[ESS11]

# UNIVERSITY OF BOLTON

# SCHOOL OF ENGINEERING

# B.ENG (HONS) MOTORSPORT ENGINEERING

# SEMESTER TWO EXAMINATION 2018/2019

# ENGINEERING SCIENCE II

# MODULE NO: MSP5016



**Date: Friday 24th May 2019** **Time: 14:00 – 16:00**

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**INSTRUCTIONS TO CANDIDATES:**

**There are SIX questions.**

**Answer ANY FOUR questions ONLY**

**All questions carry equal marks.**

**Marks for parts of questions are shown in brackets.**

**Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.**

**CANDIDATES REQUIRE:**

**Formula Sheet (attached).**

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Q1: A two dimensional state of stress occurs on the surface of a new shaft component that is being designed with a composite skin made from multiple layers of graphite reinforced epoxy. The stress components are $\boldsymbol{\sigma_y}$ = 50 MPa (tensile), $\boldsymbol{\sigma_x}$ = 60 MPa (compressive) and $\boldsymbol{\tau_{xy}} = \boldsymbol{\tau_{yx}}$ = 30 MPa.

a) Determine via calculation:

(i) The magnitude of the principal stresses. **(4 marks)**

(ii) The angular position of the principal planes in relation to the X-axis **(3 marks)**

(iii) The magnitude of the maximum shear stress. **(3 marks)**

b) Illustrate on a sketch of the element:

(i) The initial orientation of the stresses **(3 marks)**

(ii) The orientation of the principal stresses in their planes. **(3 marks)**

(iii) The orientation of the plane where the shear stress is maximum. **(3 marks)**

c) Sketch a Mohr's Stress Circle from the information provided above, labelling $\sigma_1$, $\sigma_2$ the principal stresses and the maximum shear stress $\tau_{max}$. Verify the results found in part a).

**(6 marks)**

**Total 25 Marks**

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Q2. The purpose is to design a cantilever steel beam carrying an UDL ω=60kN/m as shown in figure Q2. For that purpose, answer the following questions by applying a factor of safety of 3.

Given: E=250GPa, L=2m.

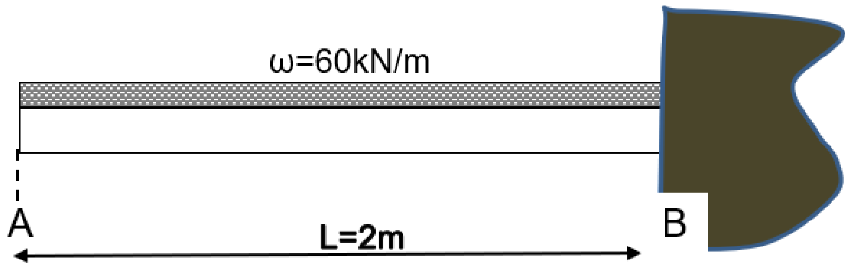


Figure Q2: Cantilever beam with UDL

a) Give the expression of the bending moment at any position along the beam in function of x. **(2 marks)**

b) Calculate the flexural rigidity (EI) of the beam if the maximum allowable deflection is not to exceed 3mm. **(4 marks)**

c) Determine the dimension of the cross-section beam if it has a hollow square cross section so that the height of the outer side is 4 times greater than the height of the inner side. **(6 marks)**

d) Calculate the allowable bending stress. **(5 marks)**

e) Calculate the allowable bending stress and deflection if the weight of the steel beam is not negligible anymore. Comment on your results. Given the density of the steel $\rho=7850 kg/m^3$.

**(8 marks)**

**Total 25 Marks**

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Q3. A straight pin ended bar transporting formula one cars has a length of 2.5m and a rectangular cross section of 240 x 240mm. The bar is made with aluminium and has an elasticity modulus of E=72GPa and a yield stress $\sigma_c$=560MPa.

a) Determine the slenderness ratio of the aluminium bar. **(4 marks)**

b) If it is subject to an axial load, what is the maximum critical load and central deflection that can occur before it reaches the yield point.

   i. Using Euler theory? **(8 marks)**

   ii. Using Ranking theory? **(8 marks)**

c) Determine the validity of Euler's formula and comment the results found in question b) above.

**(5 marks)**

**Total 25 Marks**

Q4. A closed ended cylindrical gas pressured tank of a racing car has an outer diameter of 50cm and an inner diameter of 20cm as shown in figure Q4. If the tank is subjected to an internal pressure until the outer layer reaches 220MPa, calculate:

a) The radial stress ($\sigma_R$) at the inner and outer surfaces and, the internal pressure. **(7 marks)**
b) The circumferential stress ($\sigma_C$) at the inner surfaces. **(2 marks)**
c) The longitudinal stress ($\sigma_L$) and the maximum shear stress. **(4 marks)**
d) The circumferential strain ($\varepsilon_C$), the radial strain ($\varepsilon_R$) and the longitudinal strain ($\varepsilon_L$) at the inner and outer surface. **(7marks)**
e) The final diameter and the final nominal volume of the cylinder. **(5 marks)**

Take E=230GPa, ν=0.3 and L=2m. **Total 25 Marks**

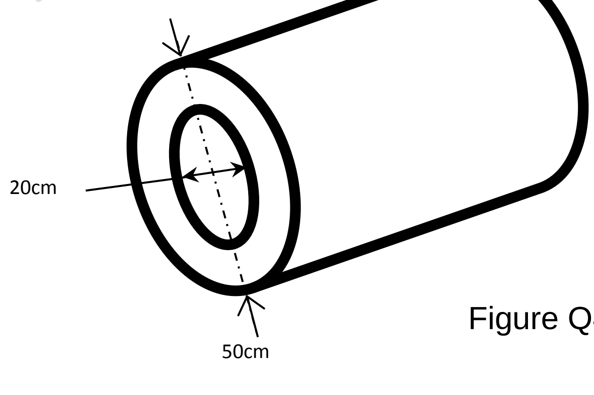


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Figure Q4

Q5. A technical racing car has a mass of 1500kg and is supported by four Identical suspension systems with elastic springs coupled with dashpot and set oscillating. It is observed that the amplitude of the oscillations reduces to 10% of its initial value after 4 oscillations over 4 seconds. Calculate the following:

a) The natural frequency of undamped vibrations (in Hertz). **(2 marks)**

b) The effective stiffness of all four springs together. **(4 marks)**

c) The critical damping coefficient. **(2 marks)**

d) The damping ratio. **(5marks)**

e) The damping coefficient. **(2marks)**

f) The frequency of damped vibrations. **(3marks)**

g) Explain as much as you can an underdamping, a critical and an overdamped system. **(7marks)**

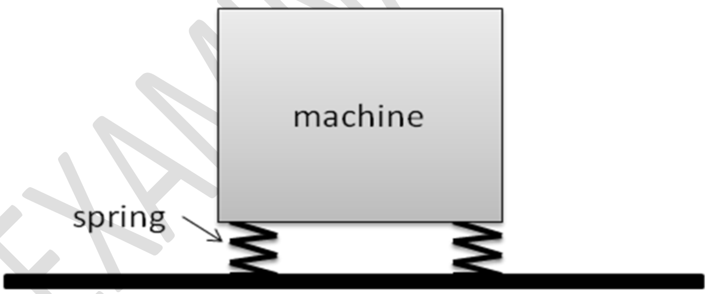


Figure Q5

**Total 25 Marks**

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Q6. A steel plate is to be used to fabricate a hydrogen cylindrical pressure tank for formula one cars. The diameter *D* of the tank is 0.6m and the wall thickness *t* is 20 mm. Due to the connection of a flange at the position of interest there are also shear stresses present related to xy with a value of 40 MPa.

a) What is the maximum internal pressure (P) allowable if the yield stress, $\sigma_{yield}$, is equal to 850 MPa and assuming a safety factor of 3? Given the Hoop stress is Pr/t and longitudinal stress is Pr/2t. **(6 marks)**

b) Draw the elemental square showing the stresses acting **(4 marks)**

c) Using this information given above calculate the principal stresses using the eigenvalues method. **(8 marks)**

d) Determine the angles relative to xy co-ordinates of the largest principal stress acting and make a sketch showing the direction of the two principal stresses. **(7 Marks)**

**Total 25 Marks**

**END OF QUESTIONS**

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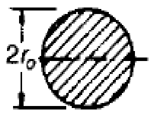
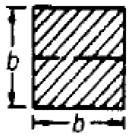
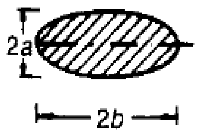
**FORMULA SHEETS**

Deflection:

$$M_{xx} = EI \frac{d^2y}{dx^2}$$

| Section Shape | $A(m^2)$ | $I_{xx}(m^4)$ |
|---|---|---|
|  | $\pi r^2$ | $\frac{\pi}{4} r^4$ |
|  | $b^2$ | $\frac{b^4}{12}$ |
|  | $\pi ab$ | $\frac{\pi}{4} a^3 b$ |

## Plane Stress:

**a) Stresses in function of the angle Θ:**

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2} sin(2\theta) + \tau_{xy}\, cos(2\theta)$$

**b) Principal stresses:**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

**Lame's equation**

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall. Note: $R_1$ = inner cylinder radius, $R_2$ = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - \nu(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - \nu(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - \nu(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

## Vibrations:

Free Vibrations:

$$f = \frac{1}{T} \qquad \omega_n = 2\pi f = \sqrt{\frac{k}{\mathrm{M}}}$$

Damped Vibrations:

$$f_d = \frac{\omega_d}{2\pi} \qquad c_c = \sqrt{4\mathrm{M}k} \qquad \zeta = \frac{c}{c_c} = \frac{c}{2k}\,\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi a\zeta}{\sqrt{1-\zeta^2}}$$

, *a* is the number of oscillations

## Stress

σ = Force/Area = F/A

## Hook’s law

σ = E·ε

ε = ∆L/L



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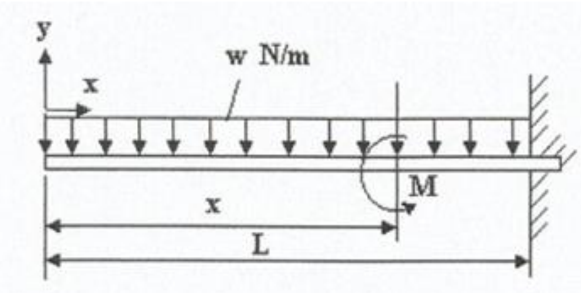
## Cantilever with uniformed distributed load (UDL)




M: maximum bending moment ($M_{max}=wL^2/2$)

Maximum bending stress:

$$\sigma_{bending} = \frac{My}{I}$$

*M: maximum bending moment*
*Y: distance from neutral axis*
*I: second moment of area*

Slope at the ends:

$$\frac{dy}{dx} = \frac{wL^3}{6EI}$$

Maximum deflection at the middle:

$$y = \frac{wL^4}{8EI}$$

## Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = (Stress\ Tensor)\begin{pmatrix} l \\ m \\ n \end{pmatrix}$$



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$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Where a, b and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \qquad l = \cos\alpha,$$
$$m = bk \qquad m = \cos\theta,$$
$$n = ck \qquad n = \cos\varphi.$$

Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$
$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$
$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

**<u>Yield Criterion</u>**

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Tresca

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

$$\sigma_{tresca} = 2 \cdot \tau_{\max}$$

$$\tau_{\max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}; \frac{|\sigma_1 - \sigma_3|}{2}; \frac{|\sigma_3 - \sigma_2|}{2}\right)$$

$$\frac{\sigma_{von\ Mises}}{\sigma_{Tresca}} = \frac{\sqrt{3}}{2}$$

**<u>Quadratic equation: $ax^2+bx+c=0$</u>**

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**<u>Allowable stress:</u>** $\boldsymbol{\sigma_{allowable}}$

$$\boldsymbol{\sigma_{allowable} = \frac{\sigma_{yield}}{Factor\ Of\ Safety}}$$

**<u>Struts:</u>**

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

<u>Euler validity</u>

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi \sqrt{\frac{E}{\sigma_{yield}}}$$



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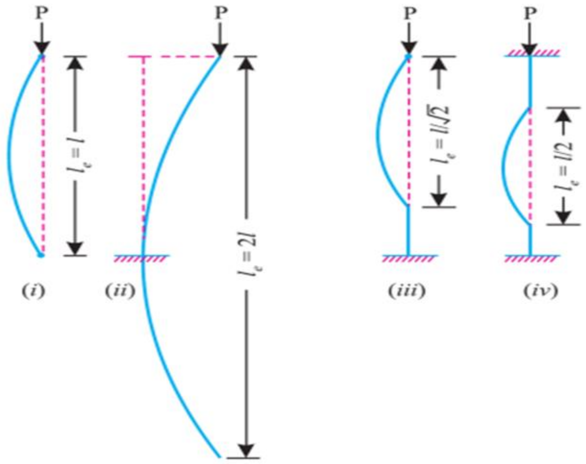


(i) *Both ends pin jointed or hinged or rounded or free.*
(ii) *One end fixed and other end free.*
(iii) *One end fixed and the other pin jointed.*
(iv) *Both ends fixed.*

| Case | End conditions | Equivalent length, $l_e$ | Buckling load, Euler |
|---|---|---|---|
| 1 | *Both ends hinged or pin jointed* or *rounded or free* | $l$ | $\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$ |
| 2. | *One end fixed, other end free* | $2l$ | $\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$ |
| 3. | *One end fixed, other end pin jointed* | $\frac{l}{\sqrt{2}}$ | $\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$ |
| 4. | *Both ends fixed or encastered* | $\frac{l}{2}$ | $\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$ |

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect.*

$$a = \frac{\sigma_c}{\pi^2 \cdot E}$$

**END OF FORMULA SHEET**

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