UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) AUTOMOTIVE PERFORMANCE ENGINEERING

SEMESTER ONE EXAMINATION 2019/2020

ENGINEERING MATHEMATICS II

MODULE NO: MSP5017

Date: Thursday 16th January 2020 **Time: 2:00pm – 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ALL SIX questions.

The maximum marks possible for each part is shown in brackets

The examination is open-book

School of Engineering
B.Eng. Automotive Performance Engineering
Semester 1: Examination 2019/20
Engineering Mathematics II
MSP5017

1. Consider the following equation:

$$e^{5x} - x^5 = 50$$

(a) Show there is a solution to the equation on the interval $[0, 1]$ of the real line. (4 marks)

(b) Use the method of bisection *once* to determine an initial approximate solution to the equation. (4 marks)

(c) With the approximation in (b) as your initial value, use the Newton-Raphson method to find a solution to the equation accurate to 4 decimal places. (6 marks)

2. Consider the following first-order ordinary differential equation:

$$\begin{cases} \dot{x} - 6x = 16e^{-2t} \\ x(0) = 1 \end{cases} \qquad (\star)$$

where $x \equiv x(t)$ and $\dot{x}$ denotes first-order differentiation of x with respect to t.

(a) Show the Laplace transform of $(\star)$ yields:

$$X(s) = \frac{s+18}{s^2 - 4s - 12}$$

where $X(s) = \mathcal{L}\{x(t)\}$ is the Laplace transform of the function $x(t)$. (8 marks)

(b) Obtain the solution to $(\star)$ by taking a partial fraction decomposition of $X(s)$ and using inverse Laplace transforms. (12 marks)

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3. Consider the following second-order ordinary differential equation for the function $x \equiv x(t)$:

$$\begin{cases} \ddot{x} + 6\dot{x} - 27x &= 54 \\ x(0) &= 2 \\ \dot{x}(0) &= 12 \end{cases}$$

where $\dot{x}$ and $\ddot{x}$ denote first and second-order differentiation of x with respect to t respectively.

(a) Show that the second-order differential equation can be written as the *system of first-order ordinary differential equations*:

$$\begin{cases} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= 54 + 27y_1 - 6y_2 \\ y_1(0) &= 2 \\ y_2(0) &= 12 \end{cases}$$

in terms of two suitably chosen new variables $y_1(t), y_2(t)$. (6 marks)

(b) Use the method of Laplace transforms to show the system of ordinary differential equations can be written as the system of *algebraic equations*:

$$\begin{aligned} sY_1(s) - Y_2(s) &= 2 \\ -27Y_1(s) + (s+6)Y_2(s) &= \frac{12s+54}{s} \end{aligned}$$

where $Y_1(s) = \mathcal{L}\{\, y_1(t)\,\}$ and $Y_2(s) = \mathcal{L}\{\, y_2(t)\,\}$. (4 marks)

(c) Solve the system of algebraic equations to show $Y_1(s) = \dfrac{2(s+3)}{s(s-3)}$. (6 marks)

(d) Take the inverse Laplace transform of $Y_1(s)$ to find the solution to the original second-order ordinary differential equation. (8 marks)

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School of Engineering
B.Eng. Automotive Performance Engineering
Semester 1: Examination 2019/20
Engineering Mathematics II
MSP5017

4. Consider the following double integral: $\mathcal{I} = \int_{x=0}^{2} \left(\int_{y=0}^{\frac{1}{2}x} xy^2 \, dy \right) dx.$

   (a) Sketch the region of integration. (4 marks)

   (b) Change the order of integration in $\mathcal{I}$, using your diagram to obtain the new limits of integration. (6 marks)

   (c) Use either expression for the integral to show that $\mathcal{I} = \frac{4}{15}$. (10 marks)

5. Laplace's equation in two spatial dimensions is given by:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

where $\phi = \phi(x, y)$ is a function of two variables.

   (a) Show, by substitution into Laplace's equation, the following function is a solution:

$$\phi(x, y) = xy - 3x^2 y + y^3$$

(6 marks)

   (b) Explain the method of it separation of variables and apply it to Laplace's equation to reduce the partial differential equation to two *ordinary differential equations.* (8 marks)

6. Write down Reynold's Transport Theorem in one dimension and show how this can be used to derive the equation of *incompressibility* of a fluid. (8 marks)

END OF PAPER