[ENG30]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) MECHANICAL ENGINEERING

SEMESTER TWO EXAMINATION 2021/2022

FINITE ELEMENT & DIFFERENCE METHODS

MODULE NO: AME6016

Date: Thursday 19th May 2020 **Time: 10:00 – 12:00**

INSTRUCTIONS TO CANDIDATES:

There are FOUR questions

Attempt ANY THREE questions.

All questions carry equal marks.
75 marks equates to 100%

Marks for parts of questions are shown in brackets.

Formula Sheet is attached in the APPENDIX at the end of the paper

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Q1

a)

An impact damper is manufactured from steel(S) and aluminum (A) 100 mm long x 200mm^2 cross-section area blocks connected in series as shown below in Fig. Q1.

Using the FEM and 1D, determine for each block the maximum displacement and the stresses in each block if the maximum load F is 20 KN**.**

(13 Marks)

b) In order to reduce the impact compressive stress a 0.1 mm gap is set up between the aluminum block and the right-hand fixture. Now determine the new stresses in each block (12 marks)

Assume for both cases **E** for steel as **210 GPa** and for aluminum **70 GPa.**

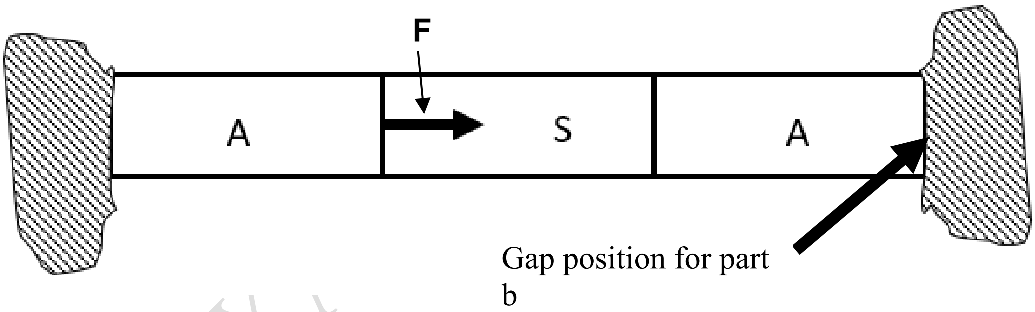

Fig Q1

Total 25 Marks

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Q2

A rotating guidance system can be modelled as shown schematically in Fig Q2. The shaft is 2L in length with a central mass, which is twice the mass of the shaft and controlling spring K.

Taking the usual notation determine an expression for the first two natural frequencies in terms of E, I, A, L and ρ including any constants derived. State any assumptions you have made. (14 marks)

Also, determine an expression for the mode associated with the first mode in terms of L and the shafts local coordinate x and sketch the shape of this mode. (7 marks)

Briefly explain the difference between the lumped mass and consistent mass approaches to solving vibration problems using FEA. (4 marks)

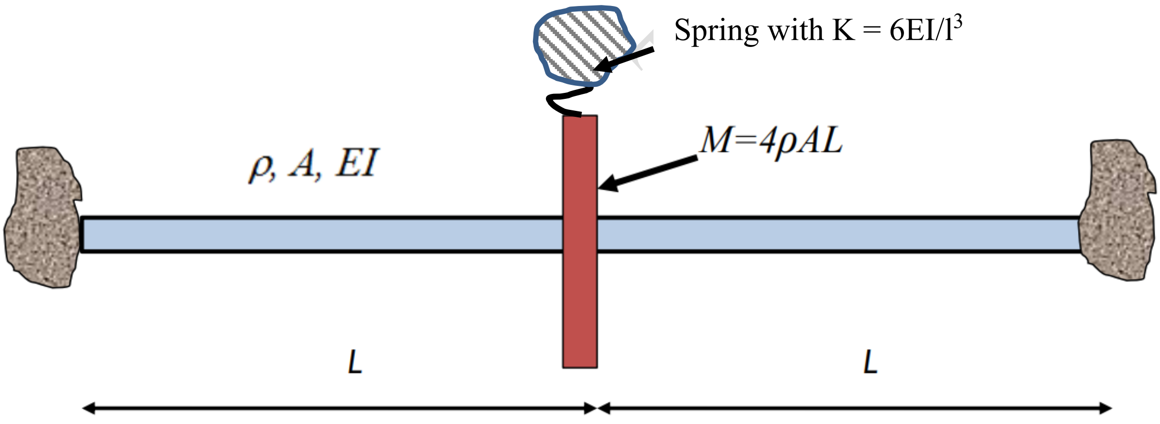

Fig Q2

Total 25 Marks

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Q3

Figure 3 below shows the arrangement for a rectangular beam, rigidly fixed at the left hand end and simply supported at the right hand end.

Figure 3

Given: E=205GNm^{-2}, I=50·10^{-8}m^{4}, L=1.2m, W=11kN.

Split the beam into four equal lengths and using the Finite Difference method of solution, where:

a) State the Boundary Conditions for the beam. (4 marks)

b) Establish the Bending Moment equations for each node on the beam. (7 marks)

c) Establish the Finite Difference equation for Bending Moment for each node. (7 marks)

d) Determine the deflection at each point-node if the reaction load at the simple support is equal to 7kN. (7 marks)

Total 25 Marks

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Q4

The radial displacement '*u*' of a thick pressure vessel is described by the following Ordinary Differential Equation *(Eq1)*:

$$\frac{d^2u}{dr} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0 \quad (Eq1)$$

And the analytical solution of *(Eq1)* is given by the equation *(Eq2)* below:

$$u = C_1 r + \frac{C_2}{r} \quad (Eq2)$$

C_1 and C_2 are constants determined by the boundary conditions.

Knowing that:

$$u(2m) = 0.008m \; and \; u(6.5m) = 0.003m$$

a) Explain the meaning of 'boundary conditions'. (4 marks)

b) Find the values of the displacement '*u*' for 4 equidistant nodes using the Finite Difference Method (FDM). (10 marks)

c) Use the analytical solution of *(Eq1)* to calculate the displacement '*u*' at the 4 equidistant nodes mentioned in a). (7 marks)

d) Calculate the relative percentage error between the analytical and the numerical results. (4 marks)

Total 25 Marks

END OF QUESTIONS

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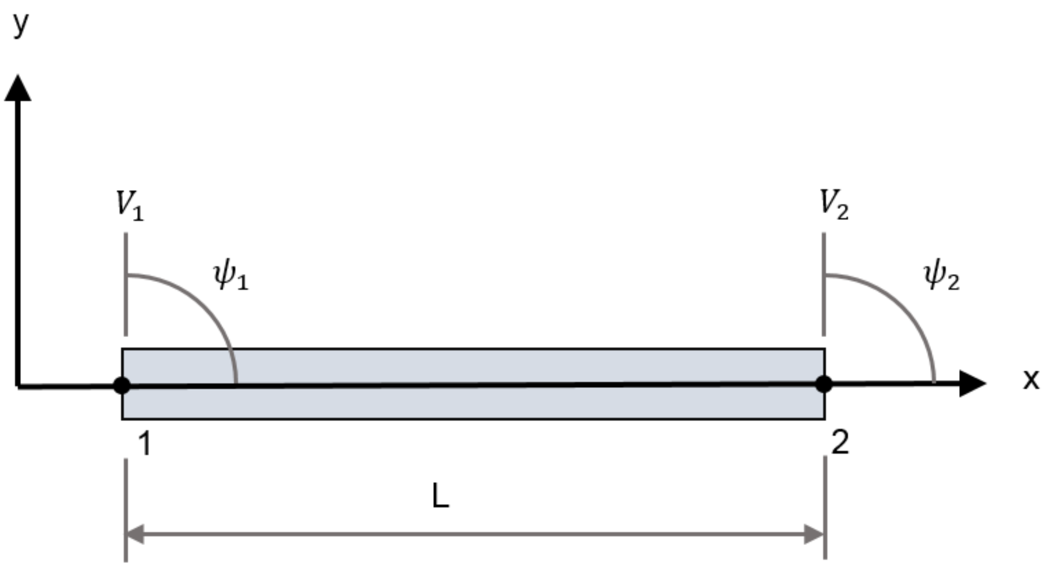
<u>FORMULAE SHEET</u>

Dynamics

$$(-\boldsymbol{\omega}^2\,[\boldsymbol{m}] + [\boldsymbol{K}])\,\{\boldsymbol{u}\} = \boldsymbol{0}$$

Finite Element Notation for 2D Beams with 2 Nodes and 4 DOF:



Element Consistent Mass Matrix

$$[m]^{\mathrm{e}} = \frac{\rho A L}{420}\begin{bmatrix} \mathbf{156} & \mathbf{22}\boldsymbol{L} & \mathbf{54} & -\mathbf{13}\boldsymbol{L} \\ & \mathbf{4}\boldsymbol{L}^{\mathbf{2}} & \mathbf{13}\boldsymbol{L} & -\mathbf{3}\boldsymbol{L}^{\mathbf{2}} \\ & & \mathbf{156} & -\mathbf{22}\boldsymbol{L} \\ & & & \mathbf{4}\boldsymbol{L}^{\mathbf{2}} \end{bmatrix}$$

Element Stiffness Matrix

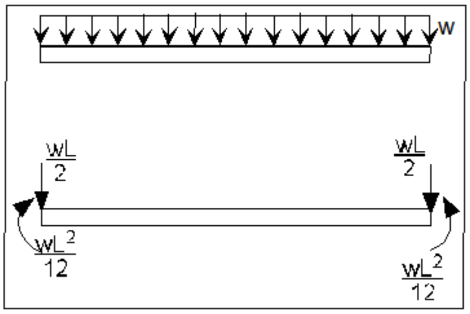
$$[K]^{\mathrm{e}} = \frac{EI}{L^3}\begin{bmatrix} \mathbf{12} & \mathbf{6}\boldsymbol{L} & -\mathbf{12} & \mathbf{6}\boldsymbol{L} \\ & \mathbf{4}\boldsymbol{L}^{\mathbf{2}} & -\mathbf{6}\boldsymbol{L} & \mathbf{2}\boldsymbol{L}^{\mathbf{2}} \\ & & \mathbf{12} & -\mathbf{6}\boldsymbol{L} \\ & & & \mathbf{4}\boldsymbol{L}^{\mathbf{2}} \end{bmatrix}$$

Element Displacement Functions

$$v(x) = \left[1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3},\, x - \frac{2x^2}{L} + \frac{x^3}{L^2},\, \frac{3x^2}{L^2} - \frac{2x^3}{L^3},\, -\frac{x^2}{L} + \frac{x^3}{L^2}\right]$$

Beam bending stress at x along the beam element is given by, $\sigma(x, y) = -Ey\frac{\partial v^2(x)}{\partial x^2}$

Equivalent nodal force due to Uniformly distributed load w



1- D Bar Element

$$\boldsymbol{U(x) = \left(1 - \frac{x}{L}\right) U_1 \; + \left(\frac{x}{L}\right) U_2}$$

1-D Beam Deflection Equation

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_i}{\Delta x}$$

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_i} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

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