[ESS13]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) MECHANICAL ENGINEERING

SEMESTER 2 EXAMINATION 2019

FINITE ELEMENT & DIFFERENCE METHODS

MODULE NO: AME6016

Date: Tuesday 21 May 2019 **Time: 2:00pm to 4:00pm**

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

Formula Sheet is attached in the APPENDIX at the end of the paper

School of Engineering
BEng (Hons) Mechanical Engineering
Semester 2-Examination 2019
Finite Element & Difference Methods
Module No. AME6016

Q1

A 1 mm thick 2D constant strain element with a modulus of 200 GPa and a Poisson's ratio of 0.3 is shown in fig Q1 along with its associated geometry. For the loadings given determine the nodal displacements.

(10 marks)

Determine also the stresses (σ_{xx}, σ_{yy} , τ_{xy}) and strains (ε_{xx}, ε_{yy}, γ_{xy}) for this element under this condition. The formulation of the B and D matrices are given in the attached formula sheet.

(10 marks)

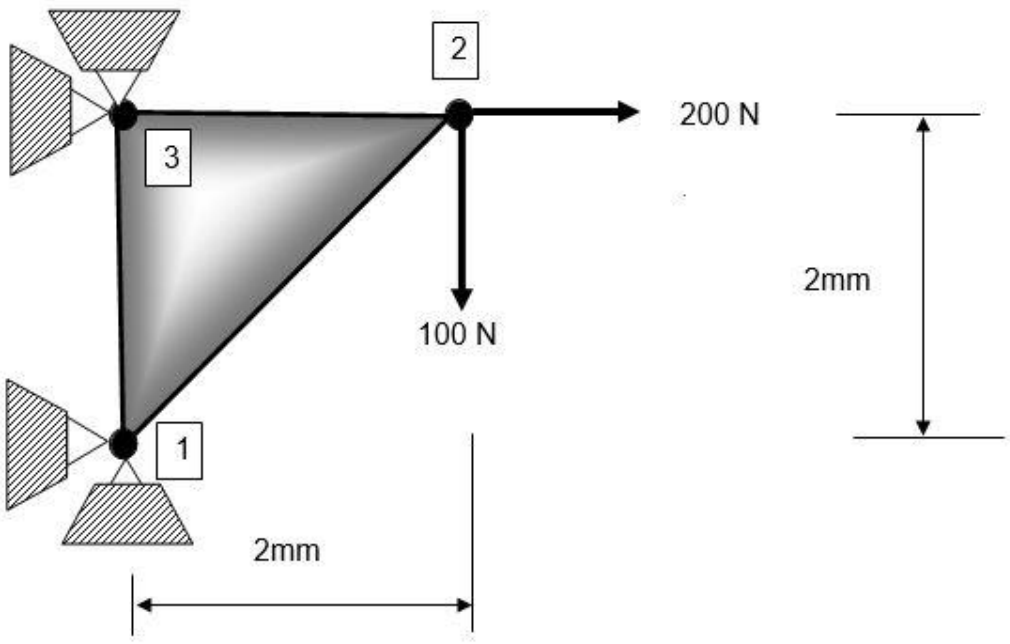


Fig Q1 Schematic set of element loading

Explain briefly, if a linear varying stress element was need why the number of nodes would need to increase to six.

(5 marks)

Total 25 Marks

PLEASE TURN THE PAGE…..

School of Engineering
BEng (Hons) Mechanical Engineering
Semester 2-Examination 2019
Finite Element & Difference Methods
Module No. AME6016

Q2

A lorry suspension can be modelled using a 2D beam element and a spring element as shown in Fig Q2. Using the beam and spring properties provided in table Q2 calculate the first two natural frequencies.

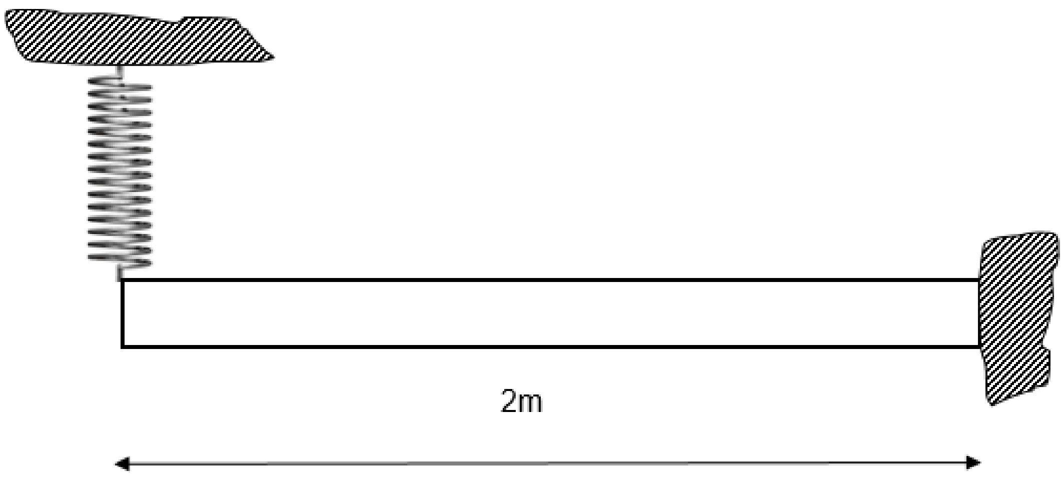
<u>Table Q2 Beam and spring properties</u>

Density (Tonnes/m^3)	7.85
Elastic Modulus (GPa)	210
Second moment of area (mm^4)	800 000
Cross sectional area (mm^2)	4000
Spring Stiffness (Nmm^{-1})	42

(15 Marks)

Determine also the model shape in terms of x (distance from the spring end) for the first mode. **(6 Marks)**

Explain briefly the difference between the consistent and lumped mass approach used to determine the dynamic behaviour of a structure. Which method would you use to model a rotating crankshaft? Explain your answer**.** **(4 Marks)**



<u>Fig Q2 Schematic of lorry suspension</u>

Total 25 Marks
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Q3

Describe the difference between the H and P methods used to analyse stresses in a finite element problem. **(5 Marks)**

Explain why for a beam under primarily a flexure load using a simple element to model the scenario would not be suitable. **(5 Marks)**

Describe the difference between nodal and elemental stresses and how this can be used to validate the stress values in an analysis. **(5 Marks)**

Explain briefly how the strain energy can used in convergence calculations and how adaptive meshing can be useful in reducing the errors in the stress values

(5 Marks)

Rayleigh damping is used to model a system under forced vibration. Describe briefly how the process works and how it relates to the equation of motion. Produce a simple sketch of how the damping varies with frequency. **(5 Marks)**

Total 25 Marks

PLEASE TURN THE PAGE…..

Q4

The square cross-section beam 250x250mm shown below in Figure Q4 is rigidly fixed at end A and simply supported at end B. Two point bending are applied along the length of the beam as shown in in figure Q4. The beam span AB is 4L in total.

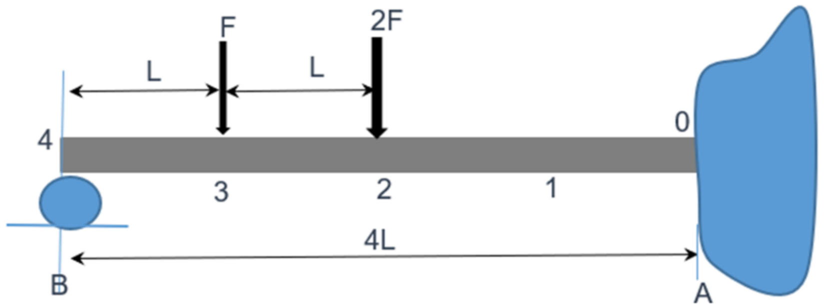


Figure Q4

The following data is given:

E = 240 GNm^{-2}, density=7850kg/m^3, L = 2m and F = 110 kN

In answering the questions below, you should split the beam into four equal sections and using the Finite Difference method of solution, where:

$$\left.\frac{dy}{dx}\right|_{x=x_i} = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_i} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2}$$

a) State the boundary conditions of the beam. **(2 marks)**

b) Establish the bending moment equations for each node on the beam. **(6 marks)**

c) Establish the Finite Difference equation for each node. **(5 marks)**

d) Calculate the reaction load at the simple support. **(6 marks)**

e) Calculate the deflection at the mid-point of the beam and the maximum bending stress. **(6 marks)**

Total 25 Marks

PLEASE TURN THE PAGE…..

School of Engineering
BEng (Hons) Mechanical Engineering
Semester 2-Examination 2019
Finite Element & Difference Methods
Module No. AME6016

Q5

Figure Q5 below shows the arrangement for a beam of length L simply supported at both ends. The beam supports three points loads F_1, F_2 and F_3 applied at point 1, 2 and 3, respectively one, two and three quarters along the length of the beam.

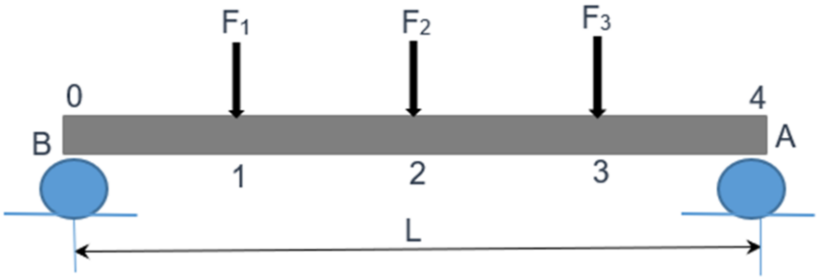


Figure Q5

The beam has 300x300mm square cross-section.

The following data is given:

E = 250 GNm^{-2}, density=8 tonnes/m^3, L = 8m, F_1=100kN, F_2=80kN and F_3=70kN.

Split the beam into four equal lengths and using the Finite Difference Method and solution where:

$$\left.\frac{dT}{dx}\right|_{x=x_i} = \frac{T_{i+1} - T_{i-1}}{2\Delta x}$$

$$\left.\frac{d^2T}{dx^2}\right|_{x=x_i} = \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta x^2}$$

a) State the boundary conditions of the beam. **(2 marks)**

b) Determine the reaction loads at the supports. **(4 marks)**

c) Establish the bending moment equations for each node along the beam. **(6 marks)**

d) Establish the Finite Difference equations for each node. **(4 marks)**

e) Calculate the maximum deflection of the beam and the maximum bending stress. **(5 marks)**

f) How would you calculate the deflection at each point using matrix method? **(4 marks)**

Total 25 Marks

PLEASE TURN THE PAGE…..

Q6

A circular solid metal beam of a spacecraft turbine, 4m long, has one cross-section end B connected to a high temperature fluid at 800ºC and the other cross-section end A is in perfect contact with low air temperature of 50°C as shown in the figure Q6 below. The rest of the beam is externally exposed to a combustion gas at 300ºC. The thermal conductivity of the bar is 50 W/m.ºC and the coefficient of convection due to the gas flow is estimated at 90 W/m^2.ºC.

Given: The radius of the beam r is 350mm.

The boundary condition is stated as follows: $T(x = 0) = 800\ ℃,\ T(x = 4m) = 50°C$

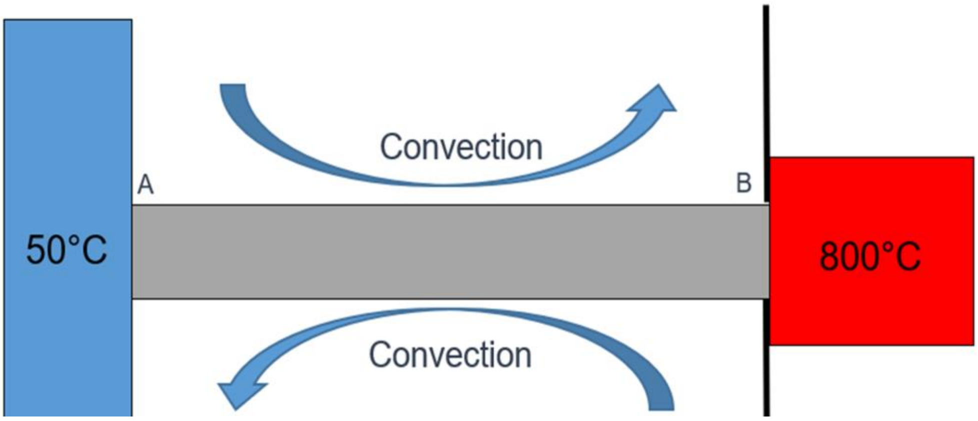


Figure Q6

a) Explain the types of heat transfer and give an example for each type to illustrate. **(6 Marks)**

b) State the thermal balance equation of the system for the unsteady state and the steady state condition and explain each term. **(6 marks)**

c) Using the steady state balance equation, divide the length into 5 sub-intervals and use the Finite Difference formula to write the numerical equation for each node. **(8 marks)**

d) Derive the matrix equation that represents the above simultaneous equations and write the matrix equation for the solution without solving it. **(5 marks)**

Total 25 Marks

END OF QUESTIONS

PLEASE TURN THE PAGE FOR FORMULA SHEETS…

School of Engineering
BEng (Hons) Mechanical Engineering
Semester 2-Examination 2019
Finite Element & Difference Methods
Module No. AME6016

FORMULAE SHEET

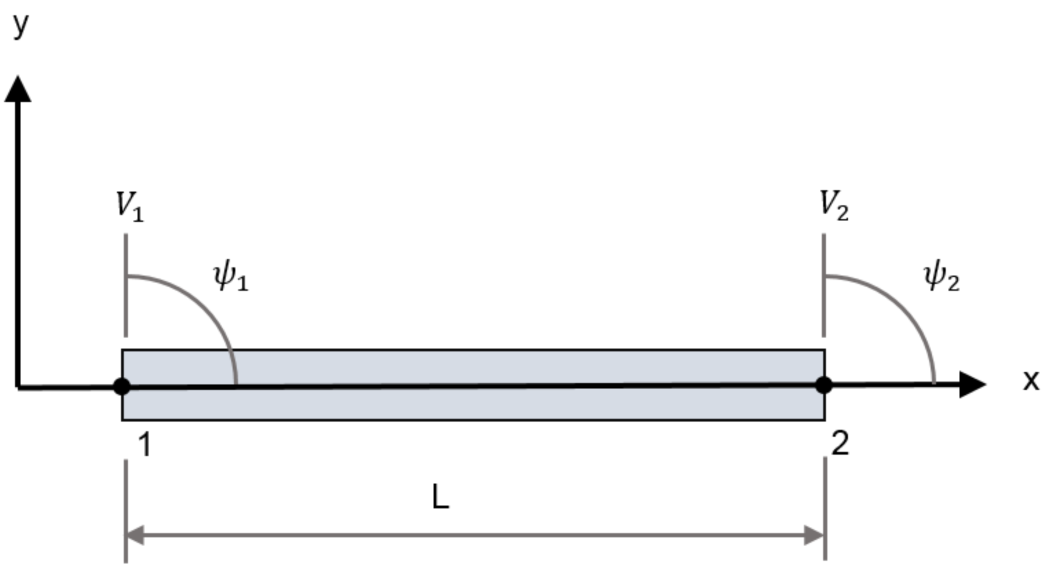
Bending stress

$$\sigma = \frac{My}{I}$$

Dynamics

$$(-\boldsymbol{\omega}^2\,[\boldsymbol{m}] + [\boldsymbol{K}])\,\{\boldsymbol{u}\} = \boldsymbol{0}$$

Finite Element Notation for 2D Beams with 2 Nodes and 4 DOF:



Element Consistent Mass Matrix

$$[m]^e = \frac{\rho AL}{420}\begin{bmatrix} \mathbf{156} & \mathbf{22}\boldsymbol{L} & \mathbf{54} & -\mathbf{13}\boldsymbol{L} \\ & \mathbf{4}\boldsymbol{L}^2 & \mathbf{13}\boldsymbol{L} & -\mathbf{3}\boldsymbol{L}^2 \\ & & \mathbf{156} & -\mathbf{22}\boldsymbol{L} \\ & & & \mathbf{4}\boldsymbol{L}^2 \end{bmatrix}$$

Element Stiffness Matrix

$$[K]^e = \frac{EI}{L^3}\begin{bmatrix} \mathbf{12} & \mathbf{6}\boldsymbol{L} & -\mathbf{12} & \mathbf{6}\boldsymbol{L} \\ & \mathbf{4}\boldsymbol{L}^2 & -\mathbf{6}\boldsymbol{L} & \mathbf{2}\boldsymbol{L}^2 \\ & & \mathbf{12} & -\mathbf{6}\boldsymbol{L} \\ & & & \mathbf{4}\boldsymbol{L}^2 \end{bmatrix}$$

PLEASE TURN THE PAGE…..

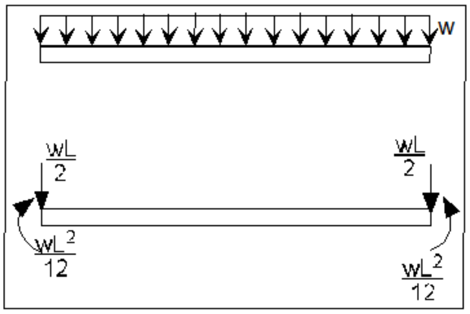
School of Engineering
BEng (Hons) Mechanical Engineering
Semester 2-Examination 2019
Finite Element & Difference Methods
Module No. AME6016

Element Displacement Functions

$$v(x) = \left[1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}, x - \frac{2x^2}{L} + \frac{x^3}{L^2}, \frac{3x^2}{L^2} - \frac{2x^3}{L^3}, -\frac{x^2}{L} + \frac{x^3}{L^2}\right]$$

Beam bending stress at x along the beam element is given by, $\sigma(x, y) = -Ey\frac{\partial v^2(x)}{\partial x^2}$

Equivalent nodal force due to Uniformly distributed load w



1-D Beam Deflection Equation

$$EI\frac{d^2y}{dx^2} = M(x)$$

2 D Constant strain triangular element (CST)

D matrix =

$$\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

PLEASE TURN THE PAGE…..

For small strains and small rotations, we have

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

In matrix form, we write

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \qquad \text{or} \qquad \varepsilon = \mathbf{Du}$$

We write

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$N_1 = \frac{1}{2A}\{(x_2 y_3 - x_3 y_2) + (y_2 - y_3)x + (x_3 - x_2)y\}$$

$$N_2 = \frac{1}{2A}\{(x_3 y_1 - x_1 y_3) + (y_3 - y_1)x + (x_1 - x_3)y\}$$

$$N_3 = \frac{1}{2A}\{(x_1 y_2 - x_2 y_1) + (y_1 - y_2)x + (x_2 - x_1)y\}$$

$$A = \frac{1}{2}\det\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$ (The area of the triangle)

Using the strain-displacement relation , we have

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \mathbf{Bd} = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

where $x_{ij} = x_i - x_j$ and $y_{ij} = y_i - y_j$ (i, j = 1, 2, 3).

The element stiffness matrix for the CST $\mathbf{k} = \int_V \mathbf{B}^T\mathbf{E}\mathbf{B}\,dV = tA(\mathbf{B}^T\mathbf{E}\mathbf{B})$

END OF FORMULA SHEETS

END OF PAPER