UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

MALAYSIA - KTG

B.ENG. (HONS) MECHANICAL ENGINEERING

SEMESTER 2 EXAMINATION 2018/2019

FINITE ELEMENT AND DIFFERENCE SOLUTIONS

MODULE NO: AME 6006

Date: Monday 13th May 2019 **Time: 3 Hours**

INSTRUCTIONS TO CANDIDATES:

There are FOUR questions.

Answer ALL questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Finite Element and Difference Solutions
Module No. AME6006

Q1. Figure Q1 shows a frictionless pin-jointed structure consisting of three members. Pin-joints 1, 3, and 4 are fixed to rigid surfaces as shown in Figure Q1. A point force of 50 kN is applied to pin-joint 2 at an angle of 60° as shown. All members have a cross-sectional area of either A_1 (150 mm^2) or A_2 (200 mm^2). Young's modulus for all members is 200 GN/m^2.

(a) Write down the element stiffness matrix for each member, and then assemble the overall stiffness matrix for the whole structure, clearly showing all its coefficients. (12 marks)

(b) Calculate the displacement of point 2. (6 marks)

(c) Describe briefly (without performing any calculations) how the strain in each element can be calculated. (7 marks)

For a pin-jointed element, the element stiffness matrix $[k_e]$ can be written as follows:

$$[k_e] = \left(\frac{AE}{L_e}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos\theta\sin\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where A_e is the cross-sectional area of the element, E is Young's modulus, and L_e is the length of the element. The angle θ is defined as the angle of inclination of the element measured anti-clockwise from the horizontal axis.

Question 1 continued over the page.

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Question 1 cont'd…

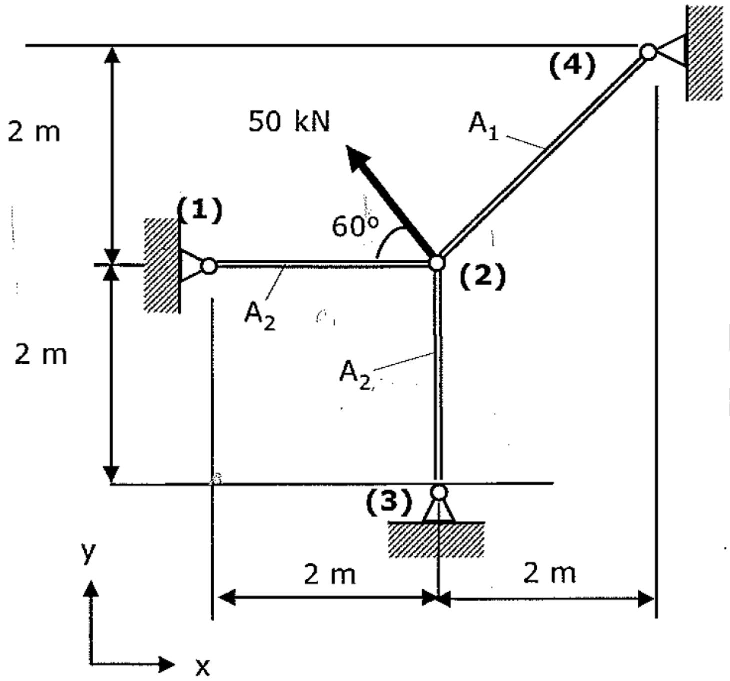

Figure Q1

Total 25 marks

Q2. Figure Q2 shows a rectangular plate ABCD of uniform thickness t which is rigidly clamped at the edges $x = 0$ (i.e. AD) and $y = 0$ (i.e. AB) and free along the edges $x = a$ (line BC). The plate is loaded by a uniform transverse pressure p (in the z-direction). The transverse displacement w, can be approximated by the following expression:

$$w = Cx^2y^2$$

where C is an arbitrary constant.

(a) Using the principle of minimum total potential energy, determine the value of C. (12 marks)

(b) Is the above displacement function for w suitable for this problem? Give reasons.

(6 marks)

(c) Write down another possible approximation for w, and explain why it would be suitable for this problem.

The strain energy per unit area of the plate is: (7 marks)

$$\frac{1}{2}D\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]$$

where the flexural rigidity D is defined as follows:

$$D = \frac{Et^3}{12(1-\nu^2)}$$

E is Young's modulus and ν is the Poisson ratio.

Question 2 continued over the page.

Question 2 cont'd…

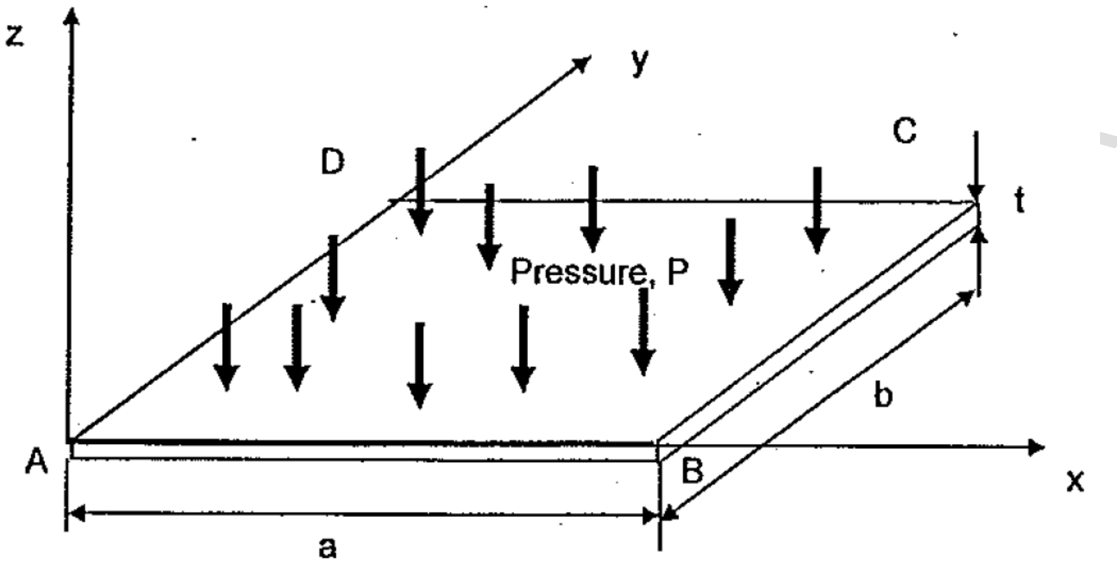

Figure Q2

Total 25 marks

Q3. A circular plate uniform thickness t and radius R is rigidly clamped around its outer edge and loaded by a uniform transverse pressure P as shown in Figure Q3. The transverse displacement w can be approximated by the following function:

$$w = c\left[1 - 3\left(\frac{r}{R}\right)^2 + 2\left(\frac{r}{R}\right)^3\right]$$

where c is a constant and r is the radial distance from the centre of the plate.

(a) Using the principle of minimum total potential energy, evaluate the value of c. (12 marks)

(b) Is the above displacement function for w suitable for this problem? Give reasons. (6 marks)

(c) Write down another possible approximation for w, and explain why it would be suitable for this problem. (7 marks)

The strain energy per unit area of the plate is:

$$\frac{1}{2}D\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]$$

where the flexural rigidity D is defined as follows:

$$D = \frac{Et^3}{12(1-\nu^2)}$$

E is Young's modulus and ν is the Poisson ratio.

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Question 3 cont'd…

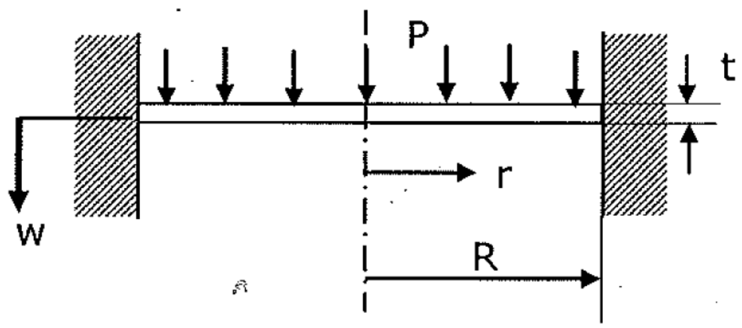

Figure Q3

Total 25 marks

Q4. Figure Q4 shows a typical thin-shell axisymmetric conical element. This type of element can be used for the analysis of axisymmetric internally pressurised thin conical shells. The deformation of the element is axisymmetric and dependent on the local coordinate s along the element. The nodal variables for this element are the meridional displacement u, the normal displacement v, and the slope θ $(= dv/ds)$. The meridional and normal displacements of the element can be approximated by the following polynomial expression:

$$u = C_1 + C_2 s$$
$$v = C_3 + C_4 s + C_5 s^2 + C_6 s^3$$

where s is the local coordinate, and C_1 to C_6 are constants.

(a) Express the nodal variables.

(9 marks)

(b) Using the principle of minimum total potential energy, show that the element stiffness matrix can be written in matrix form as follows:

(11 marks)

$$K_e = 2\pi([A]^{-1})^T \left(\int_{L_e} [X]^T [D][X] r\, ds \right) [A]^{-1}$$

where $[A]$ is a matrix containing the coordinates of the nodal points of the element and $[D]$ is a matrix containing the material parameters, defined as follows:

$$[\sigma] = [D][\varepsilon]$$

where $[\sigma]$ and $[\varepsilon]$ are the stress and strain vectors, respectively. Write down explicit expressions only for $[A]$ and $[X]$.

(c) Discuss whether a quadratic polynomial expression for v in terms of s (instead of a cubic variation) can be used for this element. Give reasons.

(5 marks)

The strain energy per unit area of shell mid-surface is $\frac{1}{2}[\sigma]^T[\varepsilon]$ and the strain vector is defined as follows:

Question 4 continued over the page.
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Question 4 cont'd…

$$[\varepsilon] = \begin{bmatrix} \dfrac{du}{ds} \\ \dfrac{1}{r}(u \sin a + v \cos a) \\ \dfrac{d^2v}{ds^2} \\ \dfrac{1}{r}\left(\sin a \dfrac{dv}{ds}\right) \end{bmatrix}$$

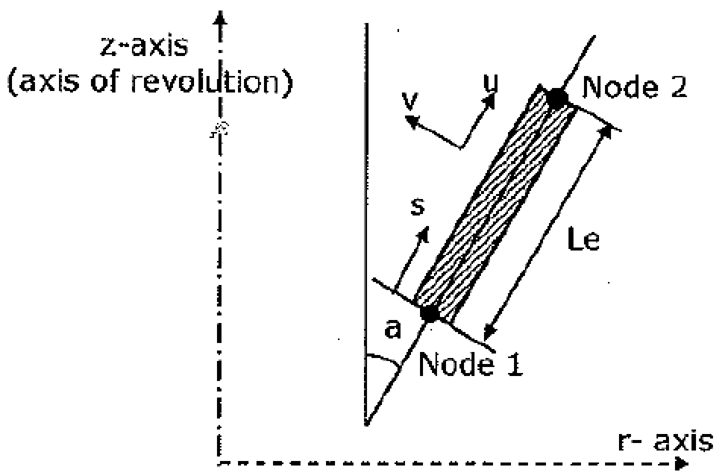

Figure Q4

Total 25 marks

END OF QUESTIONS