[ENG27]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MOTORSPORT ENGINEERING

SEMESTER TWO EXAMINATION 2021/2022

ENGINEERING SCIENCE II

MODULE NO: MSP5016

Date: **Friday 20th May 2022** **Time: 10:00 – 12:00**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer FOUR questions.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE:

Formula Sheets (attached after questions).

Q1. Beam deflection

A cantilever beam AB of length L shown in Figure Q1 is needed. It will be used in aerospace to support a wing of a military plane. It has to carry a uniformly distributed load of intensity w_o, which includes the weight of the beam and a factor of safety of 2.5 has to be applied.

Given: E=200GPa, L=4m, w_o=50kN/m.

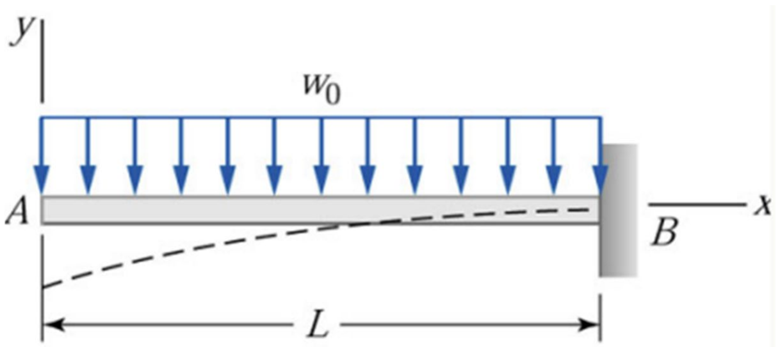

Figure Q1

a) Calculate the flexural rigidity (EI) of the beam if the maximum allowable deflection is not to exceed 3.5 cm. (6 marks)

b) Determine the dimension of the cross-section beam if it has a solid circular cross section. (8 marks)

c) Calculate the maximum bending moment and the maximum bending stress. (7 marks)

d) Is the beam safe if its yield stress σ_{yield} of the material used for the manufacturing process is 840 MPa? (4 marks)

Total 25 Marks

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Q2. Thick cylinder

A thick cylinder has an outside diameter of 110 mm and an inside diameter of 65 mm. It is pressurised internally until the outside layer has a circumferential stress of 300 MPa.

If E = 200 GPa and ν = 0.28 - Calculate:

a) The pressure difference between the inside and outside walls of the cylinder. (7 marks)

b) The circumferential stress on the outside and on the inside. (5 marks)

c) The longitudinal stress. (3 marks)

d) The circumferential strain in the inside and the outside layer. (5 marks)

e) The change in inner and outside diameter. (5 marks)

Total 25 Marks

Q3. Struts

A straight steel alloy bar of thickness 4 mm and width 12 mm is axially loaded until buckling occurs. The steel alloy bar is free at both ends.

Given: The yield stress is 350 MN/m^2.

a) Calculate the slenderness ratio of the bar when E = 70 GN/m^2 and the Euler buckling load is 95 N. (6 marks)

b) Calculate the length of the bar (4marks)

c) Find the maximum central deflection. (8 marks)

d) Using Rankine-Gordon Strut theory, what is the maximum load that the straight steel alloy bar can support without buckling? (7 marks)

Total 25 Marks

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Q4: Eigenvalues and Principal Stresses

Test procedures on an aluminium engine block have indicated the following stress state in the x and y directions: 90 MPa and 50 MPa in tension. Due to thickening of the section at the position of interest there are possibilities of a shear stress present, one related to xy with a value of 30 MPa.

a) Draw the square element showing the stresses acting. (4 marks)

b) Calculate the principal stresses. (8 marks)

c) Determine the angles relative to xy co-ordinates of the principal stresses acting in the elements and make a sketch showing the direction of these stresses. (7 marks)

d) If the proof stress of the material in tension is 370 MPa and the material follows the von Mises yield criterion, determine the factor of safety associated with this point in the block. (6 marks)

Total 25 Marks

Q5: Mohr circle and Principal Stresses

A pressure vessel made of a ductile material is subjected to a two dimensional stress system as shown on its element shown in Figure Q5 (shown over the page).

a) Determine via calculation:

(i) The magnitude of the principal stresses. (4 marks)

(ii) The angular position of the principal planes in relation to the X-axis (2 marks)

(iii) The magnitude of the maximum shear stress. (2 marks)

b) Sketch a Mohr's Stress Circle from the information provided in Figure Q5, labelling σ_1, σ_2 the principal stresses and the maximum shear stress τ_{max}. Verify the results found in part a). (8 marks)

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Question 5 continued…

c) Illustrate on a sketch of the element:

(i) the orientation of the principal planes relative to the direction of σ_X,

(3 marks)

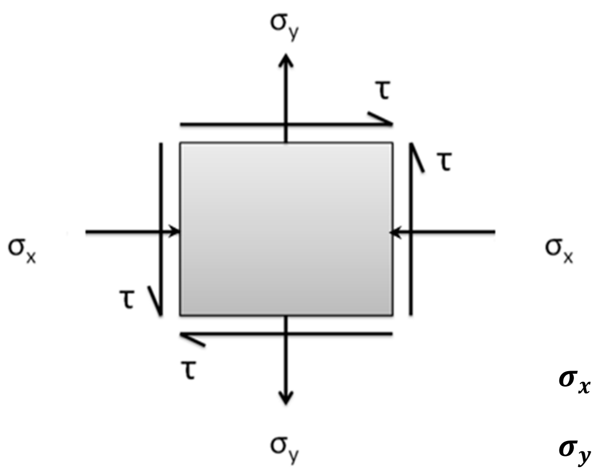
(ii) the orientation of all the stresses at the angle where the shear stress is maximum.

(3 marks)

(iii) What is the meaning of the principal stresses and the maximum shear stress in design perspective?

(3 marks)

Total 25 Marks



$$\sigma_x = -60\ MPa$$

$$\sigma_y = 70\ MPa$$

$$\tau_{xy} = \tau_{yx} = 45\ MPa$$

Figure Q5

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Q6: Application of Beam theory and FEA

A part of a car frame can be simplified into a clammed-clammed beam as shown in Figure Q6. The length, L, breath, b, height, h and thickness, t of the beam are 2 m, 10 mm, 20 mm and 2 mm, respectively. The beam is made up of Aluminum Alloy 6063-T6 and the properties of which are given in Table Q3. The beam is under a uniformly distributed load (UDL), w of 560 N/m. The FEA technique was run (see the results at the appendix).

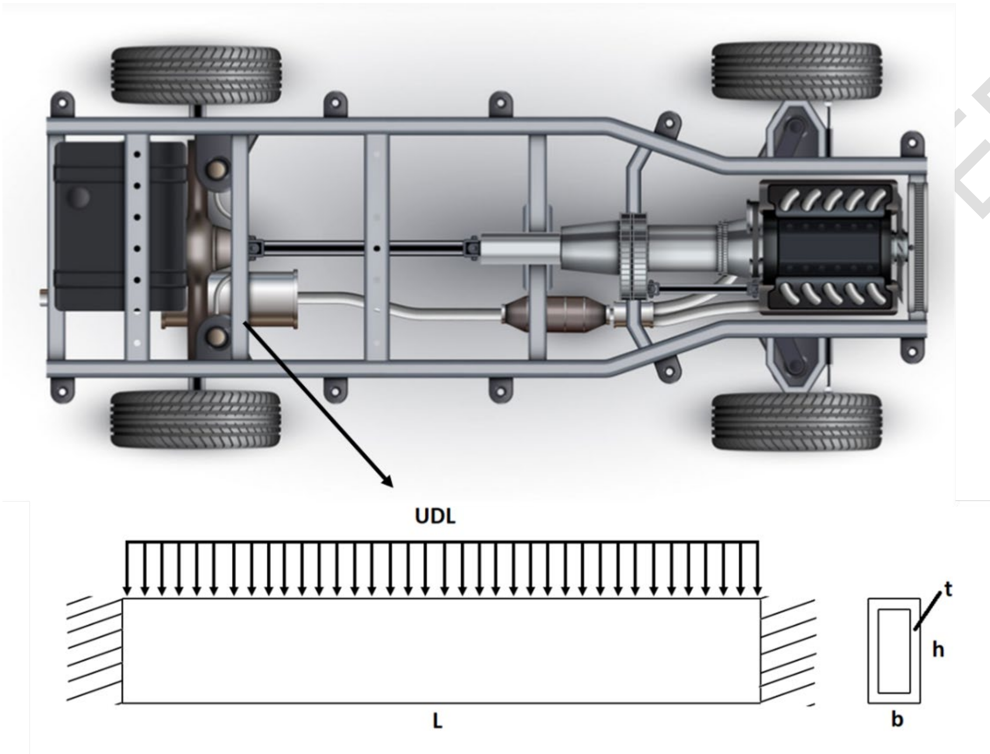

Figure Q6: Car Frame containing a clammed-clammed beam under UDL.

Table Q3: Properties of Aluminium Alloy 6063-T6

Young's Modulus, E	69 GPa
Poisson's ratio, ν	0.33
Yield Strength, σ_{ys}	215
Ultimate Strength, σ_{UTS}	240

a) Find out the maximum shear force, $F_{s(max)}$ using analytical technique, and compare the given results from the FEA technique. (4 Marks)

b) Find out the maximum bending moment, M_{max} using analytical technique, and compare the given results from the FEA technique. (4 Marks)

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Question 6 continued…

c) Find out the maximum bending stress, σ_{max} using analytical technique, and compare the given results from the FEA technique. (6 Marks)

d) Find the maximum deflection, D_{max} using analytical technique, and compare the given results from the FEA technique. (6 Marks)

e) Determine the safety factors SF_1 and SF_2 for Yield Strength and Ultimate Strength of the material, respectively. Comment on the results. (5 Marks)

Equations:

Second Moment of area,

$$I = \frac{bh^3}{12} - \frac{(b-2t)(h-2t)^3}{12}$$

Perpendicular distance to the centroid,

$$y = \frac{h}{2}$$

Maximum Shear Force,

$$F_{s(max)} = \frac{wL}{2}$$

Maximum Bending Moment,

$$M_{max} = \frac{wL^2}{12}$$

Maximum Bending Stress,

$$\sigma_{max} = \frac{M_{max}y}{I}$$

Maximum Deflection,

$$D_{max} = \frac{wL^4}{384EI}$$

Safety Factors,

$$SF_1 = \frac{\sigma_{ys}}{\sigma_{max}} \qquad SF_2 = \frac{\sigma_{UTS}}{\sigma_{max}}$$

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Question 6 continued….

Appendix: FEA results

FEA Results	
Maximum Shear Force(magnitude), F_{max}(N)	560
Maximum Bending Moment(magnitude), M_{max}(N.m)	187
Maximum Bending Stress, σ_{max}(Pa)	$404*10^6$
Maximum Deflection, D_{max}(mm)	73.37

Total 25 Marks

END OF QUESTIONS

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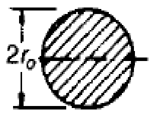
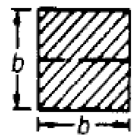
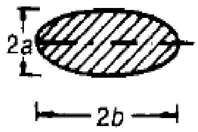
FORMULA SHEET

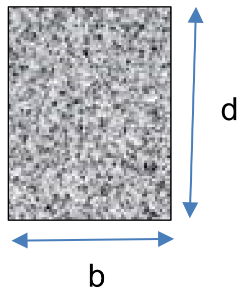
Deflection:

$$M_{xx} = EI\,\frac{d^2y}{dx^2}$$

Section Shape	$A(m^2)$	$I_{xx}(m^4)$
	πr^2	$\frac{\pi}{4}r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4}a^3b$

For solid rectangular Cross-section

$$I_{xx} = \frac{bd^3}{12}$$



Plane Stress:

a) **Stresses in function of the angle Θ:**

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2}sin(2\theta) + \tau_{xy}\,cos(2\theta)$$

b) **Principal stresses:**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Lame's equation

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall. Note: R_1 = inner cylinder radius, R_2 = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - v(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - v(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - v(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

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Vibrations:

Free Vibrations:

$$f = \frac{1}{T} \qquad \omega_n = 2\pi f = \sqrt{\frac{k}{\mathrm{M}}}$$

Damped Vibrations:

$$f_d = \frac{\omega_d}{2\pi} \qquad c_c = \sqrt{4\mathrm{M}k} \qquad \zeta = \frac{c}{c_c} = \frac{c}{2k}\,\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi a\zeta}{\sqrt{1-\zeta^2}}$$, *a* is the number of oscillations

Stress

σ = Force/Area = F/A

Hook's law

σ = E·ε

ε = ΔL/L

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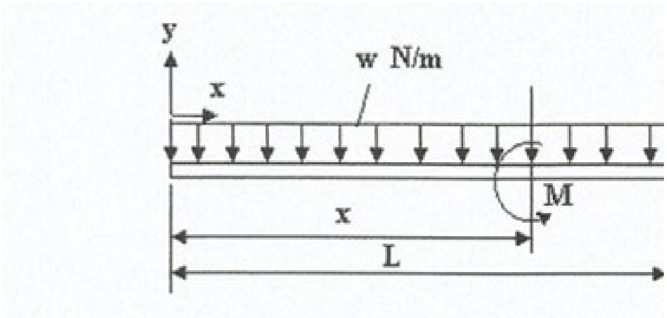
Cantilever beam with UDL:



M: maximum bending moment ($M_{max}=\omega L^2/2$)

Maximum bending stress:

$$\sigma_{bending} = \frac{My}{I}$$

M: maximum bending moment
Y: distance from neutral axis
I: second moment of area

Slope at the ends:

$$\frac{dy}{dx} = \frac{\omega L^3}{6EI}$$

Maximum deflection at the middle:

$$y = -\frac{\omega L^4}{8EI}$$

<u>Yield Criterion</u>

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

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<u>Quadratic equation: $ax^2+bx+c=0$</u>

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Allowable stress: $\boldsymbol{\sigma_{allowable}}$

$$\boldsymbol{\sigma_{allowable}} = \frac{\boldsymbol{\sigma_{yield}}}{\boldsymbol{Factor\ Of\ Safety}}$$

Struts:

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

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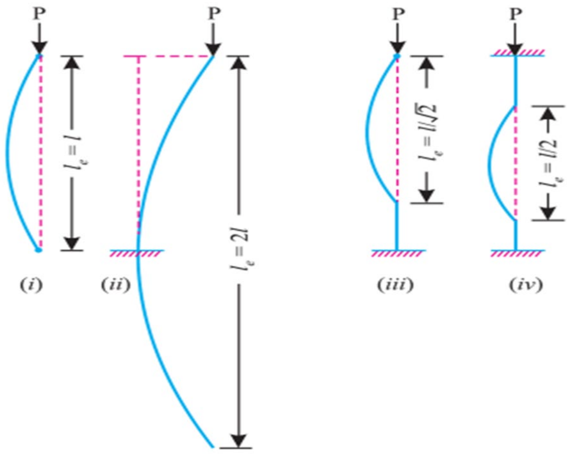
Euler validity

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi\sqrt{\frac{E}{\sigma_{yield}}}$$



(i) *Both ends pin jointed or hinged or rounded or free.*
(ii) *One end fixed and other end free.*
(iii) *One end fixed and the other pin jointed.*
(iv) *Both ends fixed.*

Case	*End conditions*	*Equivalent length, l_e*	*Buckling load, Euler*
1	*Both ends hinged or pin jointed* or *rounded or free*	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	*One end fixed, other end free*	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	*One end fixed, other end pin jointed*	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	*Both ends fixed or encastered*	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect.*

$$\boldsymbol{a = \frac{\sigma_c}{\pi^2 \cdot E}}$$

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