[ESS17]

UNIVERSITY OF BOLTON

SCHOOL ENGINEERING

BEng (HONS) CIVIL ENGINEERING

SEMESTER 1 EXAMINATION 2019/2020

MATHEMATICS & STRUCTURAL ANALYSIS

MODULE NO: CIE4011

Date: Wednesday 15th January 2020 **Time: 10:00am – 1:00pm**

INSTRUCTIONS TO CANDIDATES:

There are SEVEN questions.

Answer ALL SEVEN questions.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

Supplementary formulae sheets are provided on pages 8-9 at the rear of the question paper.

Lined Graph Paper is available for use.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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SECTION A: STRUCTURAL ANALYSIS

Question 1

Figure Q1shows a simply supported beam with a pin support at A, and a roller support at D. The beam is carrying one vertical point load at B, and a uniform distributed load (UDL) between C and D.

i) Calculate and state the support reactions at A and D. **(2 marks)**

ii) Draw the Shear Force Diagram, showing values at A, B, C, and D. **(5 marks)**

iii) Draw the Bending Moment Diagram, showing values at A, B, C, and D. **(10 marks)**

iv) Calculate the maximum bending moment value in the beam, and state its position along the beam**.** **(3 marks)**

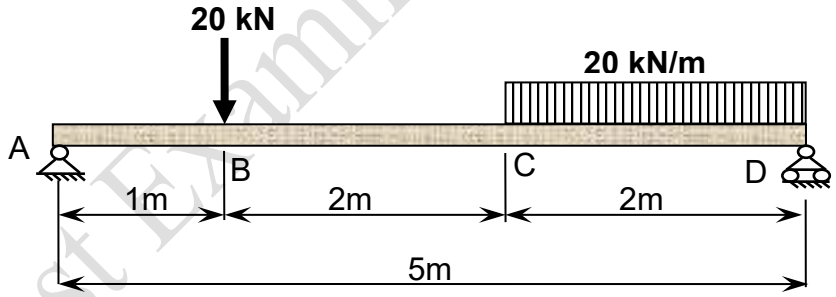

Figure Q1

Total 20 marks

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Question 2

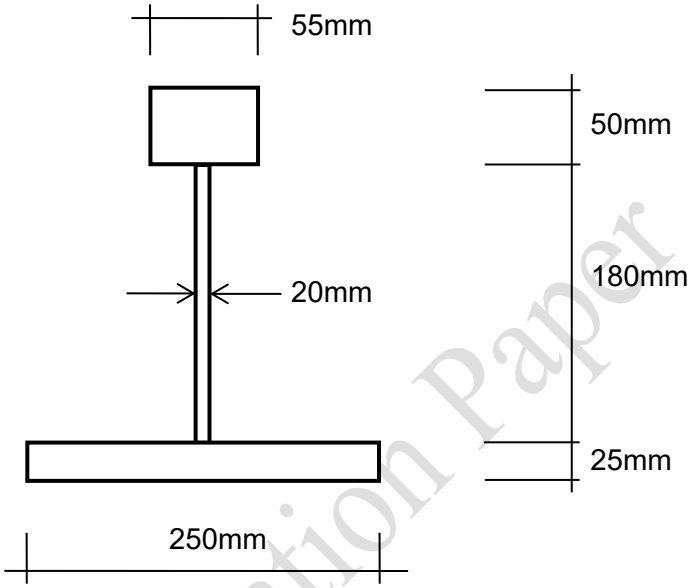

Figure Q2 (i)

Figure Q2 (i) shows a cross-section of an asymmetrical steel beam.

a) Determine the position of the horizontal neutral axis of the beam.

(6 marks)

b) What is the value of the second moment of area I about the horizontal neutral axis of the beam section?

(9 marks)

Q2 continues next page…

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Q2 continued…

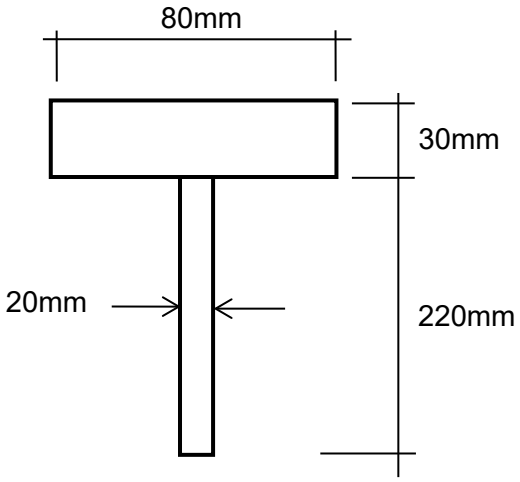

Figure Q2 (ii)
Section through cantilever tee beam

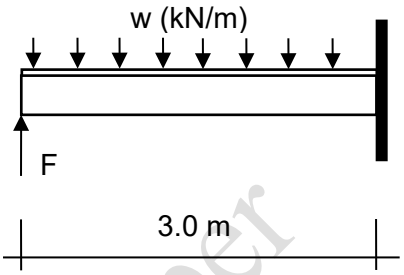

Figure Q2 (iii)
Elevation on cantilever tee beam

Figure Q2 (ii) shows a cross-section of a different asymmetrical cast iron tee beam with a cantilever span of 3.0 m; also see elevation in Figure Q2 (iii). The allowable bending stresses in the tee beam are shown the table below:

	Maximum stress (N/mm^2)
Tension	255.0
Compression	255.0

The geometrical properties of the tee beam are shown in the table below:

Distance of the horizontal neutral axis of the tee beam above the bottom of the section	154.12 mm
Second moment of area (I)	4219 cm^4

c) What is the maximum UDL (w) that can be applied vertically downward to the cantilever tee beam without exceeding the allowable bending stress in the tee beam (ignore force F)?

(9 marks)

d) What is the maximum force (F) that can be applied vertically upward to the cantilever tee beam without exceeding the allowable bending stress in the tee beam (ignore UDL (w))?

(6 marks)

Total 30 marks

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END OF SECTION A
PLEASE TURN THE PAGE FOR SECTION B…

SECTION B: MATHEMATICS

Question 3

Solve the following system of simultaneous linear equations:

$$3p + 2q + r = 44$$

$$2p - 2q + r = 10$$

$$p + q - r = 14$$

(10 marks)

Question 4

(a) The half-life is the amount of time it takes for uranium to lose half of its radioactivity. The number of radioactive atoms N at time t is given by

$$N = N_0 e^{-\frac{t}{k}}$$

where t is time in billions of years, N_0 is the number of radioactive atoms at t = 0, and *k* is a constant.

Show clearly that the half-life t_H , the time when $N = \frac{1}{2} N_0$, is given by

$$-k \ln \frac{1}{2}$$

(4 marks)

The half-life of uranium-238 is approximately 4.5 billion years.

(b) Find k **(3 marks)**

(c) Find t_Q, the time, in billions of years, when $N = \frac{1}{4} N_0$. **(3 marks)**

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Question 5

Using logarithms, solve the following simultaneous equations:

$$3^{x+y+2} = 30$$

$$x - y + 2 = 0$$

Give your answers correct to 2 decimal places.

(10 marks)

Question 6

A function is defined by the following formula:

$$f(x) = 16 - (x-2)^2$$

The value of *x* ranges between -2 and 6.

Copy and complete the following table for values of *x* and *f*(*x*):

x	-2	-1	0	1	2	3	4	5	6
f(*x*)									

(1 mark)

Let *A* be the area bounded above by the curve of the graph of the function, below by the *x*-axis, and on the left and right by the ordinates at $x = -2$ and $x = 6$.

Estimate the area *A* using:

(i) the trapezium rule with four strips **(3 marks)**

(ii) the trapezium rule with eight strips **(3 marks)**

(iii) Simpson's rule with four strips. **(3 marks)**

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Question 7

(a) Using Pascal's triangle, expand and simplify the following:

$$(3 - x)^4$$

(2 marks)

(b) Write down and simplify the first four terms of the binomial expansion for:

$$(3 - x)^{\frac{1}{2}}$$

(4 marks)

(c) Use your answer to (b) to find an approximate value to three decimal places for

$$\sqrt{3 - e}$$

(4 marks)

END OF QUESTIONS

PLEASE TURN THE PAGE FOR SUPPLEMENTARY FORMULAE SHEETS

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Formula sheet for structural analysis

Simply supported and cantilever beams

Typical units		M_{max} (kNm)	$R_{support}$ (kN)	$Deflection_{max}$ (mm)
Simply supported beam length L	Point load P at centre	$\frac{PL}{4}$	$\frac{P}{2}$	$\frac{PL^3}{48EI}$
Simply supported beam length L	UDL w along full length	$\frac{wL^2}{8}$	$\frac{wL}{2}$	$\frac{5wL^4}{384EI}$
Cantilever beam length L	Point load P at tip	PL	P	$\frac{PL^3}{3EI}$
Cantilever beam length L	UDL w along full length	$\frac{wL^2}{2}$	wL	$\frac{wL^4}{8EI}$

Shape properties

Typical units	Area A (mm^2)	Elastic section modulus W_{el} (mm^3)	Plastic section modulus W_{pl} (mm^3)	2nd moment of area I (mm^4)
Rectangle with side lengths b and h	bh	$\frac{bh^2}{6}$	$\frac{bh^2}{4}$	$\frac{bh^3}{12}$

Stresses

Bending stress = $\frac{My}{I}$

Bending stress = $\frac{M}{S}$

Bending stress = $\frac{M}{z}$

Radius of gyration = $\sqrt{\frac{I}{A}}$

Axial stress = $\frac{P}{A}$

Average shear stress = $\frac{V}{A}$

Complex shapes

Centroid

$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$ and $\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$

Parallel axes theorem

$I_{xx} = \sum(I_0 + Ay^2)$

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Mathematical Formulae

Logarithms

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log(a^p) = p \log a$$

Trapezium Rule

$$h\left(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n\right)$$

Simpson's Rule with four strips

$$\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

The Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$
$$+ \frac{n(n-1)(n-2)(n-3)}{4!}x^4 + \cdots$$

END OF FORMULA SHEETS

END OF PAPER