[ESS29]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) ELECTRICAL & ELECTRONICS ENGINEERING

SEMESTER ONE EXAMINATION 2019/2020

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Wednesday 15th January 2020 Time: 14:00 – 16:00

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Question 1 is COMPULSORY. You must answer QUESTION 1 and ANY OTHER THREE questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

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<u>PART – A</u>: Answer <u>ALL QUESTIONS</u> in this part

Question 1

i). A geostationary satellite is usually subject to two forces, these forces are

a) Centripetal force

b) Centrifugal force

c) Gravitational force

d) Directional force

*Select any **<u>Two</u>** correct ones* **[2 Marks]**

ii). For a satellite to be stationary, the two opposing forces must be

a) measureable

b) equal in magnitude

c) complementary

d) rotational

*Select any **<u>One</u>** correct answer* **[2 Marks]**

iii). For a satellite to remain stationary with respect to the Earth's surface, the satellite's angular velocity must be the same as that of the Earth's own angular velocity around its own axis, i.e. $\omega = \frac{2\pi}{T}$, where T is the period of the sidereal day. How long is T ?

a) 23 hours 56 minutes 4.1 seconds

b) 23 hours 56 minutes 41 seconds

c) 23 hours 41 minutes 5.6 seconds

d) 23 hours 41 minutes 56 seconds

*Select any **<u>One</u>** correct answer* **[3 Marks]**

Question 1 continues over the page…..

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Question 1 continued….

iv). As an entrant communication engineer to Inmarsat UK, you are required to provide advice on how to optimize the power consumption of a satellite communication system.

When considering the transponder, which module of the transponder system would be your first priority ?

a) Duplexer

b) Low Noise Amplifier

c) High Power Amplifier

d) Bandpass filter

*Select any **One** correct answer* **[3 Marks]**

v). A Radar System operates by shinning its own electromagnetic energy on objects and receiving responses. The echoes are usually received through the principles of

a) Echoing

b) Reflection

c) Doppler effect

d) Illumination

*Select any **One** correct answer* **[3 Marks]**

vi) In radar communication systems, Doppler Effect happens twice for any signal transmitted by the moving source to a stationary observer, identify where

a) from radar to the target

b) from target to the radar

c) inside the radar system

d) in the transponder

*Select any **Two** correct answer* **[3 Marks]**

Question 1 continues over the page….

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Question 1 continued….

vii) A far-field observer observes an electromagnetic wave as a resolvable plane wave given as

$$E(R) = E_0 e^{j\varphi}$$

where E_0 and φ are the magnitude and phase of the electromagnetic field received at distance R, identify which part of the wave is affected by Doppler shift ?

a) amplitude

b) distance

c) phase

d) magnitude

*Select any **<u>One</u>** correct answer* **[3 Marks]**

viii) At the receiver of a satellite system, which of the following significantly affects the frequency of the signal received

a) frequency of the transmitted signal

b) Doppler effect

c) phase

d) magnitude

Select any <u>One</u> correct answer **[3 Marks]**

Question 1 continues over the page….

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Question 1 continued….

ix) Which of the following is **NOT** the role of a duplexer ?

a) allow signals to travel in one direction

b) separate the path of transmitted signal from that of received signal

c) increase the amplitude of received signals

d) makes it possible to connect a single antenna to transmitter and receiver simultaneously

Select any ***One*** *correct answer* **[3 Marks]**

Total 25 marks

END OF PART A

PART – B: Answer ANY THREE (3) questions in this part

Question 2

a) The electric field of a traveling electromagnetic wave is given by

$$E(x,t) = 10\cos\left(\omega t + \frac{\pi x}{15} + \frac{\pi}{6}\right) \quad V/m$$

Determine:

1. The direction of wave propagation; **[5 marks]**
2. Its wavelength λ and **[5 marks]**
3. Its phase velocity and phase number. **[5 marks]**

Assuming the wave frequency $0.5x10^7$Hz.

b) A series RL circuit is connected to a voltage source given by $v_s = 150cos\omega t\ V$. Find:

1. The phasor current I and **[4 marks]**
2. The instantaneous current $i(t)$ assuming R=400 Ω , L=3 mH, and f=15.916 kHz. **[6 marks]**

Total 25 marks

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Question 3

a) Given $V = x^2y + xy^2 + xz^2$ find
   1. The gradient of V and **[3 marks]**
   2. Evaluate the gradient at (1,-1,2). **[2 marks]**

b) Find ∇xA at (2,0,3) in cylindrical coordinates for the vector field

$$A = \hat{r}10e^{-2r}cos\phi + \hat{z}10sin\phi$$ **[7 marks]**

c) Derive a formula for the capacitance per unit length of the coaxial cable shown in figure Q2 below **[13 marks]**

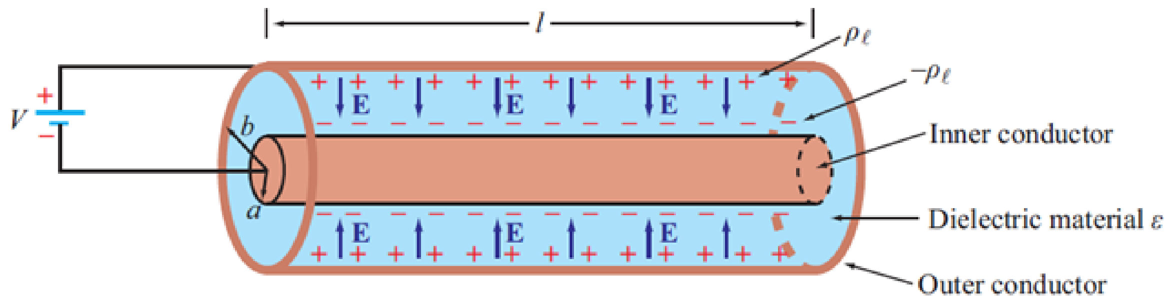

Figure Q2 Coaxial cable length=l, applied voltage=V, inner and outer radii are a, and b.

Total 25 marks

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Question 4

a) Two point charges with q1=2X10^{-5} C and q2=-4X10^{-5} C are located in free space at points with Cartesian coordinates (1,3,-1) and (-3,1,-2) respectively. Find:
   1. The electric field **E** at (3,1,-2) and **[5 marks]**
   2. The force on a 8X10^{-5} C charge located at that point. All distances in metres. **[5 marks]**

b) 1. Derive a formula for the inductance of the coaxial cable shown in figure Q3 below **[8 marks]**

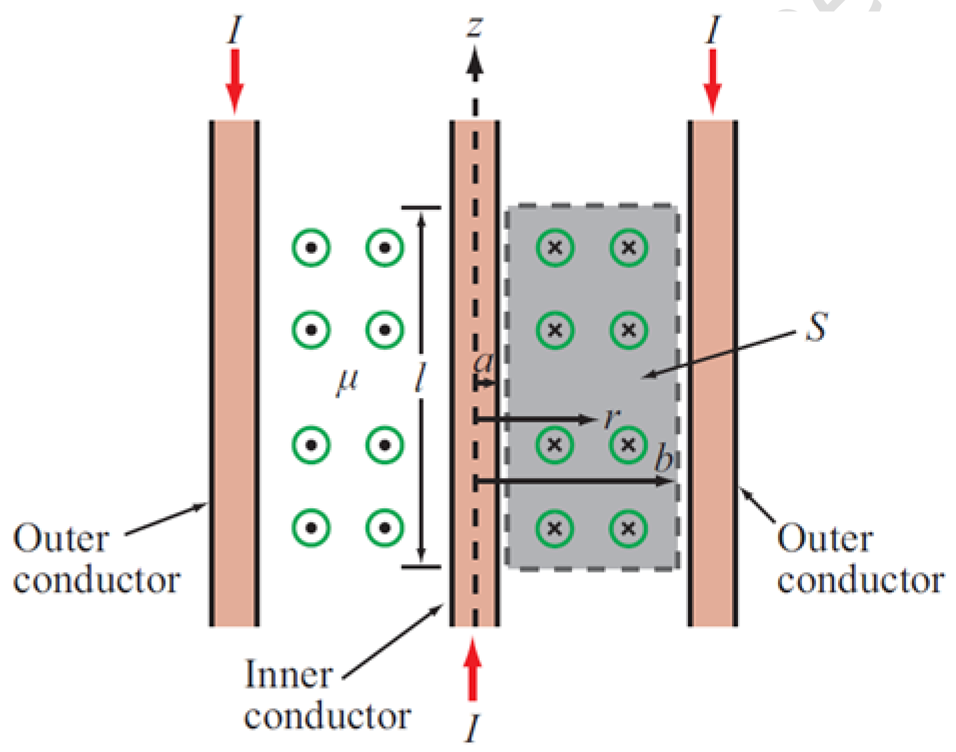

Figure Q3 Cross-sectional view of a coaxial cable

2. Then calculate the magnetic energy stored in this cable. **[7 marks]**

Total 25 marks

Question 5

a) State the reciprocity theorem of an antenna **[4 Marks]**

b) Identify any three properties of an antenna **[6 Marks]**

c) The British Telecom (BT) is planning to expand its network coverage to include the evolving 5G standard that will operate on the 3.6GHz frequency. In order to bypass a river and shadowing caused by a forest, BT plans to station two microwave antennas on top of hills 1.3×10^3m apart. If the effective area of the antenna is 26×10^3m^2, calculate the

i) received power that maximizes the transmit power **[2 marks]**

ii) Operating wavelength of transmitted signals **[3 Marks]**

iii) Transmit antenna gain **[5 marks]**

iv) Received antenna gain **[5 Marks]**

assume that $c=3\times10^8$m/s.

Total 25 marks

Question 6

a) What is a transmission line ? **[2 Marks]**
b) Give three examples of transmission lines. **[6 Marks]**
c) Given that the reflected voltage of a transmission line is

$$V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$$

where V_0^+ is the incidence voltage, Z_0 is the characteristic impedance and Z_L is the load impedance. If the normalized load impedance is $\bar{Z}_L = \frac{Z_L}{Z_0}$, show that the reflection coefficient is

$$\Gamma = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1}$$

[12 Marks]

Question 6 continues over the page….

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Question 6 continued….

d) A 100Ω transmission line is connected to a load consisting of a 50Ω resistor in series with a 10-pF capacitor. Find the reflection coefficient at the load for a 100MHz signal if the characteristic impedance is (0.5-j1.59)Ω.
Covert your answer to polar form **[5 Marks]**

Total 25 marks

END OF QUESTIONS

FORMULA SHEETS OVER THE PAGE….

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Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\tilde{Z}$, where $z(t) = \mathfrak{Re}\ [\tilde{Z}e^{j\omega t}]$.

$z(t)$		$\tilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x + \phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x + \phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0 - \pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\tilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\tilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0 - \pi/2)}$

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Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\quad |\mathbf{A}| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\quad \mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\quad \mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $\quad d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos\phi + \hat{\mathbf{y}} \sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos\phi + A_y \sin\phi$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}} \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos\phi - A_\phi \sin\phi$ $A_y = A_r \sin\phi + A_\phi \cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin\theta\cos\phi + \hat{\mathbf{y}} \sin\theta\sin\phi + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos\theta\cos\phi + \hat{\mathbf{y}} \cos\theta\sin\phi - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$	$A_R = A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta$ $A_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin\theta\cos\phi + \hat{\boldsymbol{\theta}} \cos\theta\cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}} \sin\theta\sin\phi + \hat{\boldsymbol{\theta}} \cos\theta\sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_x = A_R \sin\theta\cos\phi + A_\theta \cos\theta\cos\phi - A_\phi \sin\phi$ $A_y = A_R \sin\theta\sin\phi + A_\theta \cos\theta\sin\phi + A_\phi \cos\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin\theta + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \cos\theta - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r \sin\theta + A_z \cos\theta$ $A_\theta = A_r \cos\theta - A_z \sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_r = A_R \sin\theta + A_\theta \cos\theta$ $A_\phi = A_\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{12}} \ , \ \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3} \ , \ \mathbf{E} = \frac{\mathbf{F}}{Q}, \ \mathbf{E} = \int\frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \ , \ \mathbf{E} = \int\frac{\rho_S dS}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \ , \ \mathbf{E} = \int\frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n, \ \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho \ , \ Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_v \rho_v dv, \ \nabla\cdot\mathbf{D} = \rho_v, \ W = -Q\int_A^B \mathbf{E}\cdot d\ell, \ V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\ell, \ V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint \mathbf{E}\cdot d\ell = 0, \ \nabla\times\mathbf{E} = 0, \ \mathbf{E} = -\nabla V, \ W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k, \ W_E = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int \varepsilon_0 E^2 dv, \ \mathbf{J} = \rho_v\mathbf{u}, \ I = \int_S \mathbf{J}\cdot d\mathbf{S}, \ \mathbf{J} = \sigma\mathbf{E},$$

$$R = \frac{\mathrm{V}}{\mathrm{I}} = \frac{\int \mathbf{E}\cdot d\mathbf{l}}{\int \sigma\mathbf{E}\cdot d\mathbf{S}}, \mathbf{D} = \varepsilon\,\mathbf{E}, \ \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t}, \ E_{1t} = E_{2t}, \ D_{1n} - D_{2n} = \rho_S, \ D_{1n} = D_{2n}, \ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}, \ \nabla^2 V = 0, \ C = \frac{Q}{V} = \frac{\varepsilon\oint \mathbf{E}\cdot d\mathbf{S}}{\int \mathbf{E}\cdot d\mathbf{l}}, \ W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}, C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}, C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}, RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12}F/m \quad , \mu_o = 4\pi X10^{-7}\ H/m$$

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MAGNETOSTATICS:

$$\mathbf{H}=\int_L \frac{Id\mathbf{l}\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_S \frac{\mathbf{K}dS\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_v \frac{\mathbf{J}dv\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\frac{I}{4\pi\rho}(\cos\alpha_2-\cos\alpha_1)\mathbf{a}_\phi,\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{a}_\phi=\mathbf{a}_\ell\times\mathbf{a}_\rho,$$

$$\oint \mathbf{H}\cdot d\mathbf{l}=I_{enc},\ \nabla\times\mathbf{H}=\mathbf{J},\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{H}=\frac{1}{2}\mathbf{K}\times\mathbf{a}_n,\ \mathbf{B}=\mu\mathbf{H},\ \Psi=\int_S \mathbf{B}\cdot d\mathbf{S},\ \oint\mathbf{B}\cdot d\mathbf{S}=0,\ \nabla\cdot\mathbf{B}=0,\ \mathbf{H}=-\nabla V_m,$$

$$\mathbf{B}=\nabla\times\mathbf{A},\ \mathbf{A}=\int_L\frac{\mu_0 Id\mathbf{l}}{4\pi R},\ \mathbf{A}=\int_S\frac{\mu_0\mathbf{K}dS}{4\pi R},\ \mathbf{A}=\int_v\frac{\mu_0\mathbf{J}dv}{4\pi R},\ \Psi=\oint_L\mathbf{A}\cdot d\mathbf{l},\ \mathbf{F}=Q(\mathbf{E}+\mathbf{u}\times\mathbf{B}),\ d\mathbf{F}=Id\mathbf{l}\times\mathbf{B},\ \mathbf{B}_{1n}=\mathbf{B}_{2n},$$

$$(\mathbf{H}_1-\mathbf{H}_2)\times\mathbf{a}_{n12}=\mathbf{K},\ \mathbf{H}_{1t}=\mathbf{H}_{2t},\ \frac{\tan\theta_1}{\tan\theta_2}=\frac{\mu_1}{\mu_2},\ L=\frac{\lambda}{I}=\frac{N\psi}{I},\ M_{12}=\frac{\lambda_{12}}{I_2}=\frac{N_1\psi_{12}}{I_2},\ W_m=\frac{1}{2}\int\mathbf{B}\cdot\mathbf{H}dv=\frac{1}{2}\int\mu H^2dv$$

WAVES AND APPLICATIONS:

$$V_{emf}=-\frac{d\psi}{dt}\quad,V_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\quad,\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\quad,V_{emf}=\oint_L\mathbf{E}_m\cdot d\mathbf{l}=\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$V_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}+\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}\quad,\mathbf{J}_d=\frac{\partial\mathbf{D}}{dt}\quad,\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{dt}\quad,\beta=\frac{2\pi}{\lambda},\ \underline{\gamma}=\alpha+j\beta$$

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]},\qquad \beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]},\ \mathbf{E}(z,t)=E_0e^{-\alpha z}\cos(\omega t-\beta z)\mathbf{a}_x$$

$$|\underline{\eta}|=\frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}},\qquad \tan 2\theta_\eta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{H}=\frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t-\beta z-\theta_\eta)\mathbf{a}_y,\ \tan\theta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{a}_E\times\mathbf{a}_H=\mathbf{a}_k$$

$$\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=120\pi\approx 377\Omega,\ p(t)=\mathbf{E}\times\mathbf{H},\ p_{ave}(z)=\frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}^*{}_s),\ p_{ave}(z)=\frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta\mathbf{a}_z,\ P_{ave}=\int_S p_{ave}\cdot d\mathbf{S},$$

$$\Gamma=\frac{E_{ro}}{E_{io}}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1},\ \tau=\frac{E_{to}}{E_{io}}=\frac{2\eta_2}{\eta_2+\eta_1},\ s=\frac{|\mathbf{E}_1|_{max}}{|\mathbf{E}_1|_{min}}=\frac{|\mathbf{H}_1|_{max}}{|\mathbf{H}_1|_{min}}=\frac{1+|\Gamma|}{1-|\Gamma|},\qquad k_i\sin\theta_i=k_t\sin\theta_t,$$

$$\Gamma_\parallel=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \tau_\parallel=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \sin^2\theta_{B\parallel}=\frac{1-\mu_2\varepsilon_1/\mu_1\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2},$$

$$\Gamma_\perp=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \tau_\perp=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \sin^2\theta_{B\perp}=\frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

$\boldsymbol{\omega = \beta c}$

$$S=\frac{|V_{max}|}{|V_{min}|}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma=\frac{Z_L-Z_0}{Z_L+Z_0}$$

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Antenna and Radar formula

Dipole
Solid angle:

$$\Omega_p = \iint_{4\pi} F(\theta, \phi)\, d\Omega$$

Directivity:

$$D = \frac{4\pi}{\Omega_p} \quad \text{or} \quad D = \frac{4\pi A_e}{\lambda^2}$$

Shorted dipole

$$S_0 = \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2.$$

Hertzian monopole

$$R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

Half –wave dipole

$$\widetilde{E}_\theta = j\,60 I_0 \left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}\left(\frac{e^{-jkR}}{R}\right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}.$$

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

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$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

For Transmission line

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless $(R' = G' = 0)$	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Distortionless line

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

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<u>Open-circuited line</u>

$$\widetilde{V}_{\text{oc}}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos \beta d,$$

$$\widetilde{I}_{\text{oc}}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0} \sin \beta d,$$

$$Z_{\text{in}}^{\text{oc}} = \frac{\widetilde{V}_{\text{oc}}(l)}{\widetilde{I}_{\text{oc}}(l)} = -jZ_0 \cot \beta l.$$

<u>Short-circuited line</u>

$$\widetilde{V}_{\text{sc}}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin \beta d,$$

$$\widetilde{I}_{\text{sc}}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0} \cos \beta d,$$

$$Z_{\text{sc}}(d) = \frac{\widetilde{V}_{\text{sc}}(d)}{\widetilde{I}_{\text{sc}}(d)} = jZ_0 \tan \beta d.$$

$$j\omega L_{\text{eq}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \geq 0$$

$$\frac{1}{j\omega C_{\text{eq}}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \leq 0$$

$$Z_{\text{in}} = Z_o \left[\frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \right]$$

$$Z_{\text{in}} = Z_o \left[\frac{Z_L + Z_o \tanh \gamma \ell}{Z_o + Z_L \tanh \gamma \ell} \right]$$

$$V_o = \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g} V_g, \qquad I_o = \frac{V_g}{Z_{\text{in}} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

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For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt}G_{dr}}{4\pi}\left[\frac{\lambda}{4\pi r_1 r_2}\right]^2 \sigma P_{\text{rad}}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

$$P_{\text{rec}} = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

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