

UNIVERSITY OF BOLTON

OFF-CAMPUS DIVISION

MALAYSIA - KTG

B.ENG. (HONS) MECHANICAL ENGINEERING

SEMESTER 2 EXAMINATION 2018/2019

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME 5002

Date: Monday 13th May 2019

Time: 3 Hours

INSTRUCTIONS TO CANDIDATES:

There are FOUR questions.

Answer ALL questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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- Q1. A solid steel rod with the diameter of 25 mm is placed concentrically in a copper tube of the outer diameter 45 mm and inner diameter of 35 mm, as shown in Figure Q1. The rod and the tube are of the same length and welded to rigid end plates.

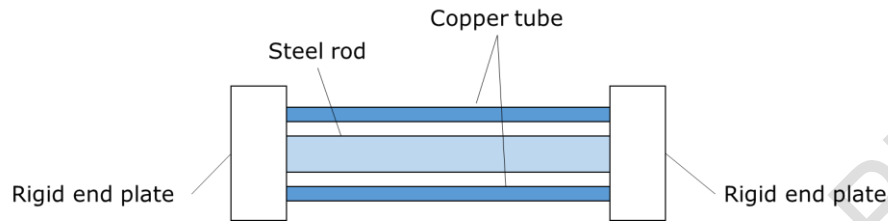


Figure Q1

- (a) Solve the stresses in the rod and tube if the temperature of the assembly is raised by 80°C . Account whether the stresses are tensile or compressive. Ignore the thermal expansion. (10 marks)
- (b) If an axial compressive force of 35 kN is applied to the rigid end plates, while the temperature is maintained at 80°C :
- (i) Evaluate the resultant stresses in the steel rod and the copper tube. (7 marks)
- (ii) Justify whether the stresses are tensile or compressive. (8 marks)

Use $E = 207 \text{ GN/m}^2$ and $\alpha = 11 \times 10^{-6}/^{\circ}\text{C}$ for steel; $E = 103 \text{ GN/m}^2$ and $\alpha = 17.5 \times 10^{-6}/^{\circ}\text{C}$ for copper

Total 25 marks

Please turn the page

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- Q2. A long closed ended cylindrical pressure vessel has an outer diameter of 800 mm and an inner diameter of 500 mm as shown in Figure Q2. If the vessel is subjected to an internal pressure of 150 MPa and an external pressure of 70MPa, determine the following:

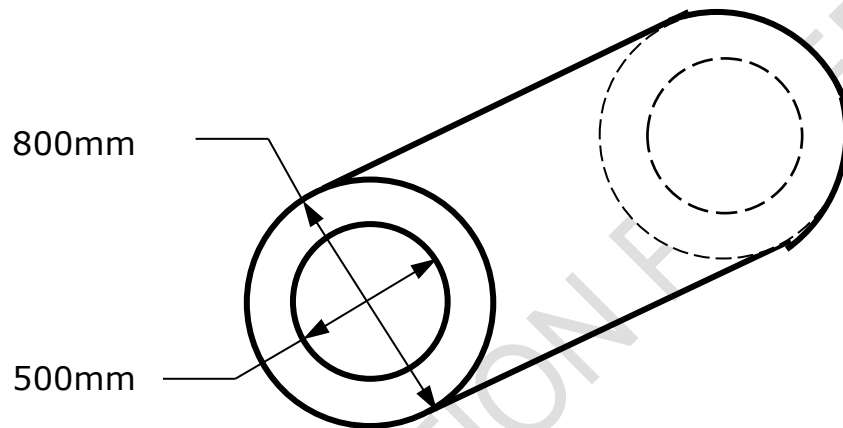


Figure Q2

- (a) The radial stress (σ_R) at the inner and outer surfaces. (7 marks)
- (b) The circumferential stress (σ_C) at the inner and outer surfaces. (9 marks)
- (c) The circumferential strain (ϵ_C) at the inner surface if the longitudinal stress (σ_L) is 90 MPa compressive. (9 marks)

[Take $E=215\text{GPa}$ and $\nu = 0.4$].

Total 25 marks

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Q3. The cross section of a cantilever section shown in Figure Q3 is 1.6 m long and is loaded at its free end with 8 kN. Evaluate:

- (a) The position of the centroid. (4 marks)
- (b) I_x , I_y , and I_{xy} about the x - y axes through. (6 marks)
- (c) The principal second moments of area. (7 marks)
- (d) The directions of the principal axes. (8 marks)

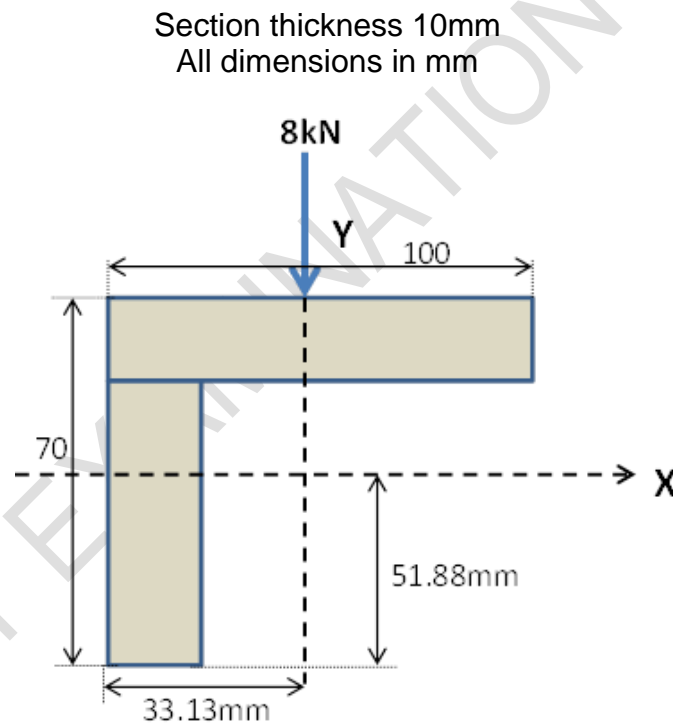


Figure Q3

Total 25 marks

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- Q4. A machine of mass 1800 kg is supported by four identical elastic springs and set oscillating. It is observed that the amplitude reduces to 15% of its initial value after 7 oscillations. It takes 3 seconds to do them. Calculate the following:
- (a) The natural frequency of undamped vibrations (in Hertz). (5 marks)
 - (b) The effective stiffness of all four springs together. (4 marks)
 - (c) The critical damping coefficient that will prevent oscillatory motion. (4 marks)
 - (d) The damping ratio. (4 marks)
 - (e) The damping coefficient. (4 marks)
 - (f) The frequency of damped vibrations. (4 marks)

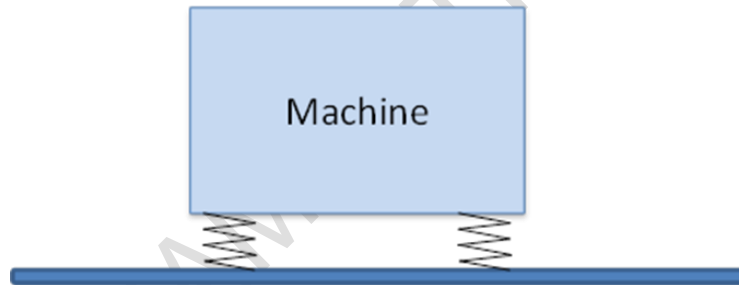


Figure Q4

Total 25 marks




END OF QUESTIONS

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Formula Sheet

1. Deflection

$$M_{xx} = EI \frac{d^2y}{dx^2}$$

Section shape	A (m ²)	I_{xx} (m ⁴)
	πr^2	$\frac{\pi}{4} r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4} a^3 b$

2. Plane stress

Stresses in function of the angle θ :

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos(2\theta) + \tau_{xy} \sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos(2\theta) - \tau_{xy} \sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2} \sin(2\theta) + \frac{\sigma_x - \sigma_y}{2} \cos(2\theta)$$

Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

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$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

3. Lamé's equation

$$\sigma_c = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = \frac{1}{E} [\sigma_c - \nu(\sigma_r + \sigma_l)]$$

$$\varepsilon_r = \frac{1}{E} [\sigma_r - \nu(\sigma_c + \sigma_l)]$$

$$\varepsilon_l = \frac{1}{E} [\sigma_l - \nu(\sigma_c + \sigma_r)]$$

4. Vibrations

Free vibrations:

$$f = \frac{1}{T}$$

$$\omega_n = 2\pi f = \sqrt{\frac{k}{M}}$$

Damped vibration:

$$f_d = \frac{\omega_d}{2\pi}$$

$$c_c = \sqrt{4Mk}$$

$$\delta = \frac{c}{c_c} = \frac{c}{2k} \omega_n$$

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$$\omega_d = \omega_n \sqrt{1 - \delta^2}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\delta}{\sqrt{1 - \delta^2}}$$

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t$$

$$x = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2} \sin\left[\omega_n t + \tan^{-1}\left(\frac{x_0 \omega_n}{\dot{x}_0}\right)\right]$$

$$X = \frac{F_0/k}{\{[1 - (\omega/\omega_n)^2]^2 + [2\zeta \omega/\omega_n]^2\}}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta \omega/\omega_n}{1 - (\omega/\omega_n)^2}\right]$$

$$x_p = X \sin(\omega t - \phi)$$

$$F_{tr} = kx_p + c\dot{x}_p$$

$$F_{tr,max} = \sqrt{(kX)^2 + (c\omega X)^2}$$

5. Differential equation

Homogeneous form:

$$a\ddot{y} + b\dot{y} + cy = 0$$

Characteristic equation:

$$a\lambda^2 + b\lambda + c = 0$$

If $b^2 - 4ac > 0$, λ_1 and λ_2 are distinct real numbers then the general solution of the differential equation is:

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

A and B are constant.

If $b^2 - 4ac = 0$, $\lambda_1 = \lambda_2 = \lambda$ are distinct real numbers then the general solution of the differential equation is:

$$y(t) = e^{\lambda t}(A + Bx)$$

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A and B are constant.

If $b^2 - 4ac < 0$, λ_1 and λ_2 are complex numbers then the general solution of the differential equation is:

$$y(t) = e^{\alpha t} [A \cos(\beta t) + B \sin(\beta t)]$$

$$\alpha = -\frac{b}{2a}$$

$$\beta = \frac{\sqrt{4ac - b^2}}{2a}$$

A and B are constant.

6. Asymmetrical bending

$$I_{u,v} = \frac{1}{2}(I_{xx} + I_{yy}) \pm \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$$

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

$$I_{xy} = Ahk$$

$$I_u + I_v = I_{xx} + I_{yy}$$

$$\sigma = \frac{M_v U}{I_v} + \frac{M_u V}{I_u}$$

$$\sigma_{bending} = \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

7. Stress

$$\sigma = \frac{F}{A}$$

8. Hooke's law

$$E = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = \frac{\Delta L}{L}$$

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9. Beam bending equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

10. Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = (\text{Stress tensor}) \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

where a , b , and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \quad l = \cos \alpha$$

$$m = bk \quad m = \cos \theta$$

$$n = ck \quad n = \cos \varphi$$

11. Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

12. Yield criterion

Von Mises:

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$$\sigma_{vm} = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Tresca:

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

$$\sigma_{tr} = 2\tau_{max}$$

$$\tau_{max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}; \frac{|\sigma_1 - \sigma_3|}{2}; \frac{|\sigma_3 - \sigma_2|}{2}\right)$$

$$\frac{\sigma_{vm}}{\sigma_{tr}} = \frac{\sqrt{3}}{2}$$

13. Quadratic equation: $ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

14. Allowable stress

$$\sigma_{allowable} = \frac{\sigma_{yield}}{\text{Factor of safety}}$$

15. Strut

$$k = \sqrt{\frac{I}{A}}$$

Euler validity:





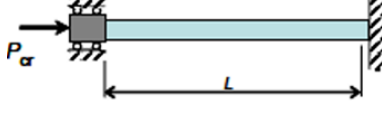
$$\sigma_E = \frac{n\pi^2 E}{(L/k)^2}$$

Rankine-Gordon:

$$\sigma_R = \frac{\sigma}{1 + c/n (L/k)^2}$$

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$$\text{Slenderness ratio} = SR = \frac{L_e}{k} \geq \pi \sqrt{\frac{E}{\sigma_{yield}}}$$

Description	Schematic	Critical buckling load P_c	Effective length L_{eff}
Free-fixed		$P_c = \frac{\pi^2 EI}{4l^2}$	$2l$
Hinged-hinged		$P_c = \frac{\pi^2 EI}{l^2}$	l
Hinged-hinged, initially curved		$P_c = \frac{\pi^2 EI}{l^2}$	l
Fixed-hinged		$P_c = \frac{2.045\pi^2 EI}{l^2}$	$0.7l$
Fixed-fixed		$P_c = \frac{4\pi^2 EI}{l^2}$	$\frac{l}{2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c A}{1 + a \left(\frac{l_e}{k}\right)^2}$$

We find

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$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a \left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a(l_e/k)^2$ has thus been introduced to *take into account the buckling effect*.

$$a = \frac{\sigma_c}{\pi^2 E}$$

16. Composite materials

$$\sigma = \frac{My}{I}$$

$$E = \eta V_f E_f + (1 - V_f) E_m$$

$$\sigma = E \varepsilon$$