[ENG06]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

MSc CIVIL ENGINEERING

SEMESTER TWO EXAMINATION 2021/2022

ADVANCED STRUCTURAL MODELLING, ANALYSIS AND DESIGN

MODULE NO: CIE7002

Date: Monday 16th May 2022 **Time: 14:00 – 17:00**

INSTRUCTIONS TO CANDIDATES:

There are THREE questions.

Answer ALL Questions.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

Extracts from EC3 for Question 3 are provided in Appendices A and B.

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Question 1

a) List three types of common mistakes that will cause a singular stiffness matrix. (5 Marks)

b) For the beam shown in Figure Q1, derive the member stiffness matrices for elements 1-2 and 2-3 in the global axes. (5 Marks)

c) Assemble the global stiffness matrix and global force vector and write the global system of equations in the global axes. (5 Marks)

d) Apply the boundary conditions, and write the reduced global stiffness matrix for the beam. (5 Marks)

e) Solve the reduced global stiffness matrix and find the angular rotations of nodes 2 and 3. (5 marks)

f) Find the support reactions at nodes 1, 2, and 3. (5 Marks)

g) Draw the bending moment diagram of the beam. (5 marks)

The two beams have the same rigidity of $\mathbf{EI = 1 \ kN.m^2}$

Total 35 Marks

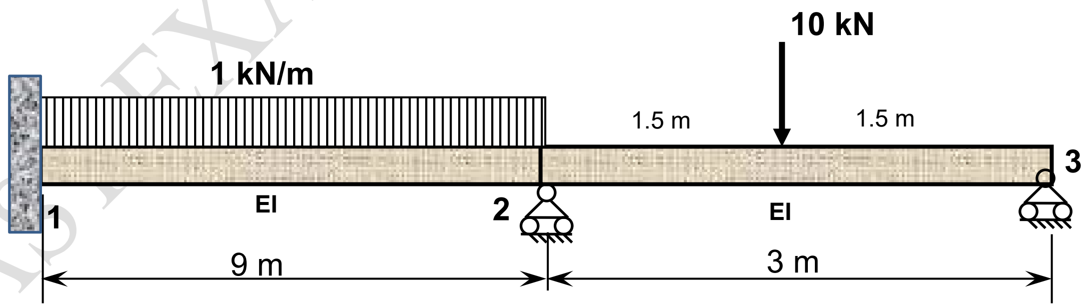

Figure Q1

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Question 1 continued….

The stiffness matrix for a beam of length L, rigidity EI, and excluding axial effect, is given by:

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

The force-displacement equation for a beam is:

$$\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{\delta}\} + \{\boldsymbol{F_e F}\}$$

Fixed end forces for beam with uniform load ***w***: $\{\boldsymbol{F_e F}\} = \begin{Bmatrix} -\boldsymbol{wl}/\boldsymbol{2} \\ +\boldsymbol{wl^2}/\boldsymbol{12} \\ -\boldsymbol{wl}/\boldsymbol{2} \\ -\boldsymbol{wl^2}/\boldsymbol{12} \end{Bmatrix}$

Fixed end forces for beam with point load ***P*** at mid-span: $\{\boldsymbol{F_e F}\} = \begin{Bmatrix} -\boldsymbol{P}/\boldsymbol{2} \\ +\boldsymbol{PL}/\boldsymbol{8} \\ -\boldsymbol{P}/\boldsymbol{2} \\ -\boldsymbol{PL}/\boldsymbol{8} \end{Bmatrix}$

Inverse matrix (2x2) of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

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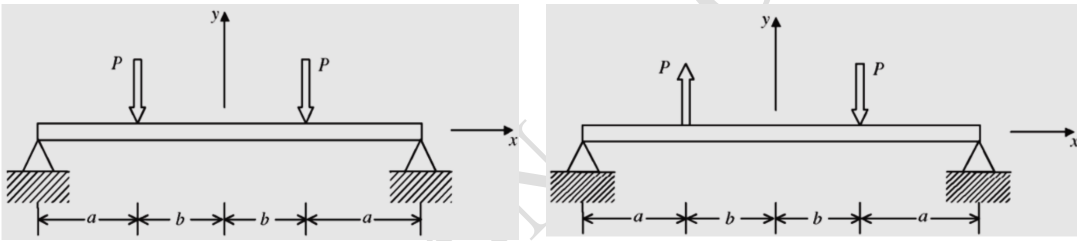
Question 2

a) Explain what is meant by Skyline of global stiffness matrix and describe how the Skyline concept is efficient when used in solving finite element problems.
(5 marks)

b) What are the advantages of using symmetry in FEM analysis? (3 Marks)

c) Identify the symmetry and anti-symmetry lines in the two-dimensional beams shown in Figure Q2(a).
Discuss the advantages and disadvantages of whether it is possible to reduce the complete structure to one half or one quarter before laying out a finite element model. If either option is possible, draw the reduced finite element model indicating, with rollers or fixed supports, which kind of displacement boundary conditions you would specify on the symmetry and anti-symmetry lines.
(7 Marks)



Beam 1 Beam 2

Figure Q2(a)

d) For the tapered bar shown in Figure Q2(b):

i) Develop the finite element stiffness matrix for a 2-node bar element.
You will have to integrate:

$$[\boldsymbol{K}] = \int_0^{\mathbf{L}} \mathbf{B}^{\mathbf{T}}(\mathbf{x})\mathbf{E}\mathbf{A}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{dx}$$

Question 2 continued next page ….

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Question 2 continued….

The shape function of a 2-node bar element is: $\boldsymbol{N} = \left[\left(\mathbf{1} - {}^{x}/_{L}\right), \left({}^{x}/_{L}\right)\right]$

The displacement function vector is $\boldsymbol{u}(\boldsymbol{x}) = [\boldsymbol{N}]\begin{Bmatrix} \boldsymbol{u_1} \\ \boldsymbol{u_2} \end{Bmatrix}$ and $\boldsymbol{B}(\boldsymbol{x}) = \frac{d\boldsymbol{N}}{dx}$

The cross-section area of the bar varies with x as:

$\boldsymbol{A}(\boldsymbol{x}) = \mathbf{0.2} - \mathbf{0.02}\boldsymbol{x}$ where $\boldsymbol{x}$ is measured in metres

The Young's modulus of the bar is E = 200 MPa.

This allows for a general element formulation.
Comment on the form of the obtained stiffness matrix.

(10 marks)

ii) Assume that the stiffness matrix of the bar is given by:

$$[\boldsymbol{K}] = \mathbf{6000}\begin{bmatrix} +\mathbf{1} & -\mathbf{1} \\ -\mathbf{1} & +\mathbf{1} \end{bmatrix} \quad \text{(kN/m)}$$

Apply the boundary conditions shown in Figure Q2(b) and calculate the displacements in the x-direction at node 2 and at the middle of the element due to application of an axial load F = 300 kN. (5 marks)

iii) Compare the values of displacement at node 2 and middle of the element obtained in part (ii) against the exact solution.
Comment on the accuracy to be expected from the stiffness relations obtained based on the assumption made to approximate the variation of displacements within the element. Use the results of part (ii) to support your arguments.

(5 Marks)

The exact analytical solution of the displacement is given by:

$$\boldsymbol{u}(\boldsymbol{x}) = -\frac{\mathbf{10}\boldsymbol{LF}}{\boldsymbol{E}}\left[\boldsymbol{ln}\left(\mathbf{1} - \frac{\boldsymbol{x}}{\mathbf{2}\boldsymbol{L}}\right)\right]$$

Total 35 Marks

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Question 2 continued….

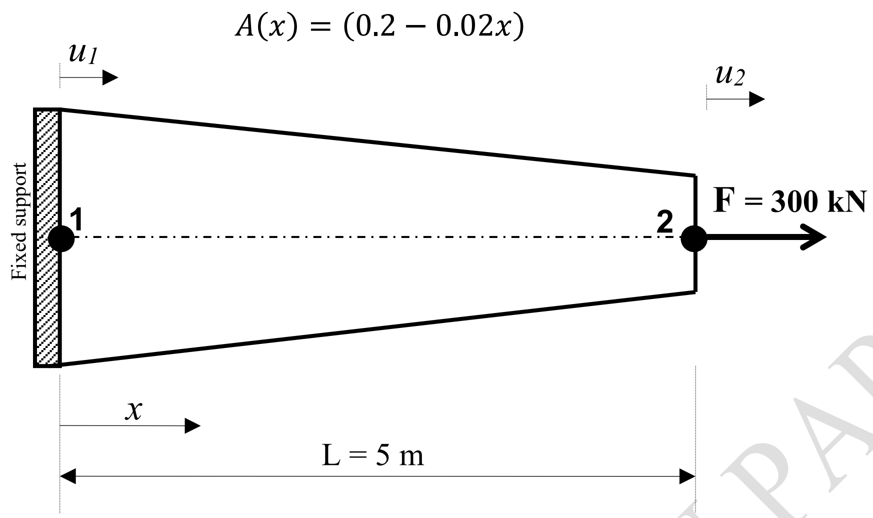

Figure Q2(b)

Question 3

A three-storey steel building is shown in Figure Q3 (page 7). The design factored vertical total loads acting on the frame are shown in the figure.

a) Calculate the global initial sway imperfections factor **ϕ** of the frame.
(2 Marks)

b) Calculate the equivalent horizontal forces (EHF) due to the sway imperfections in each floor level.
(3 Marks)

c) By using the Eurocode 3 method, compute the sensitivity factor $\boldsymbol{\alpha_{cr}}$, and check whether second order effects have to be taken into account or not. Comment on the results.
(11 Marks)

The design wind load acting on the steel frame is 10 kN/m.
The lateral deflections due to the horizontal loads at each floor level are:

Level	Lateral Displacement (mm)
Roof	12.7
2nd Floor	8.9
1st Floor	5.5

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Question 3 continued….

d) The middle column at the ground floor is subjected to an axial design load of N_{Ed} = 1500 kN.
The column is pinned at the top and fixed at the bottom ($L_{cr} = 0.85L$).
i) Determine the buckling resistance of the column. (11 Marks)
ii) In case the column is fixed at the top, assess without any further calculation, the effect of this on its buckling capacity.
(3 Marks)

Column size is UC 203x203x60 of steel grade S275 and Class1.
A= 76.4 cm^2
h = 209.6 mm
b = 205.8 mm
t_w = 9.4 mm
t_f = 14.2 mm
I_y = 6120 cm^4
I_z = 2060 cm^4
E = 210 kN/mm^2
f_y = 275 N/mm^2 , Partial safety factor $\gamma_{M1} = 1$

Critical load $N_{cr} = \dfrac{\pi^2 EI}{L_{cr}^2}$

Total 30 Marks

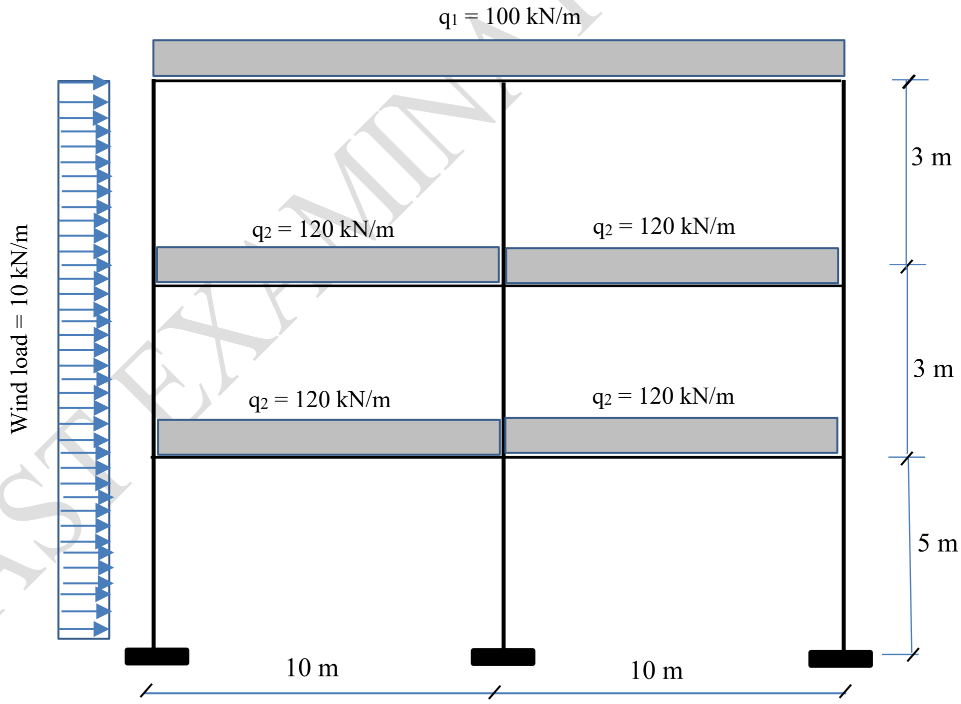

Figure Q3

Extracts from EC3 to be used with Question 3(a), (b), and (c) are included in Appendix A and Extracts from EC3 to be used with Question 3(d) are included in Appendix B over the page….

END OF QUESTIONS

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Appendix A

Formulae sheet for instability of steel structures using Eurocode 3 to be used with Question 3(a), (b), and (c).

Global initial sway imperfections

$$\emptyset = \emptyset_0 \alpha_h \alpha_m$$

$$\alpha_0 = \frac{1}{200}$$

$\alpha_h = \frac{2}{\sqrt{h}}$; with $0.66 \leq \alpha_h \leq 1$, h is the height of the structure.

$\alpha_m = \sqrt{0.5(1 + \frac{1}{m})}$, m is the number of columns in a row.

Equivalent horizontal force at each floor level, EHF = $\emptyset \times Design\ Vertical\ Load$

Sensitivity to sway, α_{cr}

$$\alpha_{cr} = \left(\frac{H_{Ed}}{V_{Ed}}\right)\left(\frac{h}{\delta_{H,Ed}}\right)$$

H_{Ed} is the horizontal force.
V_{Ed} is the total deign vertical load on the structure on the bottom of the storey.
$\delta_{H,Ed}$ is the horizontal sway at the top of the storey due to the applied horizontal loads.
h is the storey height.

$\mathbf{3 \leq \alpha_{cr} \leq 10}$

sway effects cannot be ignored, wind and equivalent horizontal loads to be increased by the amplification factor:

$$\frac{\mathbf{1}}{\mathbf{1-{}^{1}/\alpha_{cr}}}$$

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Appendix B
Extract from EC3 to be used with Question 3(d).

6.3 Buckling resistance of members

6.3.1 Uniform members in compression

6.3.1.1 Buckling resistance

(1) A compression member shall be verified against buckling as follows:

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1,0 \qquad (6.46)$$

where

N_{Ed} is the design value of the compression force
$N_{b,Rd}$ is the design buckling resistance of the compression member.

(3) The design buckling resistance of a compression member should be taken as:

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \quad \text{for Class 1, 2 and 3 cross-sections} \qquad (6.47)$$

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} \quad \text{for Class 4 cross-sections} \qquad (6.48)$$

where χ is the reduction factor for the relevant buckling mode.

NOTE For determining the buckling resistance of members with tapered sections along the member or for non-uniform distribution of the compression force second-order analysis according to 5.3.4(2) may be performed. For out-of-plane buckling see also 6.3.4.

(4) In determining A and A_{eff} holes for fasteners at the column ends need not to be taken into account.

6.3.1.2 Buckling curves

(1) For axial compression in members the value of χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ should be determined from the relevant buckling curve according to:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \text{ but } \chi \leq 1,0 \qquad (6.49)$$

where $\Phi = 0,5\left[1 + \alpha(\bar{\lambda} - 0,2) + \bar{\lambda}^2\right]$

$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}}$ for Class 1, 2 and 3 cross-sections

$\bar{\lambda} = \sqrt{\frac{A_{eff} f_y}{N_{cr}}}$ for Class 4 cross-sections

α is an imperfection factor
N_{cr} is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties.

(2) The imperfection factor α corresponding to the appropriate buckling curve should be obtained from Table 6.1 and Table 6.2.

Table 6.1 — Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

(3) Values of the reduction factor χ for the appropriate non-dimensional slenderness $\bar{\lambda}$ may be obtained from Figure 6.4.

(4) For slenderness $\bar{\lambda} \leq 0,2$ or for $\frac{N_{Ed}}{N_{cr}} \leq 0,04$ the buckling effects may be ignored and only cross-sectional checks apply.

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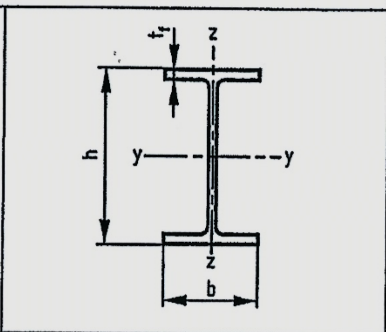
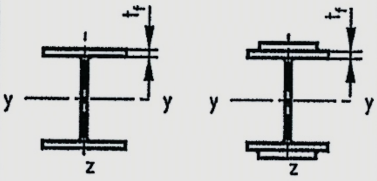
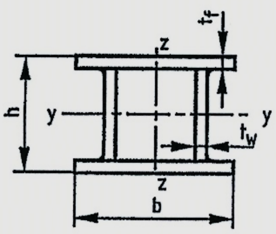
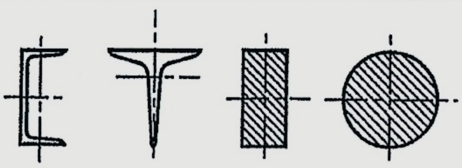
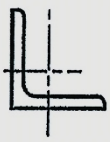
Table 6.2 — Selection of buckling curve for a cross-section

Cross section		Limits		Buckling about axis	Buckling curve: S 235, S 275, S 355, S 420	Buckling curve: S 460
Rolled sections		$h/b > 1{,}2$	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
		$h/b > 1{,}2$	$40 \text{ mm} < t_f \le 100$	y – y z – z	b c	a a
		$h/b \le 1{,}2$	$t_f \le 100$ mm	y – y z – z	b c	a a
		$h/b \le 1{,}2$	$t_f > 100$ mm	y – y z – z	d d	c c
Welded I sections		$t_f \le 40$ mm		y – y z – z	b c	b c
		$t_f > 40$ mm		y – y z – z	c d	c d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0{,}5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U, T and solid sections				any	c	c
L sections				any	b	b

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