PAST EXAMINATION PAPER

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BSc(Hons) MATHEMATICS

SEMESTER 2 EXAMINATIONS 2018/19

NUMERICAL ANALYSIS

MODULE NO: MMA5004

Date: Monday 20 May 2019 **Time: 14.00 - 16.15**

INSTRUCTIONS TO CANDIDATES:

1. **There are FOUR questions.**
2. **Answer ALL questions.**
3. **Maximum marks for each part/question are shown in brackets.**

School of Engineering
BSc(Hons) Mathematics
Semester 2 Examinations 2018/19
Numerical Analysis
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1. (a) (i) Write down the second order Lagrange interpolating polynomial.

(2 marks)

(ii) The integral $I(X) = \int_0^X f(x)\,dx$ has the following values:

$$I(0.2) = 0.030$$
$$I(0.4) = 0.093$$
$$I(0.8) = 0.230$$

Use Lagrange interpolation with three points to estimate $I(0.6)$ quoting your estimate to two decimal places.

(8 marks)

(b) (i) An nth degree Chebyshev polynomial may be defined recursively as

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

Given that $T_0(x) = 1$, $T_1(x) = x$, obtain $T_2(x)$ and $T_3(x)$ and show that the roots of $T_3(x) = 0$ are

$$x_1 = \frac{\sqrt{3}}{2}, \; x_2 = 0, \; x_3 = \frac{-\sqrt{3}}{2}.$$

(5 marks)

(ii) Show that the substitution $z = \dfrac{2x}{b-a} - \dfrac{a+b}{b-a}$ transforms the limits $[a,b]$ of the integral

$$\int_a^b f(x)\,dx$$

to $[-1,1]$.

(2 marks)

(iii) Use $\displaystyle\int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}}\,dx \simeq \frac{\pi}{3}\sum_{i=1}^{3} f(x_i)$ where x_i are the roots of $T_3(x) = 0$, and the substitution from b (ii) to evaluate

$$\int_1^2 \frac{e^{2x-3}\,dx}{(3x-x^2-2)^{\frac{1}{2}}}.$$

(8 marks)

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2. (a) Use the method of undetermined coefficients to obtain the values of c_0, c_1 and c_2 in the quadrature formula

$$\int_{x_0}^{x_2} f(x)\,dx \simeq \sum_{i=0}^{2} c_i f(x_i)$$

and hence show that

$$\int_{x_0}^{x_2} f(x)\,dx \simeq \frac{h}{3}(f(x_0) + 4f(x_1) + f(x_2)).$$

where $x_1 - x_0 = x_2 - x_1 = h$

(12 marks)

(b) Obtain the composite form of the quadrature formula from 3(a) for *n* strips and use this to evaluate

$$I = \int_0^1 e^{-x^2}\,dx$$

correct to two decimal places.

(13 marks)

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3. (a) The modified Newton-Raphson formula for the solution of the equation $f(x) = 0$ may be written as

$$x_{i+1} = x_i - \frac{m\,f(x_i)}{f'(x_i)} \qquad \text{(Eq1)}$$

(i) By writting the right hand side of Eq1 as $g(x_i)$, show that, if $x_i = X + \varepsilon_i$, where X is the root of $f(x) = 0$ and ε_i is the error in the ith iterate generated by Eq1, then

$$\varepsilon_{i+1} = g'(X)\varepsilon_i + g''(X)\frac{\varepsilon_i^2}{2} + O(\varepsilon_i^3)$$

(5 marks)

(ii) Show that

$$g'(X) = 1 - m + \frac{mf(X)f''(X)}{[f'(X)]^2}$$

(3 marks)

(iii) For functions of the form $f(X) = (x - X)^m h(x)$, show that $g'(X) = 0$, and clearly state what this proves.

(10 marks)

(b) The function $f(x) = 49x^3 + 105x^2 - 117x + 27 = 0$ has a root of mutiplicity 2 close to $x = 1$. Use Eq1 to find this root.

(7 marks)

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4. Crout's method to solve the matrix equation $[A]\{X\} = \{b\}$, requires that the matrix of coefficients $[A]$ is expressed as the product of the lower triangular matrix $[L]$ and the upper triangular matrix $[U]$ given by

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(a) Use Crout's method to solve the following system of equations:

$$\begin{aligned} x_1 - 3x_2 + 3x_3 &= -2 \\ 2x_1 + 5x_2 - x_3 &= 30 \\ 4x_1 + 2x_2 + 3x_3 &= 35. \end{aligned}$$

(15 marks)

(b) The value of the definite integral

$$I = \int_{x_0}^{x_1} f(x)dx$$

may be approximated by using the Trapezium Rule given by

$$I \approx \frac{h}{2}(f(x_0) + f(x_1)),\ h = x_1 - x_0$$

Show that the error in a single application of the Trapezium rule is given by

$$-\frac{h^3}{12}f''(\xi), \quad x_0 \leqslant \xi \leqslant x_1$$

(10 marks)

END OF QUESTIONS