[ESS08]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATIONS 2019/20

ENGINEERING PRINCIPLES 1

MODULE NO: AME4062

Date: Tuesday 14th January 2020 Time: 10:00 – 12:00

INSTRUCTIONS TO CANDIDATES:

1. There are SIX questions.
2. Answer TWO questions from each of the sections A and B.
3. Use a separate answer book for sections A and B.
3. Maximum marks for each part/question are shown in brackets. Each question is worth 25 marks.
4. Formula sheets (attached)

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Section A - Answer any TWO questions from this section.

1. (a) Find a vector joining the points (1,3,2) and (2,5,6).

(2 marks)

(b) The vectors $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$. Find the magnitudes of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(4 marks)

(c) Evaluate $\boldsymbol{a} \cdot \boldsymbol{b}$.

(2 marks)

(d) Find the angle between the vectors ***a*** and ***b***.

(4 marks)

(e) Find a vector which is perpendicular to both vectors ***a*** and ***b***.

(4 marks)

(f) Calculate the work done by a constant force ***F*** of 12*N* whose line of action is parallel to $2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ when it moves a bead a distance of 4*m* along a straight wire which is in the direction of the vector $-2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$.

(9 marks)

Total 25 marks

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2. (a) Simplify the fraction

$$\frac{\sin^3\theta + sin\theta\cos^2\theta}{cos\theta}$$

(4 marks)

(b) Solve the equation

$$1 + sin\theta\cos^2\theta = sin\theta$$

for $0° \leq \theta \leq 180°$.

(6 marks)

(c) Find all solutions of the equation

$$sin2\theta = sin\theta$$

for $0^{\circ} \leq \theta \leq 360^{\circ}$.

(7 marks)

(d) If $3sin\theta + 4cos\theta$ is written in the form $rsin(\theta + \alpha)$, show how to find the value of *r* and α.

(5 marks)

Find the greatest and least value of

$$12sin\theta + 16cos\theta.$$

(3 marks)

Total 25 marks

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3. (a) Simplify

$$\frac{ln\sqrt{3}}{ln27}\,.$$

(3 marks)

(b) Solve the equation

$$4^{x+2} = 3^{x-1}$$

leaving your answer in terms of natural logarithms and in its simplest form.

(5 marks)

(c) Solve the equation

$$4^{6x} - 4^{3x+1} - 5 = 0$$

leaving your answer in terms of natural logarithms and in its simplest form.

(5 marks)

(d) Use the inverse matrix method to solve the two-by-two system of linear equations

$$3x + 2y = -2$$
$$x + 4y = 6\,.$$

(5 marks)

(e) If $i^2 = -1$, show that

$$\left(\frac{1+i}{1-i}\right)^2$$

takes the same value.

(2 marks)

Show also that i^i is real.

(5 marks)

Total 25 marks

END OF SECTION A

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Section B - Answer ANY TWO questions from this section.

4. Two forces are applied to the bracket BCD as shown in **Fig. Q4**.

(a) Knowing that the control rod AB is to be made of a steel having an ultimate normal stress of 500 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.

(10 marks)

(b) The pin at C is to be made of a steel having an ultimate shearing stress of 350 MPa, Determine the diameter of the pin C for which the factor of safety with respect to shear will be 3.5

(8 marks)

(c) Determine the required thickness of the bracket supports at C knowing that the allowable bearing stress of the steel used is 300 MPa.

(7 marks)

Total 25 marks

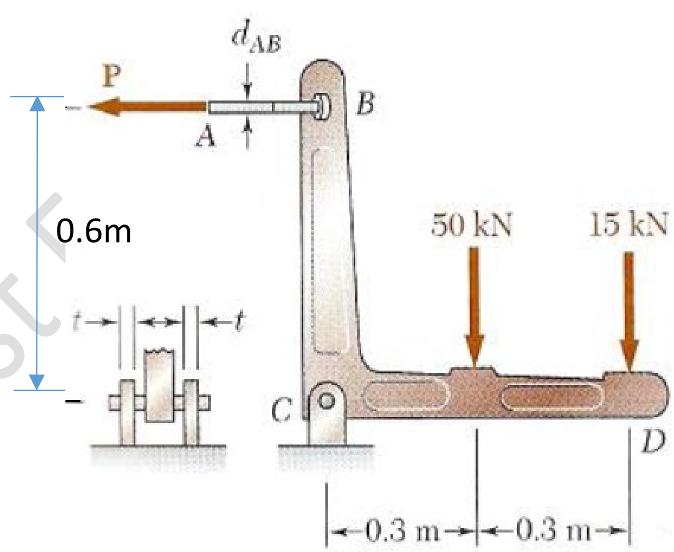

Fig. Q4

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5. A simply supported beam carries two concentrated lateral loads and a uniformly distributed load over its entire length as shown in **Fig.Q5**. Determine:

(a) Reaction forces, R1 & R2, at the supports

(5 marks)

(b) Construct the shear force diagram for the beam

(8 marks)

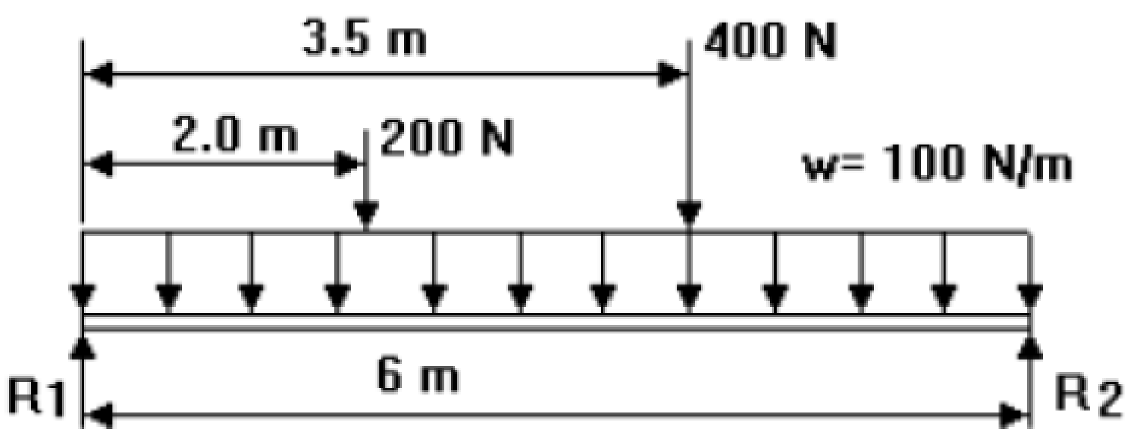
(c) Construct the bending moment diagram for the beam

(8 marks)

(d) Find the position of maximum bending moment

(4 marks)

Total 25 marks



<u>**Fig.Q5**</u>

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6. (a) Define the term torque load & its impact on bodies and mention three examples of its applications in engineering.

(5 marks)

(b) A stepped shaft has the appearance shown in **Fig.Q6**. The region *AB* is aluminium, having *G* = 28 GPa, and the region *BC* is steel, having *G* = 84 GPa. The aluminium portion is of solid circular cross section 45 mm in diameter, and the steel region is circular with 60-mm outside diameter and 30-mm inside diameter. A torsional load of 4000 N · m is applied at point B, whereas ends *A* and *C* are rigidly clamped. Determine the followings:

(i) The maximum shearing stress in each material

(15 marks)

(ii) The angle of twist at *B.*

(5 marks)

Total 25 marks

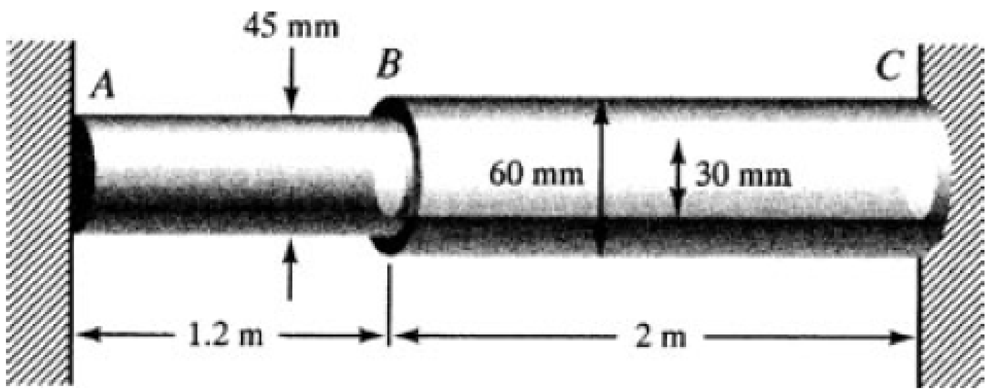

Fig.Q6

END OF QUESTIONS

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Laws of Indices

$a^m \cdot a^n$	a^{m+n}
$\frac{a^m}{a^n}$	a^{m-n}
$(a^m)^n$	a^{mn}
$\frac{1}{a^m}$	a^{-m}
$a^{\frac{1}{m}}$	$\sqrt[m]{a}$
a^0	1

Laws of Logarithms

$\log_a(b) + \log_a(c)$	$\log_a(bc)$
$\log_a(b) - \log_a(c)$	$\log_a\left(\frac{b}{c}\right)$
$\log_a(b^c)$	$c\log_a(b)$
$\log_a(1)$	0
$\log_a(a)$	1

Quadratic equations

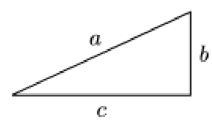
The equation:

$$ax^2 + bx + c = 0$$

has solutions:

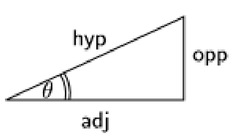
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' theorem



$$a^2 = b^2 + c^2$$

Trigonometry



$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$
$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$
$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$

Trigonometric Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(2A) = 2\cos(A)\sin(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

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Vectors

Dot or scalar product

If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\mathbf{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\mathbf{j} + b_3\boldsymbol{k}$**,** then $\boldsymbol{a} \cdot \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$

If θ is the angle between the vectors ***a*** and **b**, and $a = (a_1^2 + a_2^2 + a_3^2)^{1/2}$, the magnitude of ***a***, and similarly for the vector ***b***, then $\boldsymbol{a} \cdot \boldsymbol{b} = abcos\theta$.

Cross or vector product

If $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\mathbf{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\mathbf{j} + b_3\boldsymbol{k}$**,** then $\boldsymbol{a} \wedge \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Also, $\boldsymbol{a}\wedge\boldsymbol{b} = absin\theta\boldsymbol{n}$**,** $\boldsymbol{n}$ being a unit vector perpendicular to ***a*** and ***b*** such that (***a*****,** ***b*****,** $\boldsymbol{a}\wedge\boldsymbol{b}$) form a right-handed set.

Matrices

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^{-1} = \frac{1}{detA}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where $detA = ad - bc$.

Complex Numbers

De Moivre's Theorem

$$(cos\theta + isin\theta)^n = \cos(n\theta) + isin(n\theta)$$

Exponential form

$$e^{\pm i\theta} = cos\theta \pm isin\theta$$

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Mechanics

Notation:

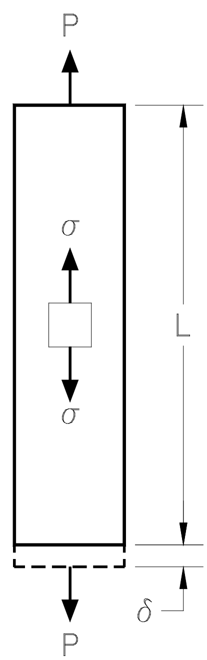
σ - normal stress (Pa) τ- shear stress (Pa) ε - normal strain (m/m) γ - shearing strain (m/m) I - area moment of inertia (m^4) J - polar area moment of inertia (m^4) E - modulus of elasticity (Pa) G - modulus of rigidity (Pa)	ν - Poisson's ratio α - coefficient of thermal expansion (/°C) M - bending moment in beams T - torque in shafts ΔT - temperature change (EC) F.S. - factor of safety = $\frac{\text{Ultimate stress}}{\text{Allowable stress}}$ $\varepsilon_t = \alpha\Delta T$ - thermal strain

Normal Stress and Strain:

If the line of action of the load, P, passes through the centroid of the resisting cross-section:

$$axial\ stress = \sigma = \frac{P}{A}$$

$$axial\ strain = \varepsilon = \frac{\delta}{L}$$



If the material is also linear, then:

$$Uniaxial\ Hooke's\ Law: \quad \varepsilon = \frac{\sigma}{E}$$

where, E is the modulus of elasticity for the material

The relationship between axial loading and deformation becomes

$$\boxed{axial\ deformation: \quad \delta = \frac{PL}{AE}}$$

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Shear Stress in Fasteners & Brackets:

a. Average Shear Stress in Fasteners

$$\tau = \frac{V}{A}$$

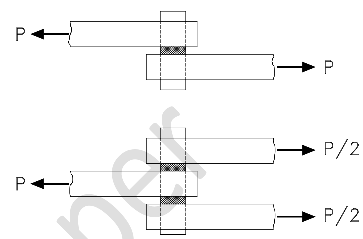
V = Shear force on pin
A = Cross sectional area of pin

Single Shear

$V = P$

Double Shear

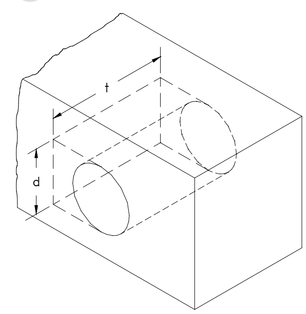
$$V = \frac{P}{2}$$



b. Bearing Stress in Fasteners

$$\sigma = \frac{P}{dt}$$

P = force
d = diameter of fastener
t = thickness of part



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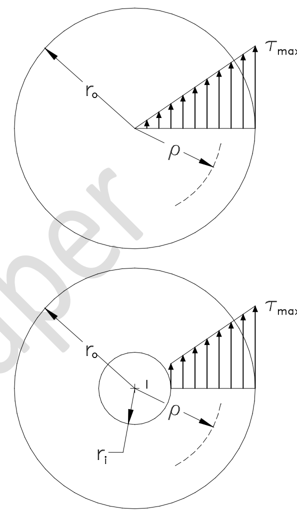
Torsion of Circular Sections:

a. Shear stress

If the shaft has a circular cross section and the material remains in the linear-elastic region, the shear stress in the shaft varies as a linear function of the distance (ρ) from the center of the shaft and is given by:

$$\textit{shear stress}: \quad \tau = \frac{T\rho}{J}$$



The maximum shear stress in the shaft is on the outer surface independent of whether the shaft is solid or hollow and is given by:

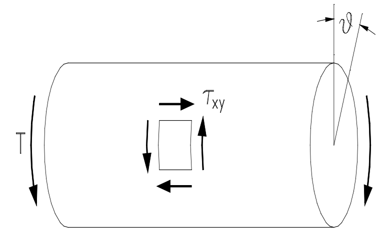
$$\max \textit{ shear stress}: \quad \tau_{\max} = \frac{T r_o}{J}$$

b. The polar area moment of inertia is:

$$\textit{solid section}: \quad J = \frac{\pi r_o^4}{2}$$

$$\textit{hollow section}: \quad J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right)$$

c. The calculated stresses act on the element as shown.



The deformation is measured by the angle of twist (θ) of one end relative to the other and is giver by:

$$\textit{angle of twist}: \quad \theta = \frac{TL}{JG}$$

where, G is the modulus of rigidity for the material and L is the length of shaft

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