[ESS09]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

B.ENG (HONS) MECHANICAL ENGINEERING

EXAMINATION SEMESTER 1 - 2019/2020

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME5012

Date: Monday 13th January 2020 **Time: 2:00 - 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are SEVEN questions.

Answer ANY FOUR questions only.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

Q1: A ductile material is used to fabricate a new submarine. The direct stresses in x and y direction are respectively 60 MPa in compression and 50 MPa in tension. There are also shear stresses present related to xy with a value of 30 MPa.

a) Sketch the elementary square describing the situation. **(2 marks)**

b) Determine via calculation:

(i) The magnitude of the principal stresses. **(4 marks)**

(ii) The angular position of the principal planes in relation to the X-axis **(3 marks)**

(iii) The magnitude of the maximum shear stress. **(3 marks)**

c) Sketch a Mohr's Stress Circle from the information provided in figure Q1, labelling σ_1, σ_2 the principal stresses and the maximum shear stress τ_{max}. Verify the results found in part a).

(8 marks)

d) Illustrate on a sketch of the element:

(i) The orientation of the principal planes. **(2 marks)**

(ii) The orientation of the plane where the shear stress is maximum. **(3 marks)**

Total 25 Marks

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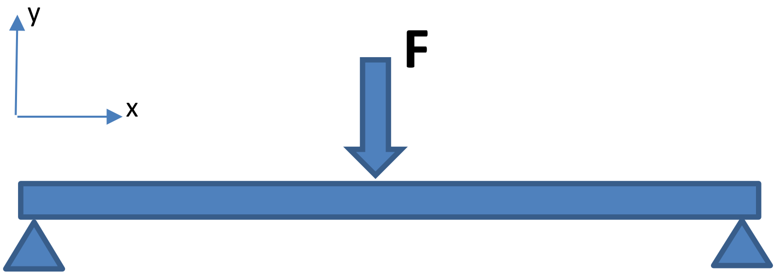
Q2. The purpose is to design a simply supported beam carrying a point load F=60kN as shown in figure below. For that purpose, some information are needed by answering the following questions knowing that a factor of safety of 3 is applied.

Given: E=250GPa, L=2m.



a) Give the expression of the bending moment at any position along the beam in function of x.

(2 marks)

b) Derive the formula of the maximum deflection y_{max} at the end A of the beam.

(6 marks)

c) Calculate the flexural rigidity (EI) of the beam if the maximum allowable deflection is not to exceed 5mm.

(4 marks)

d) Determine the dimension of the cross-section beam if it has a rectangular cross section so that the height is twice the width**.**

(5 marks)

e) Calculate the bending stress.

(4 marks)

f) Is the beam safe if its yield stress σ_{yield} is 780 MPa?

(4 marks)

Total 25 Marks

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Q3. A vertical post *AB* is embedded in a concrete foundation and held at the top by two cables with a solid diameter of 24mm. The post is a hollow aluminium tube having a length L = 2.1 m and an outer diameter d = 40 mm (see figure below). The cables are tightened equally by turnbuckles. The columns are being designed to support compressive loads P = 100 kN.

Given: E=72GPa for the modulus of elasticity and σ_c=480MPa for the proportional limit.

a) Determine the minimum required thickness t of the columns if a factor of safety of 3 is required with respect to Euler buckling theory. **(7marks)**

b) Find the maximum allowable tensile force, T_{allow}, and the maximum allowable stress, $\sigma_{allowable}$, in the cables? **(6 marks)**

c) Find the maximum allowable central deflection **(5 marks)**

d) Calculate the Rankine crushing load. **(3 marks)**

e) Comment on the validity of your results by comparing Euler and Rankine buckling theory. **(4 marks)**

Total 25 Marks

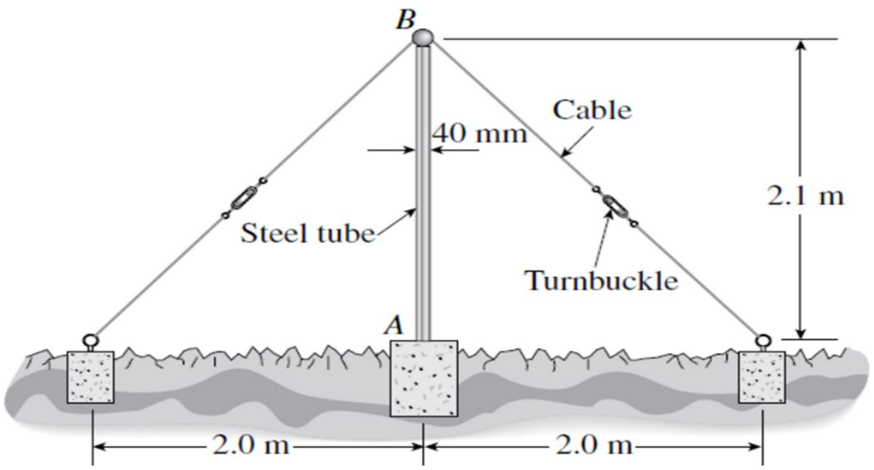

Figure Q3

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Q4. A long, closed ended cylindrical gas pressure vessel has an outer diameter of 700mm and an inner diameter of 400mm as shown in figure Q4. If the vessel is subjected to an external pressure of 10MPa and an internal pressure until the outer hoop layers reach 160MPa. Calculate:

a) The radial stress (σ_R) at the inner and outer surfaces. Give the internal pressure. **(7 marks)**

b) The circumferential stress (σ_C) at the inner surfaces. **(2 marks)**

c) The longitudinal stress (σ_L) and the maximum shear stress **(4 marks)**

d) The circumferential strain(ε_C), the radial strain (ε_R) and the longitudinal strain (ε_L) at the inner and outer surface. **(6marks)**

e) The final thickness, the final diameter and the final nominal volume of the cylinder**.** **(6 marks)**

Take E=250GPa, ν=0.3 and L=3m.

Total 25 Marks

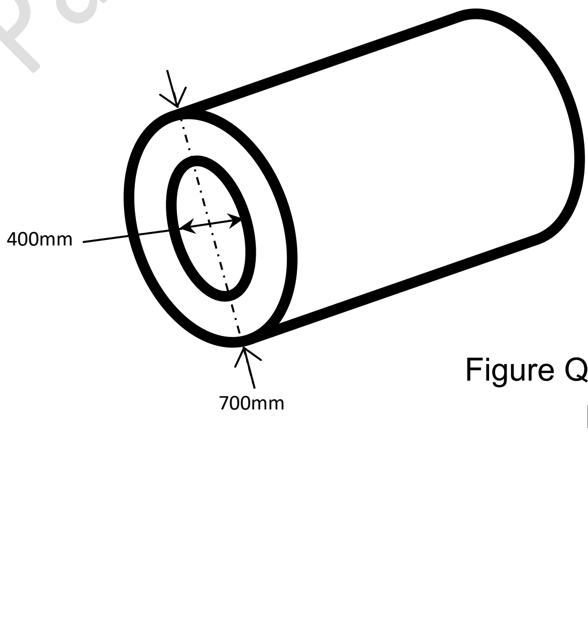

Figure Q4

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Q5. A machine of mass 1600kg is supported by four identical elastic springs coupled with a dashpot and set oscillating. It is observed that the amplitude reduces to 15% of its initial value after 3 oscillations over 7 seconds.

Calculate the following:

a) The natural frequency of undamped vibrations (in Hertz). **(2 marks)**

b) The effective stiffness of all four springs together. **(4 marks)**

c) The critical damping coefficient. **(2 marks)**

d) The damping ratio. **(5 marks)**

e) The damping coefficient. **(2 marks)**

f) The frequency of damped vibrations. **(3 marks)**

g) Explain as much as you can an underdamping, a critical and an overdamped system. **(7 marks)**

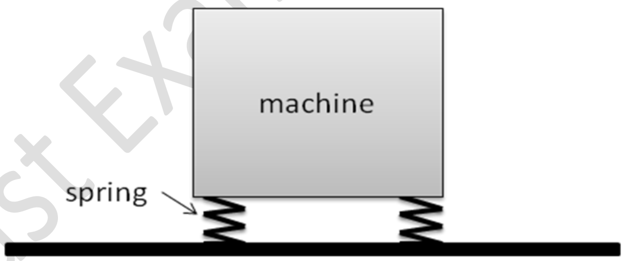

Figure Q5

Total 25 Marks

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Q6. A carbon steel plate is used to fabricate a fuselage of a plane with a diameter *D* of 4m and a wall thickness *t* of 20 mm. Due to the connection of a flange at the position of interest there are also shear stresses present related to xy with a value of 60 MPa.

a) What is the maximum internal pressure (P) allowable if the yield stress, σ_{yield}, is equal to 1200 MPa and assuming a safety factor of 3?
Given the Hoop stress is Pr/t and longitudinal stress is Pr/2t.

(5 marks)

b) Draw the elemental square showing the stresses acting

(3 marks)

c) Using this information given above calculate the principal stresses using the eigenvalues method.

(7 marks)

d) What is the factor of safety using the Von Mises criteria?

(4 Marks)

e) Determine the angles relative to xy co-ordinates of the largest principal stress acting and make a sketch showing the direction of the two principal stresses.

(6 Marks)

Total 25 Marks

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Q7. An 80 mm width pultruded beam section for a gas rig is fabricated from glass reinforced polyester resin as shown in figure Q7. The beam has a length of 3m and assumed to be simply supported at each end. The beam needs to support a point load of 20 KN. If the beam is designed to not to exceed the maximum design strain in each material.

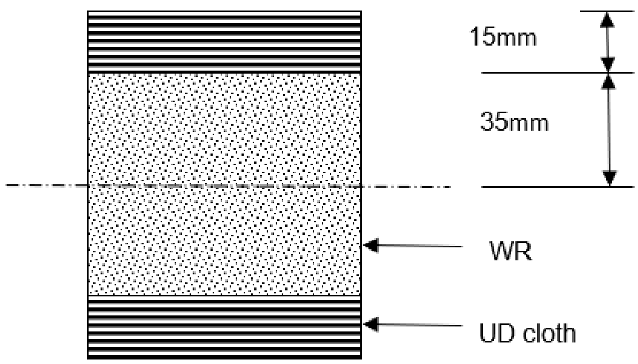
a) Calculate the maximum bending moment **(4 marks)**
b) Calculate the elasticity modulus and the design stress of each material of the beam. **(10 marks)**
c) Calculate the actual stress of each material of the beam .**(6 marks)**
d) Determine the factor of safety for the beam. **(5 marks)**

Total 25 Marks

Assume for the materials used the following values;

Material	Efficiency factor (%)	Design Strain (%)	Volume fraction (%)	Reinforcement modulus (GPa)	Matrix modulus (GPa)
UD	100	0.3	70	80	4
WR	50	0.3	30	80	4

Table Q7 Material Properties



END OF QUESTIONS

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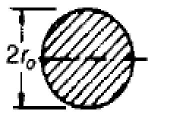
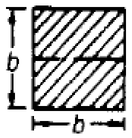
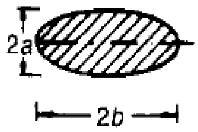
FORMULA SHEET

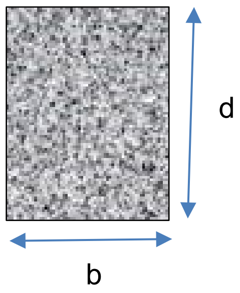
Deflection:

$$M_{xx} = EI\,\frac{d^2y}{dx^2}$$

Section Shape	$A(m^2)$	$I_{xx}(m^4)$
	πr^2	$\frac{\pi}{4} r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4} a^3 b$

For solid rectangular Cross-section

$$I_{xx} = \frac{bd^3}{12}$$



Plane Stress:

a) Stresses in function of the angle Θ:

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2} sin(2\theta) + \tau_{xy}\, cos(2\theta)$$

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b) Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Lame's equation

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall. Note: R_1 = inner cylinder radius, R_2 = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - \nu(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - \nu(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - \nu(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

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Vibrations:

Free Vibrations:

$$f = \frac{1}{T} \qquad \omega_n = 2\pi f = \sqrt{\frac{k}{\mathrm{M}}}$$

Damped Vibrations:

$$f_d = \frac{\omega_d}{2\pi} \qquad c_c = \sqrt{4\mathrm{M}k} \qquad \zeta = \frac{c}{c_c} = \frac{c}{2k}\,\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi a\zeta}{\sqrt{1-\zeta^2}}$$, a is the number of oscillations

Stress

σ = Force/Area = F/A

Hook's law

σ = E·ε

ε = ΔL/L

Composite Materials

Rule of mixture: $E_c = \eta V_F E_F + V_m E_m$

$\eta = efficiency\ factor$

$V_F = volume\ fraction\ of\ fibre$

$E_F = elasticity\ modulus\ of\ fibre$

$V_m = volume\ fraction\ of\ resin\ matrix$

$E_m = elasticity\ modulus\ of\ resin\ matrix$

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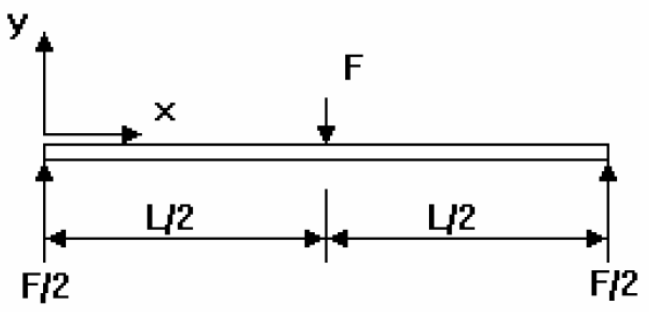
Simply supported beam:



M: maximum bending moment (M_{max}=FL/4)

Maximum bending stress:

$$\sigma_{bending} = \frac{My}{I}$$

M: maximum bending moment
Y: distance from neutral axis
I: second moment of area

Slope at the ends:

$$\frac{dy}{dx} = \frac{FL^2}{16EI}$$

Maximum deflection at the middle:

$$y = \frac{FL^3}{48EI}$$

Yield Criterion

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

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Quadratic equation: $ax^2+bx+c=0$

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Allowable stress: $\boldsymbol{\sigma_{allowable}}$

$$\boldsymbol{\sigma_{allowable}} = \frac{\boldsymbol{\sigma_{yield}}}{\boldsymbol{Factor\ Of\ Safety}}$$

Struts:

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

Euler validity

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi \sqrt{\frac{E}{\sigma_{yield}}}$$

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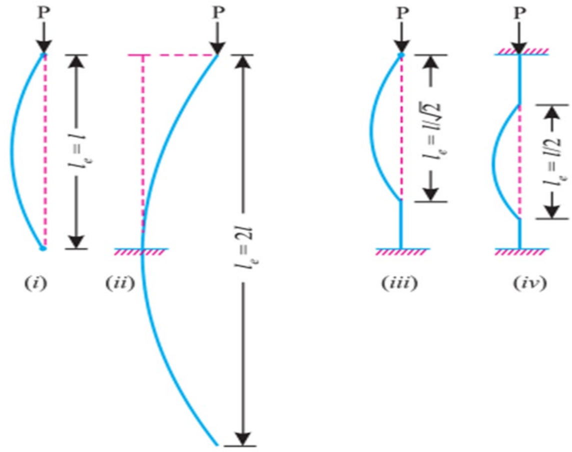

(i) *Both ends pin jointed or hinged or rounded or free.*
(ii) *One end fixed and other end free.*
(iii) *One end fixed and the other pin jointed.*
(iv) *Both ends fixed.*

Case	*End conditions*	*Equivalent length, l_e*	*Buckling load, Euler*
1	*Both ends hinged or pin jointed* or *rounded or free*	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	*One end fixed, other end free*	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	*One end fixed, other end pin jointed*	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	*Both ends fixed or encastered*	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect.*

$$\boldsymbol{a = \frac{\sigma_c}{\pi^2 \cdot E}}$$

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