

**UNIVERSITY OF BOLTON**  
**SCHOOL OF ENGINEERING**  
**BENG (HONS) IN BIOMEDICAL ENGINEERING**  
**SEMESTER ONE EXAMINATION 2019/2020**  
**ADVANCED BIOMECHATRONIC SYSTEMS**  
**MODULE NO: BME6003**

Date: Friday 17<sup>th</sup> January 2020

Time: 10:00 – 12:00

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**INSTRUCTIONS TO CANDIDATES:**

There are SIX questions. You are required to answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

**CANDIDATES REQUIRE:**

Property tables provided  
Formula Sheet (attached)

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**Q1**

(a) Explain, helped by sketches, what amplitude, gain, phase and phase shift are and why frequency response is useful for biomechatronic systems control.

**[8 marks]**

(b) Figure Q1 (b) shows a Bode plot.

i) Estimate the gain margin and the phase margin.

**[4 marks]**

ii) Explain the functions of gain margin and phase margin in systems control.

**[4 marks]**

iii) Comment on the system's stability performance.

**[2 marks]**

iv) Explain the system's Peak Resonance  $M_p$  and Bandwidth.

**[3 marks]**

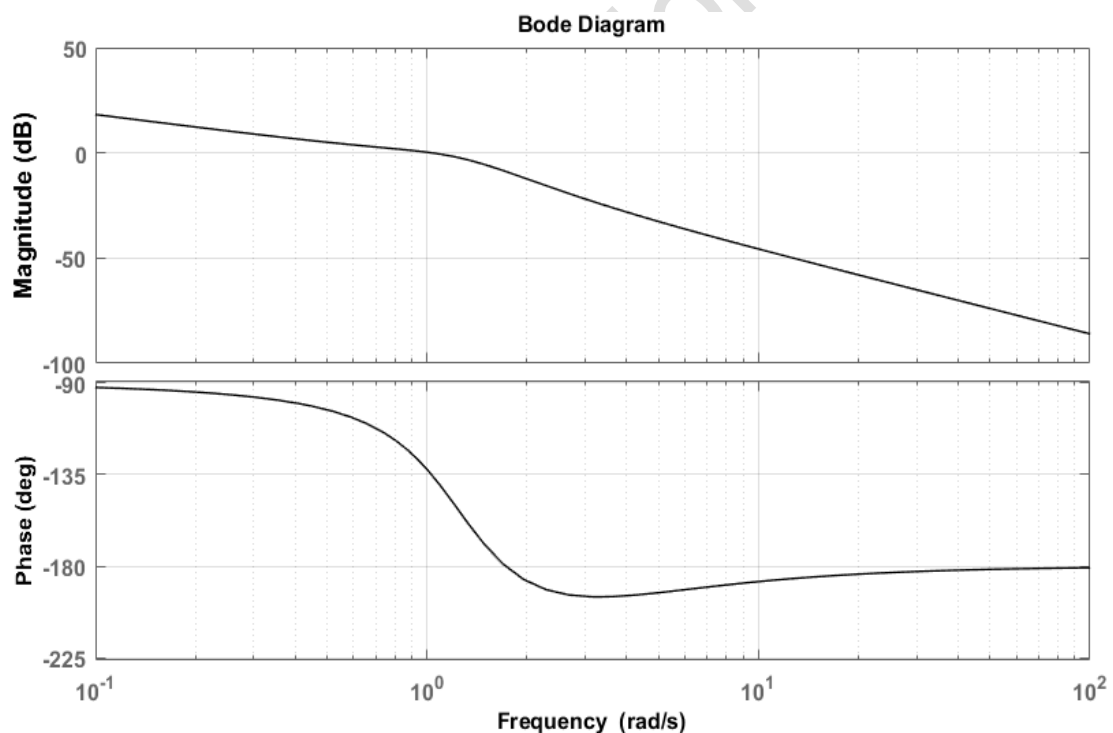


Figure Q1 (b) A Bode Plot

**Q1 continues over the page....**

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School of Engineering  
 BEng (Hons) Advanced Biomedical Engineering  
 Semester One Examination 2019/2020  
 Advanced Biomechatronic Systems  
 Module No. BME6003

**Q1 continued....**

(c) The following figure Q1 (c) shows a sinusoidal input and output of a biomedical system. Given the input frequency is 3 rad/sec. Please determine the gain and phase.

**[4 marks]**

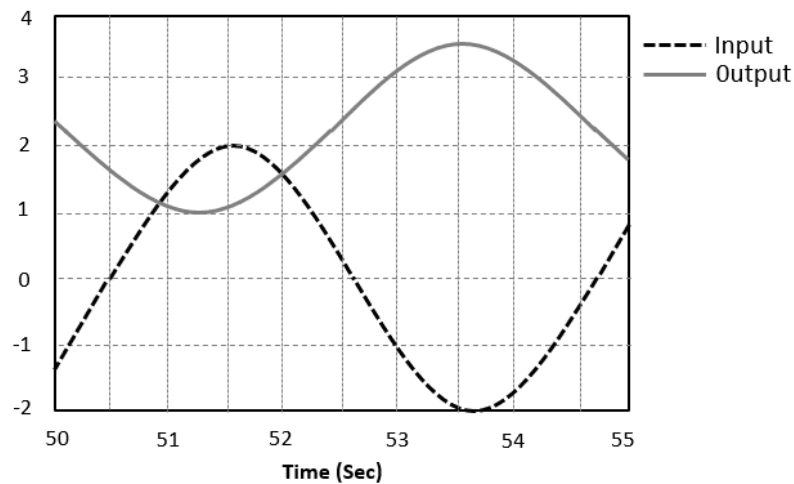


Figure Q1 (c) shows an open loop Bode plot.

**Total 25 marks**

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Past Examination

**Q2**

A simplified model of a prosthesis limb control system is shown in Figure Q2 and the control system for the prosthesis dynamics is given by:

$$G_p(s) = \frac{6}{(s + 2)(4s + 5)}$$

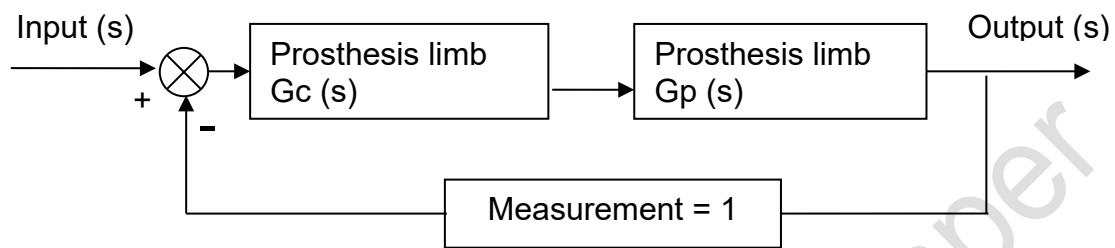


Figure Q2 A prosthesis limb

The system responses for a unit step input are required as:

- The maximum overshoot is less than 12%
- The rise time is less than 0.7 seconds
- The steady-state error is 0

(a) If  $G_c(s)$  is a PID controller with  $K_p = 5$ ,  $K_i = 2$  and  $K_d = 3$ , find the range of the gain  $K_p$  making the system to be an underdamped system for unit step input. examine the actual system's percent overshoot, rise time and steady-state error under the PID controller and check whether the design criteria have been achieved or not.

**[14 marks]**

(b) If the design criteria haven't been achieved by using the PID controller provided above, explain the procedure to modify the PID controller.

**[3 marks]**

(c) Describe, helped by sketches, how the error item is handled by proportional, integral and derivative controller.

**[8 marks]**

**Total 25 marks**

**PLEASE TURN THE PAGE....**

School of Engineering  
BEng (Hons) Advanced Biomedical Engineering  
Semester One Examination 2019/2020  
Advanced Biomechatronic Systems  
Module No. BME6003

**Q3**

- (a) Using block diagrams, briefly explain discrete time signal processing for feedback and feedforward system.

**[6 marks]**

- (b) Explain what is meant by a zero-order hold (ZOH) system.

**[4 marks]**

- (c) A controller of biomechatronic system consists of a Digital to Analogue Converter with zero order element in series with the processing centre which has a transfer function

$$G_p(s) = \frac{3}{s(3 + s)}$$

Find the sampled-data transfer function,  $G(z)$  for the digital control system. The sampling time,  $T$ , is 0.5 seconds.

**[7 marks]**

- (d) A controller has a 12 bit Analogue to Digital Converter with the signal range between 0 Volt to +10 Volt:

- (i) What is the resolution of the AD converter?

**[2 marks]**

- (ii) What integer number represented a value of +7.5 Volts?

**[2 marks]**

- (iii) What voltage does the integer 1345 represent?

**[2 marks]**

- (iv) What voltage does 010101011101 represent?

**[2 marks]**

**Total 25 marks**

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**Q4**

- (a) A typical op-amp has an open loop gain function of;  $A_v = \frac{10^6}{1+j\frac{f}{20\text{Hz}}}$  sketch it's graph and label the open loop gain in decibels and the open loop corner frequency. **[5 marks]**
- (b) If an op-amp signal conditioning circuit is shown in Figure Q4(b), sketch it's closed loop magnitude and phase graphs, indicating the closed loop gain and corner frequency if it uses the transfer function from part(a). **[10 marks]**

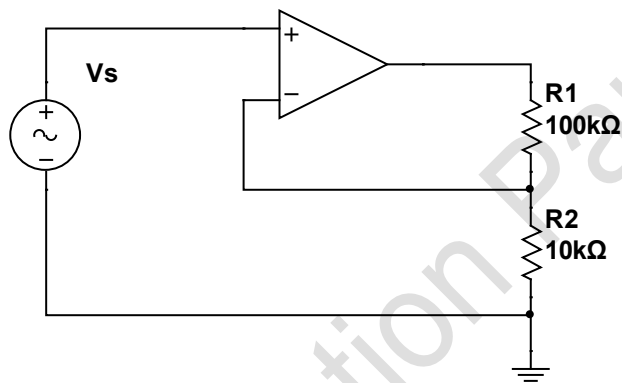


Figure Q4(b)

- (c) A high pass first order filter is connected to the input of the amplifier (in part b), sketch the new circuit and calculate the corner frequency if the high pass filter uses a resistor of  $100\text{k}\Omega$  and a capacitor of  $100\text{nF}$ . **[10 marks]**

**Total 25 marks****PLEASE TURN THE PAGE....**

**Q5**

(a) When describing an OP-Amp, what is meant by the terms; transition frequency bias currents and slew rate. **[6 marks]**

(b) For the circuit shown in Figure Q5 (b) calculate the output voltage.

**[10 marks]**

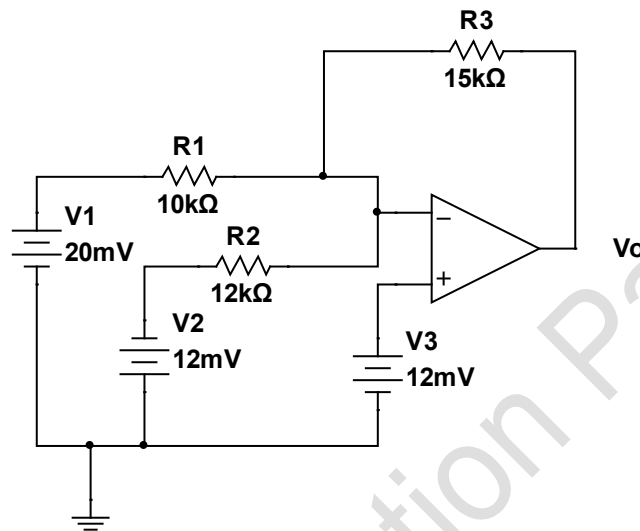


Figure Q5 (b)

(c) A biomedical signal processing circuit is shown in Figure Q5 (c), derive the equation for the output signal, if  $V_s = 10\text{mV} \sin(1000t)$  using

$$y = \sin ax \text{ then } \frac{dy}{dx} = a \cos ax.$$

**[9 marks]**

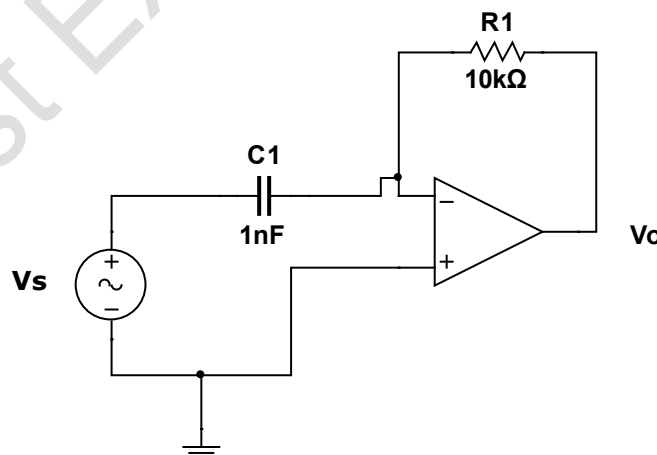


Figure Q5 (c)

**Total 25 marks**

**PLEASE TURN THE PAGE....**

School of Engineering  
 BEng (Hons) Advanced Biomedical Engineering  
 Semester One Examination 2019/2020  
 Advanced Biomechatronic Systems  
 Module No. BME6003

**Q6**

- (a) A Biomedical system has the following closed loop transfer function; sketch the magnitude and phase plots and the final plot.

**[10 marks]**

$$T(s) = \frac{10(s + 10)}{s + 5}$$

- (b) Describe what is meant by gain and phase margins.

**[2 marks]**

- (c) A low pass filter is shown in Figure Q6(d), derive the transfer function and sketch magnitude and phase.

**[6 marks]**

- (d) By deriving the transfer function for the circuit shown in Figure Q6 (d) and converting to the s-domain, sketch the pole zero diagram and indicate on the diagram all relevant points.

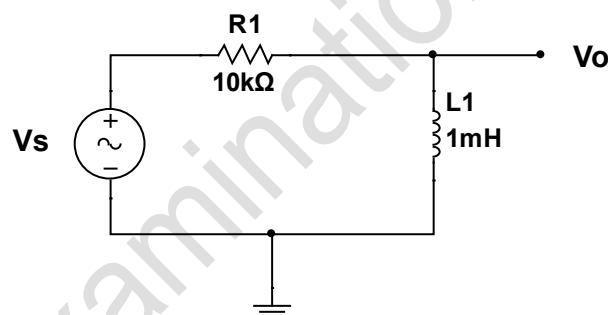
**[7 marks]**

Figure Q6 (d)

**Total 25 marks****END OF QUESTIONS****PLEASE TURN THE PAGE FOR FORMULAE SHEETS....**



### Formula Sheets

#### Blocks with feedback loop

$$G(s) = \frac{G_o(s)}{1 + G_o(s)H(s)} \quad (\text{for a negative feedback})$$

$$G(s) = \frac{G_o(s)}{1 - G_o(s)H(s)} \quad (\text{for a positive feedback})$$

#### Steady-State Errors

$$e_{ss} = \lim_{s \rightarrow 0} [s(1 - G_o(s))\theta_i(s)] \quad (\text{for an open - loop system})$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[ s \frac{1}{1 + G_o(s)} \theta_i(s) \right] \quad (\text{for the closed - loop system with a unity feedback})$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[ s \frac{1}{1 + \frac{G_1(s)}{1 + G_1(s)[H(s) - 1]}} \theta_i(s) \right] \quad (\text{if the feedback } H(s) \neq 1)$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[ -s \frac{G_2(s)}{1 + G_2(G_1(s) + 1)} \theta_d(s) \right] \quad (\text{if the system subjects to a disturbance input})$$

#### First order Systems

$$G(s) = \frac{\theta_o}{\theta_i} = \frac{G_{ss}(s)}{\tau s + 1}$$

$$\tau \left( \frac{d\theta_o}{dt} \right) + \theta_o = G_{ss} \theta_i$$

$$\theta_o = G_{ss} (1 - e^{-t/\tau}) \quad (\text{for a unit step input})$$

$$\theta_o = A G_{ss} (1 - e^{-t/\tau}) \quad (\text{for a step input with size } A)$$

$$\theta_o = G_{ss} \left( \frac{1}{\tau} \right) e^{-(t/\tau)} \quad (\text{for an impulse input})$$

School of Engineering  
 BEng (Hons) Advanced Biomedical Engineering  
 Semester One Examination 2019/2020  
 Advanced Biomechatronic Systems  
 Module No. BME6003

### Second- order Systems

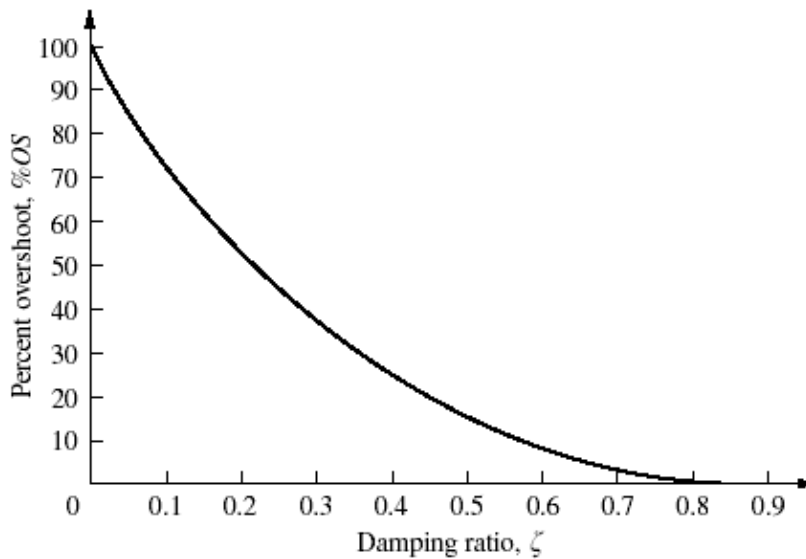
$$\frac{d^2\theta_o}{dt^2} + 2\zeta\omega_n \frac{d\theta_o}{dt} + \omega_n^2\theta_o = b_o\omega_n^2\theta_i$$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{b_o\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_d t_r = 1/2\pi \quad \omega_d t_p = \pi$$

$$\text{p.o.} = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100\%$$

$$t_s = \frac{4}{\zeta\omega_n} \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$



PLEASE TURN THE PAGE FOR MORE FORMULAE SHEETS....

School of Engineering  
 BEng (Hons) Advanced Biomedical Engineering  
 Semester One Examination 2019/2020  
 Advanced Biomechatronic Systems  
 Module No. BME6003

Table 4.1 Laplace transforms

Laplace transform	Time function	Description of time function
1		A unit impulse
$\frac{1}{s}$		A unit step function
$\frac{e^{-st}}{s}$		A delayed unit step function
$\frac{1 - e^{-st}}{s}$		A rectangular pulse of duration $T$
$\frac{1}{s^2}$	$t$	A unit slope ramp function
$\frac{1}{s^3}$	$\frac{t^2}{2}$	
$\frac{1}{s+a}$	$e^{-at}$	Exponential decay
$\frac{1}{(s+a)^2}$	$te^{-at}$	
$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	
$\frac{a}{s(s+a)}$	$1 - e^{-at}$	Exponential growth
$\frac{a}{s^2(s+a)}$	$t - \frac{(1 - e^{-at})}{a}$	
$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at} - ate^{-at}$	
$\frac{s}{(s+a)^2}$	$(1 - at)e^{-at}$	
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at} - e^{-bt}}{b-a}$	
$\frac{ab}{s(s+a)(s+b)}$	$1 - \frac{b}{b-a}e^{-at} + \frac{a}{b-a}e^{-bt}$	
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-a)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$	
$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	Sine wave
$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	Cosine wave
$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	Damped sine wave
$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	Damped cosine wave
$\frac{\omega^2}{s(s^2 + \omega^2)}$	$1 - \cos \omega t$	
$\frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$	$\frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin[\omega\sqrt{1-\zeta^2}t]$	
$\frac{\omega^2}{s(s^2 + 2\zeta\omega s + \omega^2)}$ with $\zeta < 1$	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin[\omega\sqrt{1-\zeta^2}t + \phi]$ with $\zeta = \cos \phi$	

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School of Engineering  
 BEng (Hons) Advanced Biomedical Engineering  
 Semester One Examination 2019/2020  
 Advanced Biomechatronic Systems  
 Module No. BME6003

**Table 15.1** z-transforms

Sampled $f(t)$ , sampling period $T$	$F(z)$
Unit impulse, $\delta(t)$	1
Unit impulse delayed by $kT$	$z^{-k}$
Unit step, $u(t)$	$\frac{z}{z-1}$
Unit step delayed by $kT$	$\frac{z}{z^k(z-1)}$
Unit ramp, $t$	$\frac{Tz}{(z-1)^2}$
$t^2$	$\frac{T^2z(z+1)}{(z-1)^3}$
$e^{-at}$	$\frac{z}{z - e^{-aT}}$
$1 - e^{-at}$	$\frac{z(1 - e^{-aT})}{(z-1)(z - e^{-aT})}$
$t e^{-at}$	$\frac{Tz e^{-aT}}{(z - e^{-aT})^2}$
$e^{-at} - e^{-bt}$	$\frac{(e^{-aT} - e^{-bT})z}{(z - e^{-aT})(z - e^{-bT})}$
$\sin \omega t$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$
$\cos \omega t$	$\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$
$e^{-at} \sin \omega t$	$\frac{z e^{-aT} \sin \omega T}{z^2 - 2z e^{-aT} \cos \omega T + e^{-2aT}}$
$e^{-at} \cos \omega t$	$\frac{z(z - e^{-aT} \cos \omega T)}{z^2 - 2z e^{-aT} \cos \omega T + e^{-2aT}}$

**Table 15.2** z-transforms

$f[k]$	$f[0], f[1], f[2], f[3], \dots$	$F(z)$
$1u[k]$	1, 1, 1, 1, ...	$\frac{z}{z-1}$
$a^k$	$a^0, a^1, a^2, a^3, \dots$	$\frac{z}{z-a}$
$k$	0, 1, 2, 3, ...	$\frac{z}{(z-1)^2}$
$ka^k$	0, $a^1, 2a^2, 3a^3, \dots$	$\frac{az}{(z-a)^2}$
$ka^{k-1}$	0, $a^0, 2a^1, 3a^2, \dots$	$\frac{z^2}{(z-a)^2}$
$e^{-ak}$	$e^0, e^{-a}, e^{-2a}, e^{-3a}, \dots$	$\frac{z}{z - e^{-a}}$

END OF FORMULAE SHEETS