = uniform load per unit area; ν = Poisson's ratio.
[b] For thicker plates ($h/r > 0.1$), the deflection is $w_{max} = C\left(\frac{3}{16}\right)(1-\nu^2)(pa^4/Eh^3)$, where the constant C depends on the ratio h/a as follows: $C = 1 + 5.72(h/a)^2$.

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$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = -\frac{Q_r}{D}$$

Hooke's law is expressed in terms of w, as follows

$$\sigma_r = \frac{E}{1-\upsilon^2}(\varepsilon_r + \upsilon\varepsilon_\theta) = -\frac{Ez}{1-\upsilon^2}\left(\frac{d^2w}{dr^2} + \frac{\upsilon}{r}\frac{dw}{dr}\right)$$

$$\sigma_\theta = \frac{E}{1-\upsilon^2}(\varepsilon_\theta + \upsilon\varepsilon_r) = -\frac{Ez}{1-\upsilon^2}\left(\frac{1}{r}\frac{dw}{dr} + \upsilon\frac{d^2w}{dr^2}\right)$$

Bending moment and shear force

$$M_r = -D\left(\frac{d^2w}{dr^2} + \frac{\upsilon}{r}\frac{dw}{dr}\right), D = \frac{Et^3}{12\left(1-\upsilon^2\right)}$$

$$M_\theta = -D\left(\frac{1}{r}\frac{dw}{dr} + \upsilon\frac{d^2w}{dr^2}\right)$$

$$Q_r = -\frac{1}{2\pi r}\int_0^{2\pi}\int_b^r qrdrd\theta = -\frac{1}{r}\int_b^r qrdr$$

Governing equation

$$\boxed{\nabla^4 w = \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)w = \frac{q}{D}}$$

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Related Mathematics

Cubic Equations-General form

$\sigma^3 + F_1\,\sigma^2 + F_2\sigma + F_3 = 0$ where: F_1, F_2, & F_3 are constants then the solution has three roots, say a, b & c, giving: $(\sigma-a).(\sigma-b).(\sigma-c) = 0$,

hence,

$\sigma^3 - \sigma^2\,(a+b+c) + \sigma\,(a+c)b - abc = 0$

as a general form.

If either a, b or c is known a simple quadratic equation based upon the other two unknowns can derived and solved.

Position of the Maximum moment of a propped cantilever length L is given by:

$(\sqrt{2}-1)\,L$ from the prop end

Finding determinants using cofactors

Sign of cofactor

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$$

Find determinants

$$2\begin{vmatrix} 0 & 4 \\ -1 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$$

$$2[(0\times 2)-(-1\times 4)]-4[(1\times 2)-(2\times 4)]-3[(1\times -1)-(0\times 2)]$$

$$8+24+3=35$$

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