[ESS19]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (HONS) CIVIL ENGINEERING

SEMESTER 1 EXAMINATION 2019/2020

ENGINEERING MATHEMATICS & STRUCTURES

MODULE NO: CIE5004

Date: Friday 17th January 2020 **Time: 10:00am – 1:00pm**

INSTRUCTIONS TO CANDIDATES:

There are TWO Sections, A and B.

Answer Section A in ONE Answer Booklet and Section B in the other.

Section A: Q1 to Q2 (Answer ALL Questions)

Section B: Q3 to Q4 (Answer ALL Questions)

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

A formula sheet is attached to the end of the paper

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SECTION A: STRUCTURES
(Answer ALL Questions)

Question 1

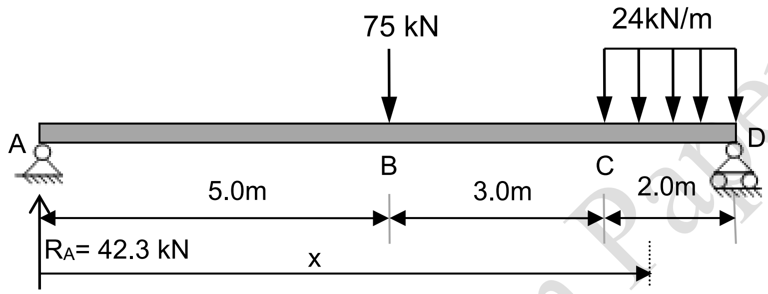

Figure Q1

Figure Q1 shows a beam ABCD which is simply supported with a span of 10 m. The beam carries one point load and one distributed load as shown in Figure Q1. The beam has uniform rigidity EI = 20,000kNm^2.

a Use the method of Macaulay to calculate

i. The rotation (slope) at A.

ii. The vertical deflection at B.

(20 marks)

b Estimate the value of x at which the rotation (slope) will be zero.

(5 marks)

Formula for the deflection of a beam: $M = -EI\dfrac{d^2v}{dx^2}$

(Total 25 marks)

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Question 2

The three pin frame shown in Figure Q2 (i) is pinned to supports at A and F, with a third pin at D. The frame is subjected to a vertical point load of 50 kN at position C and horizontal point load of 40 kN at position E.

a) Calculate the value of the support reactions at A and F. **(5 marks)**

b) Draw the axial force diagram (AFD) **(4 marks)**

c) Draw the shear force diagram (SFD) **(4 marks)**

d) Draw the bending moment diagram (BMD) **(6 marks)**

For b), c) and d) show all important values on the diagrams and produce accompanying calculations to show how these values have been derived.

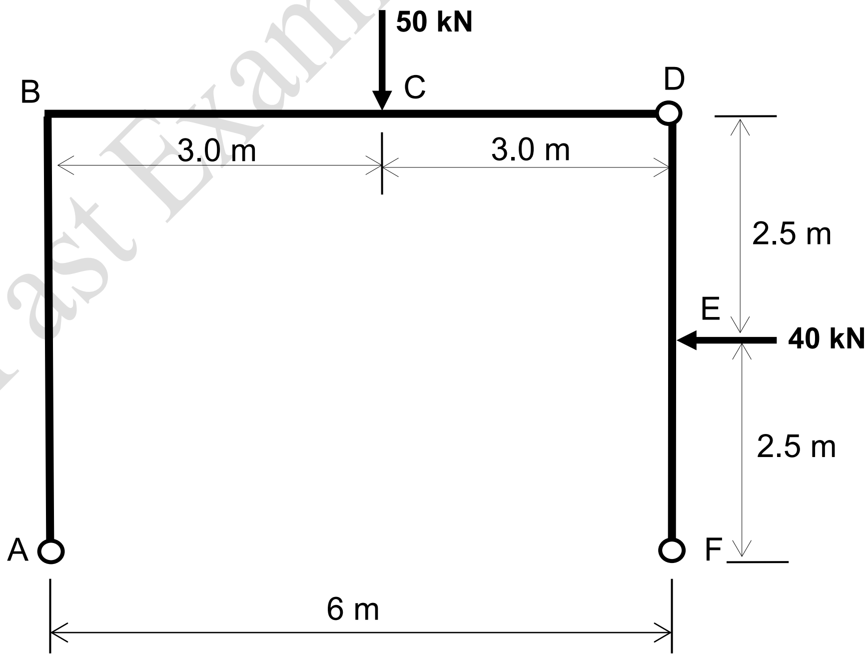

FIGURE Q2 (i)

Q2 continues over the page…

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Q2 continued…

Figure Q2 (ii) shows a very similar three pin frame, pinned to supports at A and F, with the third pin at B (no longer at D). The applied loads remain the same as Figure Q2 (i).

e) Without doing any further calculations, sketch the Bending Moment Diagram (BMD) for the three pin frame shown in Figure Q2 (ii). Do not attempt to calculate the values of the bending moments in the frame.

(6 marks)

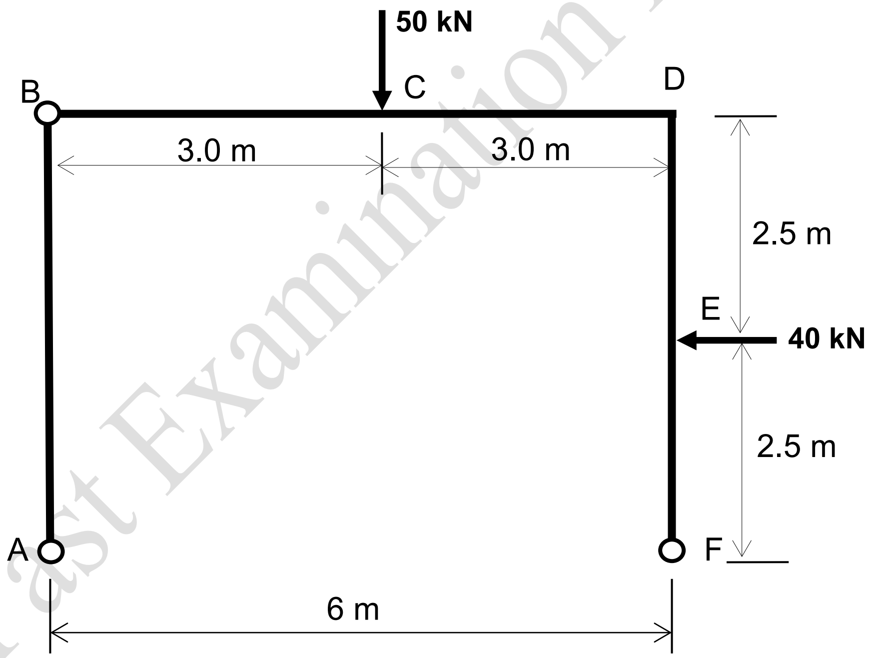

FIGURE Q2 (ii)

Total 25 marks

END OF SECTION A

PLEASE TURN THE PAGE FOR SECTION B…

SECTION B: ENGINEERING MATHEMATICS

(Answer BOTH Questions)

Question 3

(a) A service engineer can be called out to inspect drainage at one of five large properties A,B,C,D and E. On any given week, the probability the engineer will be called out to one of these properties is 0.12. The event of being called out to one property is independent of being called out to one of the other properties.

Find, to five decimal places, the probability that, on a particular day, the engineer is called out to:

(i) all five properties; **(3 marks)**
(ii) at least three properties; **(3 marks)**
(iii) four properties, given that the engineer has been called out to property A. **(4 marks)**

(b) The number of failures occurring in a machine of a certain type in a year has a Poisson distribution with mean 0.34. In a factory there are twelve of these machines. What is:

(i) the expected total number of failures in a year? **(3 marks)**
(ii) the probability, to five decimal places, that there are fewer than two failures in the factory in a year? **(4 marks)**

(c) The tensile strength of a bar made with a certain metal composite is normally distributed with a mean of 1200 $kg \cdot cm^{-2}$ and a standard deviation of 80 $kg \cdot cm^{-2}$.

(i) What proportion of these components exceed 1320 $kg \cdot cm^{-2}$ in tensile strength? **(4 marks)**
(ii) To ensure safety when the bars are used in construction, it is required that at least 95% of they have a strength greater than 1072 $kg \cdot cm^{-2}$. Do the bars meet the specifications? **(4 marks)**

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Total: 25 marks

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Question 4

(a) A manufacturer claims that the average tensile strength of material A exceeds the average tensile strength of material B by at least 12 kg. To test this claim, 50 pieces of each type of material were tested under similar conditions. Material A had an average tensile strength of 86.7 $kg \cdot cm^{-2}$ with a standard deviation of 6.28 $kg \cdot cm^{-2}$, while material B had an average tensile strength of 77.8 $kg \cdot cm^{-2}$ with a standard deviation of 5.61 $kg \cdot cm^{-2}$. Test the manufacturer's claim using a 5% level of significance. **(9 marks)**

(b) A manufacturer produces electronic components for use in computer controlled monitoring systems. A random sample of 100 components is inspected and the number of faults per component recorded. The results are given in the table below:

Number of faults per component	Frequency of occurrence
0	42
1	34
2	18
3	6

Perform a chi-squared test with a 5% level of significance to determine the validity of the assumption that the occurrence of faults in the components is Poisson. **(16 marks)**

Total 25 marks

END OF SECTION B
END OF QUESTIONS

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Mathematical Tables and Formulae

Probability Rules

The probability that two events E_1 and E_2 occur is denoted $p(E_1$ **Ç** $E_2)$ and given by:

$p(E_1$ **Ç** $E_2) = p(E_1)\, p(E_2)$.

If the two events are independent. The probability that event E_1 or event E_2 occurs is denoted $p(E_1$ **È** $E_2)$ and given by:

$p(E_1$ **È** $E_2) = p(E_1) + p(E_2) - p(E_1$ **Ç** $E_2)$.

An event E_2 is conditional if it depends upon the event E_1 with conditional probability denoted $p(E_2|\, E_1)$ given by:

$p(E_2|\, E_1) = p(E_1$ **Ç** $E_2) / p(E_1)$.

Binomial Distribution

Let X be a random variable denoting the number of successes in n independent trials where each trial has probability p of success and probability 1 − p of failure. Then X follows a binomial distribution: $X \sim B(n,p)$ and

$$p(X = k) = {}^{n}C_{k}\, p^{k}\, (1-p)^{n-k}$$

Poisson Distribution

Let X be a random variable denoting the number of events occurring in an interval where the average number of events in the interval is λ. Then X follows a Poisson distribution: $X \sim P(\lambda)$ and

$$p(X = k) = e^{-\lambda} \left(\lambda^{k} / k! \right)$$

where $k = 0,1,2,\ldots$

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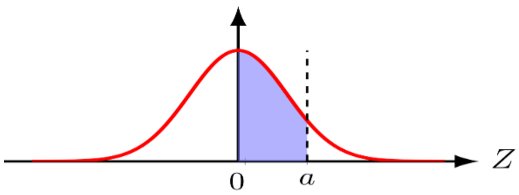
Standard Normal Distribution Table

Any random variable $X \sim N(\mu,\sigma^2)$ can be transformed into the standard normal distribution variable $Z \sim N(0,1^2)$ using:

$$Z = (X - \mu) / \sigma$$



The shaded region gives $p(Z < a)$ and yields the numerical values in the table below:

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0278	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3032	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4865	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4981	0.4982	0.4983	0.4983	0.4984	0.4984	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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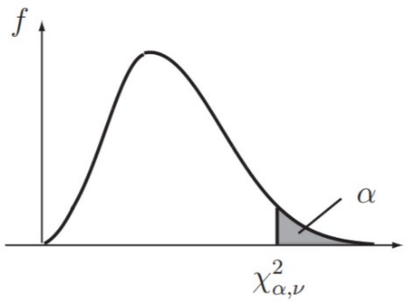
Percentage Points $\chi^2_{\alpha,\upsilon}$ of the χ^2 Distribution



α	0.995	0.990	0.975	0.950	0.900	0.500	0.100	0.050	0.025	0.010	0.005
v											
1	0.00	0.00	0.00	0.00	0.02	0.45	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.01	0.21	1.39	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	2.37	6.25	7.81	9.35	11.34	12.28
4	0.21	0.30	0.48	0.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	31.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

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Goodness-of-Fit

Given the observed frequency O_i of the i-th interval of a random sample of size n drawn from a population; the expected frequency $E_i = nP_i$ of the i-th interval under the hypothesis that the data follows a known distribution where P_i is the probability associated with interval i; we can calculate the goodness-of-fit statistic:

$$W = \sum_i [(O_i - E_i)^2 / E_i]$$

which follows a χ^2 distribution with (k-p-1) degrees of freedom where k is the number of interval and p is the number of parameters needed to describe the probability distribution of the population which we have to estimate from the data.

t-Test and z-Test Statistics

Given a sample $\{X_1, X_2, \ldots, X_n\}$ of size n drawn from a normally distributed population with mean μ and variance σ^2, the sample mean and sample variance are given by:

$$\overline{X} = (X_1 + X_2 + \cdots + X_n) / n \quad \text{and} \quad s^2 = 1/(n-1) \sum_i (X_i - \overline{X})^2$$

respectively. The t-test and z-test statistics are

$$z = (\overline{X} - \mu) / (\sigma / \sqrt{n}) \quad \text{and} \quad t = (\overline{X} - \mu) / (\sigma / \sqrt{n})$$

respectively. The t-test is adopted when the sample is small (n<30) and the variance is unknown. The z-test requires the variance to be known or assumed to be known and is used for large sample sizes (n>30).

END OF FORMULA SHEETS

END OF PAPER