OCD041

UNIVERSITY OF BOLTON

WESTERN INTERNATIONAL COLLEGE FZE

BENG(HONS) MECHANICAL ENGINEERING

SEMESTER TWO EXAMINATION 2018/2019

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

MODULE NO. AME6016

Date: Wednesday 22nd May 2019 **Time: 10:00 – 12:00**

INSTRUCTIONS TO CANDIDATES:

There are FIVE questions on the paper.

Answer any FOUR questions

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is erased or cleared prior to the examination.

CANDIDATES REQUIRE: **A Formula Sheet (attached)**

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Q1. The rectangular beam shown below in **Figure Q1** is rigidly fixed at end A and simply supported at end B. Point load **P** is applied at 3L from support A along the length of the beam. The beam span AB is 4L in total.

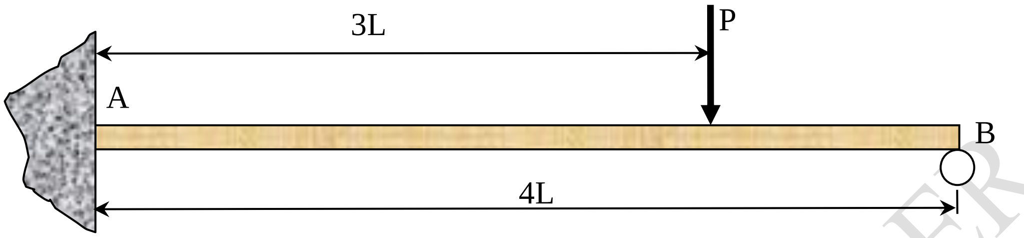

Figure Q1, Beam

Note – B is a simple roller support, whilst end A is built in.
The following data is given:
Young's modulus, E = 200 GNm^{-2}, Moment of inertia, I = 120 x 10^{-9} m^4, Beam length, L = 1.6 m and Load, P = 10 kN

In answering the questions below, you should split the beam into **four equal sections** and use the Finite Difference method of solution, where:

$$\left(\frac{dy}{dx}\right)_i \approx \frac{1}{2h}\left(y_{i+1} - y_{i-1}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_i \approx \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1}\right)$$

$$\left(\frac{d^3y}{dx^3}\right)_i \approx \frac{1}{2h^3}\left(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}\right)$$

$$\left(\frac{d^4y}{dx^4}\right)_i \approx \frac{1}{h^4}\left(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}\right)$$

a) State the Boundary Conditions for the beam. **(2 Marks)**

b) Establish the Bending Moment equations for each node on the beam. **(6 Marks)**

c) Establish the Finite Difference equations for each node. **(7 Marks)**

d) Determine the value of the reaction (R_B) at the simple support B **(6 Marks)**

e) Determine the deflection at the mid-point of the beam. **(4 Marks)**

Total 25 marks

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Q2.

a) Briefly describe the general steps of the finite element method and list five typical areas of engineering where the finite element method is applied.

(8 Marks)

b) For the spring assemblages shown in **Figure Q2** below, the spring are arranged in series and parallel with node 1, 2 and 4 as fixed and node 3 restricted in moving in x direction only. Use the direct stiffness method for problems.

Determine the following,

1) Using the connectivity table establish the global stiffness matrix for the spring assemblages. **(10 Marks)**

2) Nodal displacements at the junction 3 **(7 Marks)**

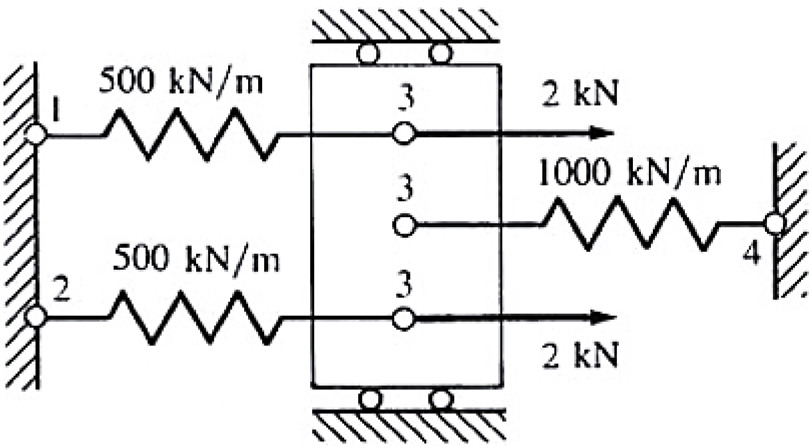

Figure Q2: Spring assemblages

Total 25 marks

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Q3.

The aluminum and steel pipes shown in **Figure Q3** below are fastened to rigid supports at ends A and B and to a rigid plate C at their junction with a load of 50 KN on the rigid plate C. Use the minimum number of two linear finite elements with three nodes. For steel, Modulus of elasticity E= 200 GPa with Area= 60 mm^2, and for Aluminium, Modulus of Elasticity E=70 GPa with Area=600 mm^2.

1) Develop the individual stiffness matrix for Steel and Aluminium **(5 Marks)**
2) Developing the global stiffness matrix using the method of superposition **(5 Marks)**
3) Determine the displacement of point C **(5 Marks)**
4) Stresses in the aluminum and steel pipes **(5 Marks)**
5) Critically analyse the results obtained in (3) and (4) **(5 marks)**

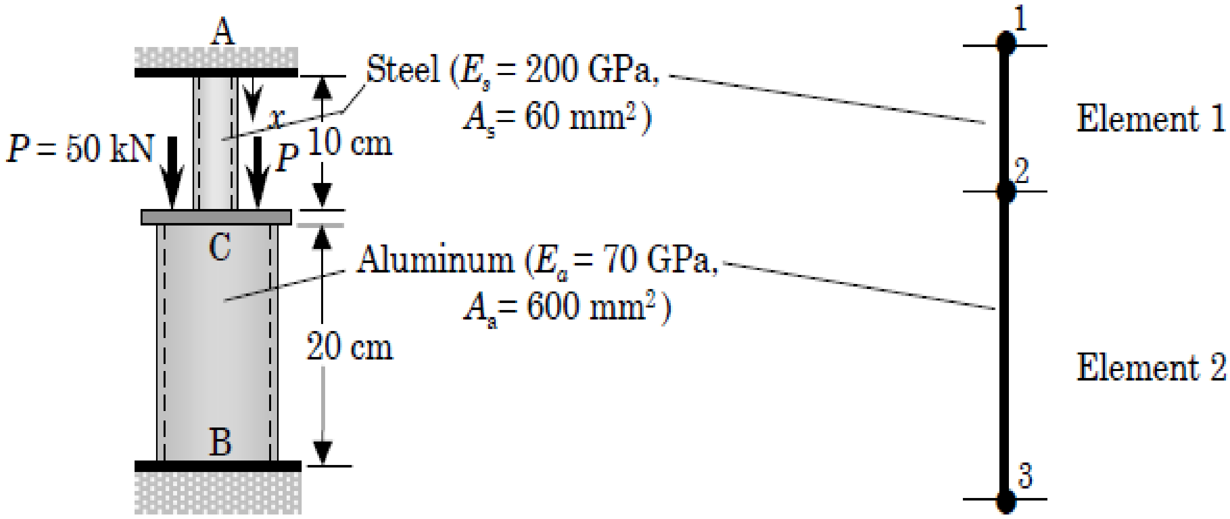

Figure Q3: Assembly of steel and Aluminum pipes

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Q4

A rigid frame shown in **Figure Q4** below consists of a horizontal and a vertical member attach at node 2. The horizontal member is 5m long and vertical member is 4m in height. Both the members are welded at point 2 with an anticlockwise moment of 600 KN-m acting at that point. Consider modulus of elasticity E= 210 GPa with area of both the members as 2 X 10^{-2} m^2, moment of Inertia for the members I= 2 X 10^{-4} m^4. For the rigid frame

1) Develop the global stiffness matrix of the elements using the method of superposition **(5 marks)**
2) Determine the displacements at all the nodes **(10 Marks)**
3) Determine rotations of the nodes. **(10 marks)**

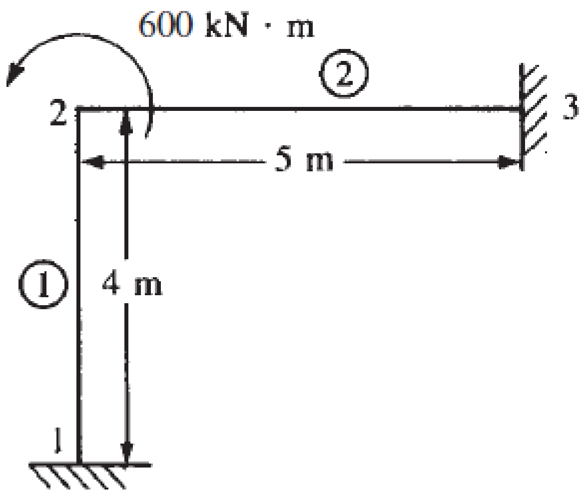

Figure Q4: Rigid Frame

Total 25 marks

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Q5.

A double-pane glass window shown in **Figure Q5** below, consists of two 4-mm thick layers of glass with coefficient of conductivity, K=0.80 W/m-^{o}C separated by a 10 mm thick stagnant air space with K=0.025 W/m-^{o}C. Assume the inside room temperature T_i`=20^{o}C with heat transfer coefficients, h_i=10 W/m^{2}–^{o}C and the outside temperature T_o= -10^{o}C with heat transfer coefficients h_o= 30 W/m^{2}-^{o}C. Assume one-dimensional heat flow through the glass.

Determine the following:

1) Individual and global stiffness matrix with superposition method **(10 Marks)**
2) Force matrix taking into account the entire element. **(5 Marks)**
3) Critically analyse the temperatures at both surfaces of the inside layer of glass and the temperature at the outside surfaces of glass. **(10 Marks)**

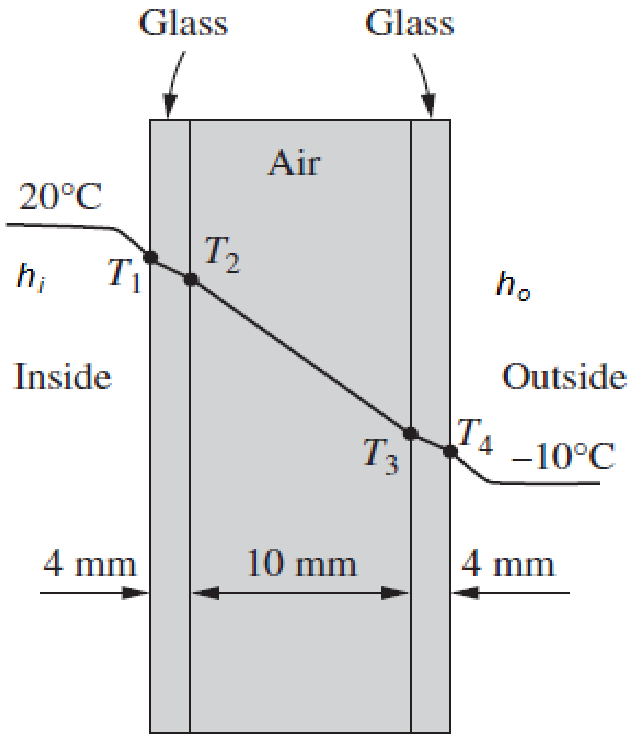

Figure Q5: Double pane glass window

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END OF QUESTIONS

Formula sheet over page

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FORMULA SHEET

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

Finite Difference Equations for Beam Deflection:

$$\left(\frac{dy}{dx}\right)_i \approx \frac{1}{2h}\left(y_{i+1} - y_{i-1}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_i \approx \frac{1}{h^2}\left(y_{i+1} - 2y_i + y_{i-1}\right)$$

$$\left(\frac{d^3y}{dx^3}\right)_i \approx \frac{1}{2h^3}\left(y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}\right)$$

$$\left(\frac{d^4y}{dx^4}\right)_i \approx \frac{1}{h^4}\left(y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}\right)$$

One dimensional heat conduction with free end convection

$$[K_h]_{end} = hA\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Force vector due to free end convection

$$[F_h]_{end} = AhT_\infty\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Stiffness matrix for the heat conduction

$$[K_c] = \frac{AK_g}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Force equilibrium condition

$\{F\} = [K]\,[T]$

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Formula sheet continued.

Elemental Force Matrix.

$$\{f\} = \frac{QAL + q^{*}PL + hT_{\infty}PL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Local stiffness matrix for spring element

$$\underline{\hat{k}} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

Beam Element Stiffness matrix.

$$\underline{\hat{k}} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Elemental Stiffness matrix for bar element

$$\mathbf{k} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Elemental stress.

$$\underline{\sigma} = \underline{C}'\underline{d} \qquad \underline{C}' = \frac{E}{L}[-C \quad -S \quad C \quad S]$$

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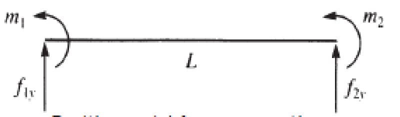
Stiffness matrix for Rigid frames

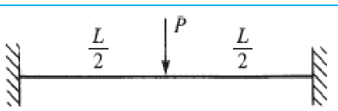
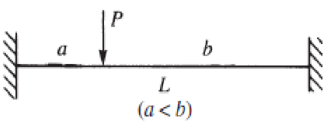
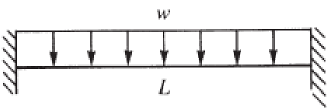
$$[k] = \frac{E}{L} \times \begin{bmatrix} AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2 + \frac{12I}{L^2}S^2\right) & -\left(A - \frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2 + \frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A - \frac{12I}{L^2}\right)CS & -\left(AS^2 + \frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2 + \frac{12I}{L^2}S^2 & \left(A - \frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2 + \frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ \text{Symmetry} & & & & & 4I \end{bmatrix}$$

Single element equivalent joint force for different types of loads



Positive nodal force conventions

Single element equivalent joint forces f_0 for different types of loads

	f_{1y}	m_1	Loading case	f_{2y}	m_2
1.	$\frac{-P}{2}$	$\frac{-PL}{8}$		$\frac{-P}{2}$	$\frac{PL}{8}$
2.	$\frac{-Pb^2(L+2a)}{L^3}$	$\frac{-Pab^2}{L^2}$		$\frac{-Pa^2(L+2b)}{L^3}$	$\frac{Pa^2b}{L^2}$
3.	$-P$	$-\alpha(1-\alpha)PL$		$-P$	$\alpha(1-\alpha)PL$
4.	$\frac{-wL}{2}$	$\frac{-wL^2}{12}$		$\frac{-wL}{2}$	$\frac{wL^2}{12}$

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